SUBJECT : MATHEMATICS

SYLLABUS

Unit I **ALGEBRA**

Groups – Examples – Cyclic Groups – Permutation Groups – Lagrange's theorem – Normal subgroups – Homomorphism – Cayley's theorem – Cauchy's theorem – Sylow's theorems – Finite Abelian Groups.

Rings – Integral Domain – Field – Ring Homomorphism – Ideals and Quotient Rings – Field of Quotients of Integral domains – Euclidean Rings – Polynomial Rings – Unique factorization domain.

Fields – Extension fields – Elements of Galois theory – Finite fields.

Vector Spaces – Linear independence of Bases – Dual spaces – Inner product spaces – Linear transformations – Rank – Characteristic roots – Matrices – Canonical forms – Diagonal forms – Triangular forms – Nilpotent transformations – Jordan form – Quadratic forms and Classification – Hermitian, Unitary and Normal transformations.

Unit II **REAL ANALYSIS**

Elementary set theory – Finite, countable and uncountable sets – Real number system as a complete ordered field – Archimedean Property – Supremum, infimum, Sequences and Series – Convergence – limit supremum – limit infimum – The Bolzano – Weierstrass theorem – The Heine – Borel Covering theorem – Continuity, Uniform Continuity, Differentiability – The Mean Value theorem for derivatives – Sequences and Series of functions – Uniform convergence.

Riemann – Stieltjes integral: Definition and existence of the integral – properties of the integral – Integral and Differentaition – Integration of vector valued functions – Sequences and Series of functions: Uniform convergence – Continuity, Integration and Differentiation.

Power series – Fourier series.

Functions of several variables – Directional derivative – Partial derivative – derivative as a linear transformation – The Inverse function theorem and The Implicit function theorem.

Unit III **TOPOLOGY**

Topological spaces – Basis – The order Topology – The product Topology – The subspace Topology – Closed sets and limit points.

Continuous functions – The box and product Topologies – The matrix Topology.

Connected spaces – Connected subspaces of the real line – Components and local connectedness – compact spaces – Compact subspaces of the real line – Limit point compactness – Local compactness.

Countability and separation Axioms – Normal spaces – The Urysohn Lemma – The Urysohn metrization theorem – The Tietze extension theorem.

Unit IV **COMPLEX ANALYSIS**

Introduction to the concept of analytic function: Limits and continuity – Analytic functions – Polynomials and rational functions – Elementary theory of power series – Maclaurin's series – Uniform convergence – Power series and Abel's limit theorem – Analytic functions as mapping – Conformality arcs and Closed curves – Analytical functions in regions – Conformal mapping – Linear transformations – the linear group, the cross ratio and symmetry.

Complex integration – Fundamental theorems – line integrals – rectifiable arcs – line integrals as functions of arcs – Cauchy's theorem for a rectangle – Cauchy's theorem in a Circular disc – Cauchy's integral formula: The index of a point with respect to a closed curve – The integral formula – Higher derivatives – Local properties of Analytic functions and removable singularities – Taylor's theorem – Zeros and Poles – The local mapping – The maximum modulus Principle.

Unit V **FUNCTIONAL ANALYSIS**

Banach Spaces – Definition and examples – Holder's inequality and Minkowski's inequality – Continuous linear transformations – The Hahn-Banach theorem – Natural imbedding of X in X** – The Open mapping and The Closed graph theorem – Properties of conjugate of an operator.

Hilbert spaces – Orthonormal bases – Conjugate space H* – Adjoint of an operator – Projections – Matrices – Basic operations of matrices – Determinant of a matrix – Determinant and Spectrum of an operator – Spectral theorem for operators on a finite dimensional Hilbert space – Regular and Singular elements in a Banach Algebra – Topological divisor of zero – Spectrum of an element in a Banach algebra – The formula for the spectral radius – Radical and semi-simplicity.

Unit VI DIFFERENTIAL GEOMETRY

Curves in spaces – Serret – Frenet formulae – Locus of centers of curvature – Spherical curvature – Intrinsic equations – Helices – Spherical Indicatrix Surfaces – Curves on a surface – Surface of revolution – Helicoids – Gaussian curvature – First and Second fundamental forms – Isometry – Meusnier's theorem – Euler's theorem- lines of curvature – Dupin's Indicatrix – Asymptotic lines – Edge of regression – Developable surfaces associated to a curve – Geodesics – Conjugate points on Geodesics.

Unit VII DIFFERENTIAL EQUATIONS

Ordinary Differential Equations

Linear differential equation with constant and variable co-efficients – Linear dependence and independence – Wronskian – Non homogeneous equations of order two and n – Initial value problems for nth order equations – Second order equations with ordinary point and regular singular points – Legendre Equations – Bessel's equation – Hermite's equation and their properties – Existence and Uniqueness of solutions to first order equations – Exact equation – Lipschitz condition – Non local existence of Solution – Approximation to Uniqueness of solutions.

Partial Differential Equations

Lagrange and Charpit methods for solving first order Partial Differential equations – Classification of Second order partial differential equations – General solution of higher order partial differential equation with constant co-efficients – Method of separation of variables for Laplace, Heat and Wave equations (upto two dimensions only).

Unit VIII CLASSICAL MECHANICS AND NUMERICAL ANALYSIS

Classical Mechanics

Generalised Co-ordinates – Lagrange's equations – Hamilton's Canonical equations – Hamilton's principle – Principle of least action – Canonical transformations – Differential forms and Generating functions – Lagrange and Poisson brackets.

Numerical Analysis

Numerical solutions of algebraic and transcendental equations – Method of iteration – Newton Raphson method – Rate of convergence – Solution of Linear algebraic equations using Gauss elimination and Gauss – Seidel methods.

Finite differences – Lagrange, Hermite and Spline Interpolation, Numerical differentiation and integration – Numerical solutions of Ordinary differential equations using Picard, Euler, Modified Euler and Runge- Kutta methods.

Unit IX **OPERATIONS RESEARCH**

Linear programming problem – Simplex Methods – Duality – Dual Simplex Method – Revised Simplex Method – Integer Programming Problem – Dynamic Programming – Non linear programming – Network Analysis – Directed Network – Max Flow Min Cut theorem – Queuing theory – Steady State solutions of M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1 models – Inventory models – Deterministic models with and without shortages – Single Price break models.

Unit X **PROBABILITY THEORY**

Sample space – Discrete Probability – Independent events – Baye's theorem – Random variables and Distribution functions (Univariate and Multivariate) – Expectation and Moments – Moment Generating function – Characteristic functions and Cumulants – Independent Random variables – Marginal and conditional distributions – Probability inequalities (Tchebyshev, Markov, Jensen) – Modes of convergence, Weak and Strong laws of large numbers – Central limit theorem (i.i.d case).

Probability distributions – Binomial, Poisson, Uniform, Normal, Exponential, Gamma, Beta, Cauchy distributions – Standard Errors – Sampling distributions of t, F and Chi square and their uses in tests of significance – ANOVA – Large sample tests for mean and proportions.