

BRINDHAVAN HR SEC SCHOOL, SUKKIRANPATTI**HALF YEARLY EXAM - 2024****10th Standard**

Date : 16-12-24

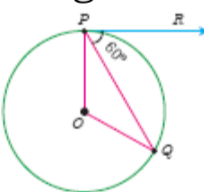
Reg.No. :

Total Marks : 100

Exam Time : 03:00 Hrs

PART - I**CHOOSE THE CORRECT ANSWER**

14 x 1 = 14

- 1) If the ordered pairs $(a + 2, 4)$ and $(5, 2a + b)$ are equal then (a, b) is
 (a) $(2, -2)$ (b) $(5, 1)$ (c) $(2, 3)$ **(d) $(3, -2)$**
- 2) $7^{4k} \equiv \underline{\hspace{2cm}} \pmod{100}$
(a) 1 (b) 2 (c) 3 (d) 4
- 3) Sum of infinite terms of G.P is 12 and the first term is 8. What is the fourth term of the G.P?
(a) $\frac{8}{27}$ (b) $\frac{4}{27}$ (c) $\frac{8}{20}$ (d) $\frac{1}{3}$
- 4) If $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$ then the value of k is
 (a) 3 **(b) 5** (c) 6 (d) 8
- 5) $y^2 + \frac{1}{y^2}$ is not equal to
 (a) $\frac{y^2+1}{y^2}$ **(b) $\left(y + \frac{1}{y}\right)^2$** (c) $\left(y - \frac{1}{y}\right)^2 + 2$ (d) $\left(y + \frac{1}{y}\right)^2 - 2$
- 6) Find the matrix X if $2X + \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 9 & 5 \end{pmatrix}$
 (a) $\begin{pmatrix} -2 & -2 \\ 2 & -1 \end{pmatrix}$ **(b) $\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$** (c) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}$
- 7) In figure if PR is tangent to the circle at P and O is the centre of the circle, then $\angle PQR$ is

(a) 120° (b) 100° (c) 110° (d) 90°
- 8) The angle of inclination made by the line joining the points $(1, -4)$ and $(2, -3)$ with x axis is
 (a) 90° (b) 30° **(c) 45°** (d) 60°
- 9) The equation of a line passing through the origin and perpendicular to the line $7x - 3y + 4 = 0$ is
 (a) $7x - 3y + 4 = 0$ (b) $3x - 7y + 4 = 0$ **(c) $3x + 7y = 0$** (d) $7x - 3y = 0$
- 10) If $\sin \theta = \cos \theta$, then $2 \tan^2 \theta + \sin^2 \theta - 1$ is equal to
 (a) $-\frac{3}{2}$ **(b) $\frac{3}{2}$** (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$
- 11) The difference between the CSA and TSA of a right circular cylinder is ----(sq.units)
 (a) πr^2 sq.units (b) $3\pi r^2$ sq.units **(c) $2\pi r^2$ sq.units** (d) $4\pi r^2$ sq.units

- 12) The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is
 (a) 1:2:3 (b) 2:1:3 (c) 1:3:2 **(d) 3:1:2**
- 13) The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is
 (a) 3 (b) 15 (c) 5 **(d) 225**
- 14) If a letter is chosen at random from the English alphabets {a, b,...,z}, then the probability that the letter chosen precedes x
 (a) $\frac{12}{13}$ (b) $\frac{1}{13}$ **(c) $\frac{23}{26}$** (d) $\frac{3}{26}$

PART -II

10 x 2 = 20

ANSWER ANY 10 QUESTIONS .QUESTION NO.28 IS COMPULSORY

- 15) a) Let $A = \{1, 2, 3, 7\}$ and $B = \{3, 0, -1, 7\}$, which of the following are relation from A to B ?
 $R_1 = \{(2, 1), (7, 1)\}$

Answer : $A = \{1, 2, 3, 7\}$, $B = \{3, 0, -1, 7\}$

$A \times B = \{(1, 3), (1, 0), (1, -1), (1, 7), (2, 3), (2, 0), (2, -1), (2, 7), (3, 3), (3, 0), (3, -1), (3, 7), (7, 3), (7, 0), (7, -1), (7, 7)\}$

$R_1 = \{(2, 1), (7, 1)\}$

Since (2, 1) and (7, 1) are not the elements of $A \times B$, R_1 is not a relation from A to B. Moreover $1 \notin B$.

b)

$R_2 = \{(-1, 1)\}$

Answer : $A = \{1, 2, 3, 7\}$, $B = \{3, 0, -1, 7\}$

$A \times B = \{(1, 3), (1, 0), (1, -1), (1, 7), (2, 3), (2, 0), (2, -1), (2, 7), (3, 3), (3, 0), (3, -1), (3, 7), (7, 3), (7, 0), (7, -1), (7, 7)\}$

$R_2 = \{(-1, 1)\}$, $(-1, 1) \notin A \times B$,

$\therefore R_2$ is not a relation from A to B.

But $(-1, 1) \in (B \times A)$ as $-1 \in B$ and $1 \in A$.

- 16) Find k if $f \circ f(k) = 5$ where $f(k) = 2k - 1$.

Answer : $f \circ f(k) = f(f(k))$

$= 2(2k - 1) - 1 = 4k - 3$

Thus, $f \circ f(k) = 4k - 3$

But, it is given that $f \circ f(k) = 5$

Therefore $4k - 3 = 5 \Rightarrow k = 2$

- 17) Which term of an A.P. 16, 11, 6, 1, ... is -54?

Answer : Given the A.P. 16, 11, 6, 1,

Here $a = 16$, $d = 11 - 16 = -5$

We have the n^{th} term of an A.P. $t_n = a + (n - 1)d$

Put $t_n = -54$

$-54 = 16 + (n - 1)(-5)$

$\frac{-70}{-5} = n - 1$

$14 = n - 1$

$14 + 1 = n$

$n = 15$

- 54 is the 15th term of 16, 11, 6, 1,

- 18) Find the 10th term of a G.P. whose 8th term is 768 and the common ratio is 2

Answer : n^{th} term of a G.P. $t_n = ar^{n-1}$

Given Common ratio $r = 2$

$$ar^{8-1} = 768$$

$$a(2)^7 = 768$$

$$10^{\text{th}} \text{ term } t_{10} = ar = ar^7 \times r^2 = 768 \times 2^2 \text{ [using (1) \& (2)]}$$

$$10^{\text{th}} \text{ term} = 3072.$$

19) Simplify

$$\frac{p^2-10p+21}{p-7} \times \frac{p^2+p-12}{(p-3)^2}$$

Answer : $\frac{p^2-10p+21}{p-7} \times \frac{p^2+p-12}{(p-3)^2}$

$$= \frac{(p-7)(p-3)}{(p-7)} \times \frac{(p+4)(p-3)}{(p-3)(p-3)} = p+4$$

20) Determine the nature of roots for the following quadratic equations

$$2x^2 - 2x + 9 = 0$$

Answer : $2x^2 - 2x + 9 = 0$

Here, $a = 2$, $b = -2$, $c = 9$

Now, $\Delta = b^2 - 4ac = (-2)^2 - 4(2)(9) = -68$

Here, $\Delta = -68 < 0$. So, the equation will have no real roots.

21) If $A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$ then verify $(A^T)^T = A$

Answer : If $A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$, $A^T = \begin{bmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{bmatrix}$

$$(A^T)^T = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix} = A$$

\therefore verified

22) What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

Answer : Let x be the length of the ladder. $BC = 4$ ft, $AC = 7$ ft.

By Pythagoras theorem we have, $AB^2 = AC^2 + BC^2$

$$x^2 = 7^2 + 4^2 \text{ gives } x^2 = 49 + 16$$

$$x^2 = 65, \text{ Hence } x = \sqrt{65}$$

The number $\sqrt{65}$ is between 8 and 8.1.

$$82 = 64 < 65.61 = 8.1^2$$

Therefore, the length of the ladder is approximately 8.1ft

23) The line through the points $(-2, a)$ and $(9, 3)$ has slope $-\frac{1}{2}$. Find the value of a .

Answer : Given points (- 2, a) and (9, 3) and Slope = $-\frac{1}{2}$.

Slope of the line = $-\frac{1}{2}$

$$\frac{y_1 - y_2}{x_1 - x_2} = -\frac{1}{2}$$

$$\frac{a - 3}{-2 - 9} = -\frac{1}{2}$$

$$\frac{a - 3}{-11} = -\frac{1}{2}$$

$$2(a - 3) = 11$$

$$2a = 11 + 6$$

$$a = \frac{17}{2}$$

24) show that $\left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2$

Answer : LHS

$$\begin{aligned} \left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right) &= \frac{1 + \tan^2}{1 + \frac{1}{\tan^2 A}} \\ &= \frac{1 + \tan^2 A}{\frac{\tan^2 A + 1}{\tan^2 A}} = \tan^2 A \dots (1) \end{aligned}$$

RHS

$$\begin{aligned} \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 &\left(\frac{1 - \tan A}{1 - \frac{1}{\tan A}}\right)^2 \\ &= \left(\frac{1 - \tan A}{\frac{\tan A - 1}{\tan A}}\right)^2 = (-\tan A)^2 = \tan^2 A \dots (2) \end{aligned}$$

From (1) and (2), $\left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2$

25) If the total surface area of a cone of radius 7cm is 704 cm^2 , then find its slant height.

Answer : Given that, radius $r = 7 \text{ cm}$

Now, total surface area of the cone = $\pi r(l + r)$ sq. units

$$\text{T.S.A} = 704 \text{ cm}^2$$

$$704 = \frac{22}{7} \times 7(l + 7)$$

$$32 = l + 7 \text{ implies } l = 25 \text{ cm}$$

Therefore, slant height of the cone is 25 cm.

26) Find the range and coefficient of range of the following data. 63, 89, 98, 125, 79, 108, 117, 68

Answer : 63, 89, 98, 125, 79, 108, 117, 68

Largest value $L = 125$

Smallest value $S = 63$

$$\therefore R = L - S = 125 - 63 = 62$$

$$\text{Co-efficient of range} = \frac{L - S}{L + S}$$

$$= \frac{125 - 63}{125 + 63}$$

$$= \frac{62}{188} = 0.329 = 0.33$$

Range = 62; coefficient of range = 0.33

27) What is the probability that a leap year selected at random will contain 53 Saturdays. (Hint: $366 = 52 \times 7 + 2$)

Answer : leap year has 366 days. So it has 52 full weeks and 2 days. 52 Saturdays must be in 52 full weeks.

The possible chances for the remaining two days will be the sample space.

$S = \{(Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun)\}$

$n(S) = 7$

Let A be the event of getting 53rd Saturday.

Then $A = \{Fri-Sat, Sat-Sun\}$; $n(A) = 2$

Probability of getting 53 Saturdays in a leap year is $P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$.

- 28) An solid mettalic spherical ball of diameter 6 cm is melted and recast into a cone with diameter of the base as 12 cm .Find the height of the cone

Answer : Radius of sphere = 3 cm

Volume of sphere = $\frac{4}{3}\pi r^3$ cu. units

$$= \frac{4}{3}\pi(3)^3$$

Radius of cone = 6 cm

height = h cm

Volume of cone = $\frac{1}{3}\pi r^2 h$ cu. units

$$= \pi(6)^2 h$$

Given that sphere is melted and cast into a cone

Volume of cone = Volume of sphere

Height of the cylinder = 3 cm.

PART -III

10 x 5 =50

ANSWER ANY 10 QUESTIONS .QUESTION NO.42 IS COMPULSORY

- 29) Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that $(A \cap B) \times C = (A \times C) \cap (B \times C)$

Answer : Given: A is the set of all natural numbers less than 8 $A = \{1,2,3,4,5,6,7\}$

B is the set of all prime numbers less than 8 $B = \{2,3,5,7\}$

'C' is the set of all even prime number

$C = \{2\}$

verify

$(A \cap B) \times C = (A \times C) \cap (B \times C)$

$A \cap B = \{2,3,5,7\}$

$(A \cap B) \times C = \{2,3,5,7\} \times \{2\}$

$= \{(2,2),(3,2),(5,2),(7,2)\} \dots\dots(1)$

$A \times C = \{1,2,3,4,5,6,7\} \times \{2\}$

$= \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$B \times C = \{2,3,5,7\} \times \{2\}$

$= \{(2,2),(3,2),(5,2),(7,2)\}$

$(A \times C) \cap (B \times C) = \{(2,2),(3,2),(5,2),(7,2)\} \dots(2)$

From (1) and (2) it is clear that

$(A \cap B) \times C = (A \times C) \cap (B \times C)$

Hence verified.

- 30) If the function $f: R \rightarrow R$ defined by

$$f(x) = \begin{cases} 2x + 7, & x < -2 \\ x^2 - 2, & -2 \leq x < 3 \\ 3x - 2, & x \geq 3 \end{cases}$$

(i) $f(4)$

(ii) $f(-2)$

(iii) $f(4) + 2f(1)$

(iv) $\frac{f(1)-3f(4)}{f(-3)}$

Answer : The function f is defined by three values in intervals I, II, III as shown by the side. For a given value of $x = a$, find out the interval at which the point a is located, there after find

$f(a)$ using the particular value defined in that interval.

(i) First, we see that, $x = 4$ lie in the third interval.

Therefore, $f(x) = 3x - 2$; $f(4) = 3(4) = 10$

(ii) $x = -2$ lies in the second interval

Therefore, $f(x) = x^2 - 2$; $f(-2) = (-2)^2 - 2 = 2$

(iii) From (i), $f(4) = 10$.

To find $f(1)$ first we see that $x = 1$ lies in the second interval.

Therefore, $f(x) = x^2 - 2 \Rightarrow f(1) = 1^2 - 2 = -1$

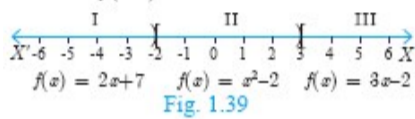
So, $f(4) + 2f(1) = 10 + 2(-1) = 8$

(iv) We know that $f(1) = -1$ and $f(4) = 10$

For finding $f(-3)$, we see that $x = -3$, lies in the first interval.

Therefore, $f(x) = 2x + 7$; thus, $f(-3) = 2(-3) + 7 = 1$

Hence, $\frac{f(1)-3f(4)}{f(-3)} = \frac{-1-3(10)}{1} = -31$



- 31) The product of three consecutive terms of a Geometric Progression is 343 and their sum is $\frac{91}{3}$. Find the three terms.

Answer : Since the product of 3 consecutive terms is given. we can take them as $\frac{a}{r}$, a , ar

Product of the terms = 343

$\frac{a}{r} \times a \times ar = 343$

$a^3 = 343$ gives $a = 7$

Sum of the terms = $\frac{91}{3}$

Hence $a \left(\frac{1}{r} + 1 + r \right) = \frac{91}{3} \quad 7 \left(\frac{1+r+r^2}{r} \right) = \frac{91}{3}$

$3 + 3r + 3r^2 = 13r$ gives $3r^2 - 10r + 3 = 0$

$(3r - 1)(r - 3) = 0$ gives $r = 3$ or $r = \frac{1}{3}$

if $a = 7$, $r = 3$ then the three terms are $\frac{7}{3}$, 7 , 21

If $a = 7$, $r = \frac{1}{3}$ then the three terms are 21 , 7 , $\frac{7}{3}$.

- 32) Find the sum of the following series
 $10^3 + 11^3 + 12^3 + \dots + 20^3$

Answer : $1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$

$= (1^3 + 2^3 + 3^3 + \dots + 20^3) - (1^3 + 2^3 + \dots + 9^3)$

$= \left[\frac{20(20+1)}{2} \right]^2 - \left[\frac{9(9+1)}{2} \right]^2$

$= \left[\frac{20 \times 21}{2} \right]^2 - \left[\frac{9 \times 10}{2} \right]^2 = (210)^2 - (45)^2$

$= 44100 - 2025 = 42075$

$10^3 + 11^3 + 12^3 + \dots + 20^3 = 42075$

- 33) Find the values of a and b if the following polynomials are perfect squares
 $4x^4 - 12x^3 + 37x^2 + bx + a$

Answer :

$$\begin{array}{r}
 2x^2 - 3x + 7 \\
 2x^2 \overline{) 4x^2 - 12x^3 + 37x^2 + bx + 9} \\
 \underline{4x^2 - 3x} \\
 -12x^3 + 37x^2 + bx + 9 \\
 \underline{(-12x^3 + 9x^2)} \\
 28x^2 + bx + a \\
 \underline{28x^2 - 42x + 49} \\
 0
 \end{array}$$

b=-42

a=49

34) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ Show that

$A(BC) = (AB)C$

Answer : $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

L.H.S. = A(BC)

$$|BC| = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} (8+0) & (0+0) \\ (2+5) & (0+10) \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 7 & 10 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 7 & 10 \end{bmatrix} = \begin{bmatrix} (8+14) & (0+20) \\ (8+21) & (0+30) \end{bmatrix} = \begin{bmatrix} 22 & 20 \\ 29 & 30 \end{bmatrix} \dots (1)$$

R.H.S = (AB)C

$$AB = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix} = \begin{bmatrix} (4+2) & (0+10) \\ (4+3) & (0+15) \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ 7 & 15 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 6 & 10 \\ 7 & 15 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} (12+10) & (0+20) \\ (14+15) & (0+30) \end{bmatrix} = \begin{bmatrix} 22 & 20 \\ 29 & 30 \end{bmatrix} \dots (2)$$

(1) = (2) \Rightarrow L.H.S = R.H.S

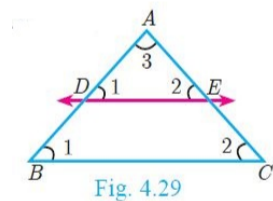
$\therefore A(BC) = (AB)C$, verified

35) State and Prove Basic Proportionality Theorem

Answer : Statement

A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.

Proof



In $\triangle ABC$, D is a point on AB and E is a point on AC

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Draw a line $DE \parallel BC$

No.	Statement	Reason
1.	$\angle ABC = \angle ADE = \angle 1$	Corresponding angles are equal because $DE \parallel BC$
2.	$\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because $DE \parallel BC$
3.	$\angle DAE = \angle BAC = \angle 3$	Both triangles have a common angle
	$\triangle ABC \sim \triangle ADE$	By AAA similarity
	$\frac{AB}{AD} = \frac{AC}{AE}$	Corresponding sides are proportional
	$\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$	Split AB and AC using the points D and E.
4.	$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$	On simplification
	$\frac{DB}{AD} = \frac{EC}{AE}$	Cancelling 1 on both sides
	$\frac{AD}{DB} = \frac{AE}{EC}$	Taking reciprocals
		Hence proved

- 36) Find the value of k, if the area of a quadrilateral is 28 sq. units, whose vertices are (-4, -2), (-3, k), (3, -2) and (2, 3)

Answer : Given vertices are (-4, -2), (-3, k), (3, -2) and (2, 3) and area of quadrilateral is 28 sq. units.

$$\text{Area of quadrilateral} = \frac{1}{2} [(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)]$$

$$\frac{1}{2} [(-4 - 3)(k - 3) - (-3 - 2)(-2 + 2)] = 28$$

$$(-7)(k - 3) - (-5)(0) = 56$$

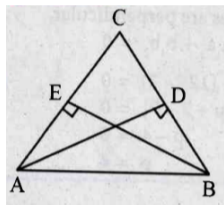
$$-7k + 21 = 56$$

$$-7k = 56 - 21 = 35$$

$$k = \frac{35}{-7} = -5$$

$$k = -5$$

- 37) A(-3, 0) B(10, -2) and C(12, 3) are the vertices of ΔABC . Find the equation of the altitude through A and B.



Answer :

Given vertices are A(-3, 0), B(10, -2) and, C(12, 3).

$$\text{Slope of BC} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-2 - 3}{10 - 12} = \frac{-5}{-2} = \frac{5}{2}$$

Altitude AD is perpendicular to BC and passing through A(-3, 0)

$$\text{Slope of AD} = -\frac{2}{5}$$

$$\text{Equation of AD } y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{2}{5}(x + 3)$$

$$5y = -2x - 6$$

$$2x + 5y + 6 = 0$$

$$\text{Slope of AC} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{0 - 3}{-3 - 12} = \frac{-3}{-15} = \frac{1}{5}$$

Altitude BE is perpendicular to AC and passing through B(10, -2). Slope of BE = 5

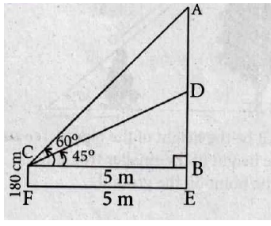
$$\text{Equation of BE } y - y_1 = m(x - x_1)$$

$$y + 2 = -5(x - 10)$$

$$y + 2 = -5x + 50$$

$$5x + y - 48 = 0$$

- 38) To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and if he is 5 m away from the wall, what is the height of the window? ($\sqrt{3} = 1.732$)

Answer :

Let CF be the height of the man; AD be the height of the window; BC is the distance between the observer and the house.

From the right triangle $\triangle CBD$

$$\tan 45^\circ = \frac{DB}{BC}$$

$$1 = \frac{DB}{5}$$

$$DB = 5\text{m} \quad \dots(1)$$

From the right triangle CBA

$$\tan 60^\circ = \frac{AB}{CB}$$

$$\sqrt{3} = \frac{AD+DB}{5}$$

$$5\sqrt{3} = AD + 5 \quad [\because \text{from (1)} DB = 5 \text{ m}]$$

$$AD = 5\sqrt{3} - 5 = 5(\sqrt{3} - 1)$$

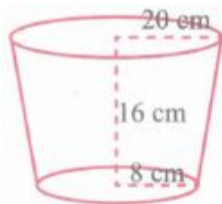
$$AD = 5(1.732 - 1)$$

$$[\text{Given } \sqrt{3} = 1.732]$$

$$= 5 \times 0.732 = 3.660$$

$$\text{Height of the window} = 3.66 \text{ m}$$

- 39) A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of Rs. 40 per litre.

**Answer :**

Given radius of lower end $r = 8 \text{ cm}$

radius of upper end $R = 20 \text{ cm}$

height $h = 16 \text{ cm}$

$$\text{Volume} = \frac{\pi h}{3} (R^2 + Rr + r^2) \text{ cu. units}$$

$$= \frac{22 \times 16}{7 \times 3} ((20)^2 + (20)(8) + (8)^2)$$

$$= \frac{22 \times 16}{21} [400 + 160 + 64]$$

$$= \frac{22 \times 16}{21} (624) = 10459.43 \text{ cm}^3$$

$$= \frac{10459.43}{1000} [\because 1000 \text{ cm}^3 = 1 \text{ litre}]$$

$$= 10.45943 \text{ litre}$$

$$\text{Cost of milk per litre} = \text{Rs. } 40$$

$$\text{Total cost} = 10.459 \times 40$$

$$= \text{Rs. } 418.36$$

- 40) A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only 32, 35, 37, 30, 33, 36, 35 and 37 pages. Find the standard deviation of the pages yet to be completed by them.

Answer : Total pages = 60

Completed pages by the students are

32, 35, 37, 30, 33, 36, 35, 37

Let the pages yet to be completed by 8 students be x_i

Completed Pages (x_i)	(x_i^2)
32	1024
35	1225
37	1369
30	900
33	1089
36	1296
35	1225
37	1369
$\Sigma x_i = 275$	$\Sigma x_i^2 = 9497$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{9497}{8} - \left(\frac{275}{8}\right)^2}$$

$$= \sqrt{1187.125 - (34.375)^2} = \sqrt{1187.125 - 1181.64063}$$

$$= \sqrt{5.48437} = 2.34$$

- 41) Find the GCD of the polynomials $13m^3 + 13m^2 - 13m + 26$ and $22m^3 - 55m^2 + 55m - 33$.

Answer : Let $f(x) = 13m^3 + 13m^2 - 13m + 26 = 13(2m^3 - 5m^2 + 5m - 3)$ and $g(x) = 22m^3 - 55m^2 + 55m - 33 = 11(m^3 + m^2 - m + 2)$

$$\begin{array}{r} x^3 + x^2 - x + 2 \quad \overset{2}{} \\ \underline{2x^3 - 5x^2 + 5x - 3} \\ 2x^3 + 2x^2 - 2x + 4 \\ \underline{-7x^2 + 7x - 7} \\ -7(x^2 - x + 1) \end{array}$$

$-7(x^2 - x + 1) \neq 0$, note that -7 is not a divisor of $g(x)$

Now dividing, $g(x) = x^3 + x^2 - x + 2$ by the new remainder $x^2 - x + 1$ (leaving the constant factor), we get

$$\begin{array}{r} x^2 - x + 1 \quad \overset{x+2}{} \\ \underline{x^3 + x^2 - x + 2} \\ x^3 - x^2 + x \\ \underline{2x^2 - 2x + 2} \\ 2x^2 - 2x + 2 \\ \underline{0} \end{array}$$

Here, we get zero remainder

Therefore, $\text{GCD}(2m^3 - 5m^2 + 5m - 3, m^3 + m^2 - m + 2) = m^2 - m + 1$.

- 42) Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Answer : When two dice are rolled together, there will be $6 \times 6 = 36$ outcomes. Let S be the sample space. Then $n(S) = 36$.

Let A be the event of getting a doublet and B be the event of getting face sum 4.

Then $A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$B = \{(1,3), (2,2), (3,1)\}$

Therefore, $A \cap B = \{(2,2)\}$

Then, $n(A) = 6$, $n(B) = 3$, $n(A \cap B) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

Therefore, $P(\text{getting a doublet or a total of 4}) = P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$$

Hence, the required probability is $\frac{2}{9}$.

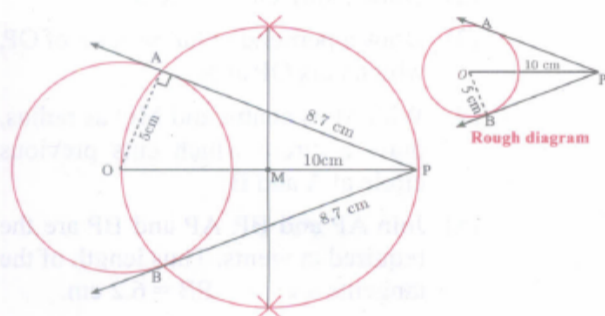
PART -IV

2 x 8 = 16

ANSWER ALL THE QUESTIONS .

- 43) a) Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents.

Answer : The distance between the point from the centre is 10 cm.



Length of the tangents $PA = PB = 8.7$ cm

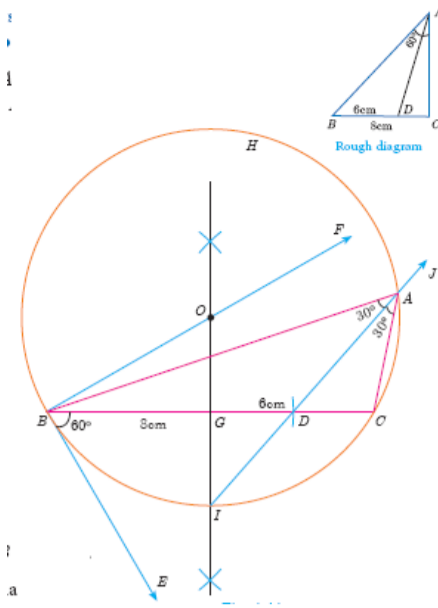
Construction:

Steps:

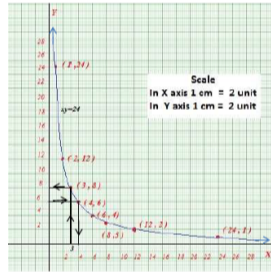
- (1) With O as centre, draw a circle of radius 5cm.
- (2) Draw a line $OP = 10$ cm.
- (3) Draw a perpendicular bisector of OP which cuts OP at M.
- (4) With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
- (5) Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA and PB = 8.7 cm

(OR)

- b) Draw a triangle ABC of base $BC = 8$ cm, $\angle A = 60^\circ$ and the bisector of $\angle A$ meets BC at D such that $BD = 6$ cm.

Answer :**Construction**Step 1 : Draw a line segment $BC = 8 \text{ cm}$.Step 2 : At B, draw BE such that $\angle CBE = 60^\circ$.Step 3 : At B, draw BF such that $\angle EBF = 90^\circ$.Step 4 : Draw the perpendicular bisector to BC , which intersects BF at O and BC at G .Step 5 : With O as centre and OB as radius draw a circle.Step 6 : From B , mark an arc of 6 cm on BC at D .Step 7 : The perpendicular bisector intersects the circle at I . Join ID .Step 8 : ID produced meets the circle at A . Now join AB and AC . Then $\triangle ABC$ is the required triangle.

- 44) a) Draw the graph $xy = 24$, $x, y > 0$, Using the graph find,
 (i) y when $x = 3$ and
 (ii) x when $y = 6$.

Answer :**1. Table**

x	24	12	8	6	4	3	2	1
y	1	2	3	4	6	8	12	24

2. Variation:

Indirect Variation

3. Equation

$$xy = k$$

$$xy = 24 \times 1 = 12 \times 2 = 8 \times 3 = \dots = 24$$

$$xy = 24$$

4. Points

(24, 1), (12, 2), (8, 3), (6, 4)

(4, 6), (3, 8), (2, 12), (1, 24)

5. Solution

From the graph

(i) If $x = 3$, then, $y = 8$ (ii) If $y = 6$ then, $x = 4$ **(OR)**

- b) Draw the graph of $y = x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$

Answer : -1