# GOVT. HIGH SCHOOL, PAITHUR

## HALF YEARLY EXAMINATION - 2024

## 10 std - MATHEMATICS

## Answer Key -

### PART - I

1) (d) $(3,-2)$
2) (a) 1
3) (a) $\frac{8}{27}$
4) (b) 5
5) (b) $\left(y+\frac{1}{y}\right)^2$
6) (b) $\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$
7) (d) $90°$
8) (a) $90°$
9) (c) $3x+7y=0$
10) (b) $\frac{3}{2}$
11) (c) $2\pi r^2$
12) (d) $3:1:2$
13) (d) 225
14) (c) $\frac{23}{26}$

### PART - II

15) $A\times B = \{1,2,3,7\} \times \{3,0,-1,7\}$

$= \{(1,3)\,(1,0),(1,-1),(1,7)\,(2,3)\,(2,0)\,(2,-1)(2,7)\,(3,3)(3,0)\,(3,-1),(3,7)\,(7,3)\,(7,0)(7,-1),(7,7)\}$

(i) $R_1 = \{(2,1)\,(7,1)\}$

here $1 \notin B$

$\therefore (2,1)$ and $(7,1) \notin A\times B$

So, $R_1$ is not a relation from A to B.

(ii) $R_2 = \{(-1,1)\}$

here $(-1,1) \in R_2$ but $(-1,1) \notin A\times B$. So, $R_2$ is not a relation from A to B.

16) $f\circ f(k) = f(f(k))$

$= 2(2k-1)-1$

$= 4k-3$

$f\circ f(k) = 4k-3$

But $f\circ f(k) = 5$

$4k-3 = 5$

$\Rightarrow \boxed{k=2}$

17) A.P is $16, 11, 6, 1, \ldots, -54$

$a=16,\ d=-5,\ l=-54$

$n = \frac{l-a}{d}+1$

$= \frac{-54-16}{-5}+1$

$= 14+1$

$\boxed{n=15}$

18) Given $t_8 = 768 \Rightarrow ar^7 = 768,\ r=2$

To find $t_{10}$,

$t_{10} = ar^9 = ar^7 \times r^2$

$= 768 \times 4$

$\boxed{t_{10} = 3072}$

19) $\frac{p^2-10p+21}{p-7} \times \frac{p^2+p-12}{(p-3)^2}$

$= \frac{\cancel{(p-7)}\,\cancel{(p-3)} \times (p+4)\cancel{(p-3)}}{\cancel{(p-7)}\,\cancel{(p-3)^2}}$

$= (p+4)$

20) $2x^2-2x+9=0$

$a=2,\ b=-2,\ c=9$

$\Delta = b^2-4ac$

$= (-2)^2 - 4(2)(9)$

$= 4-72$

$\Delta = -68 < 0$

So, the equation will have no real roots.

21) $A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$

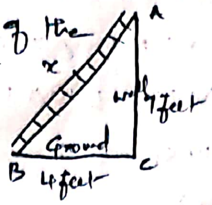
$A^T = \begin{bmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & \frac{5}{2} \\ 2 & \frac{5}{2} & 1 \end{bmatrix}$

$(A^T)^T = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$

$(A^T)^T = A$

22) Let $x$ be the length of the ladder. $BC = 4ft$, $AC = 7ft$




$AB^2 = AC^2 + BC^2$

$x^2 = 7^2 + 4^2$

$\Rightarrow x^2 = 49+16$

$x^2 = 65$

$x = \sqrt{65} = 8.1$

∴ The length of the ladder is approximately 8.1 ft.

23) The slope of the points $(-2,a)$ and $(9,3) = -1/2$

$\Rightarrow \frac{3-a}{9-(-2)} = -1/2$

$\Rightarrow \frac{3-a}{11} = -1/2$

$\Rightarrow 3-a = -11/2$

$\Rightarrow a = 3+11/2$

$\boxed{a = 17/2}$

24) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}} = \frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}}$

$= \tan^2 A$ — (1)

$\left(\frac{1-\tan^2 A}{1-\cot^2 A}\right) = \frac{1-\tan A}{1-\frac{1}{\tan A}} = \left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2$

$= (-\tan A)^2$

$= \tan^2 A$ — (2)

From (1) and (2)

$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2$

25) $r = 7cm$.

Total surface area of the cone $= \pi r(l+r)$ sq.units

TSA $= 704\ cm^2$

$704 = \frac{22}{7} \times 7(l+7)$

$32 = l+7$

$\Rightarrow \boxed{l = 25cm}$

∴ slant height of the cone $= 25cm$

26) $L = 125$, $S = 63$

Range $= L - S = 125 - 63 = 62$

Co-efficient of Range $= \frac{L-S}{L+S}$

$= \frac{125-63}{125+63}$

$= \frac{62}{188}$

$= 0.33$

27) $n(S) = 7$

$A = \{Fri-Sat, Sat-Sun\}$

$n(A) = 2$

Probability of getting 53 saturda in a leap year $= 2/7$

28) volume of the spherical ball

$V_s = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 3^3 = 36\pi\ cm^3$

Volume of the cone made from sph

$V_c = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 6^2 \times h$

$= 12\pi h\ cm^3$

$\Rightarrow V_s = V_c$

$\Rightarrow 36\pi = 12\pi h$

$h = \frac{36}{12}$

$\boxed{h = 3cm}$

PART-III

29) $A \cap B = \{2,3,5,7\}$

$(A\cap B) \times C = \{(2,2),(3,2),(5,2),(7,2)\}$ — (1)

$A \times C = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$B \times C = \{(2,2),(3,2),(5,2),(7,2)\}$

$(A\times C) \cap (B \times C) = \{(2,2),(3,2),(5,2),(7,2)\}$ — (2)

from (1) and (2)

$(A\cap B)\times C = (A\times C)\cap(B\times C)$

30) (i) $f(4) = 3(1)-2 = 10$

(ii) $f(-2) = (-2)^2 - 2 = 2$

(iii) $f(4) + 2f(1) = 10 + 2(-1)$

$= 8$

(iv) $\frac{f(1) - 3f(4)}{f(-3)} = \frac{-1-3(10)}{1}$

$= -31$

31) The product of 3 consecutive term $\frac{a}{r}, a, ar$

Product of the terms $= 343$

$$\frac{a}{r} \times a \times ar = 343$$

$$a^3 = 7^3$$

$$\boxed{a=7}$$

Sum of the terms $= \frac{91}{3}$

$$a\left(\frac{1}{r}+1+r\right) = \frac{91}{3}$$

$$\Rightarrow 7\left(\frac{1+r+r^2}{r}\right) = \frac{91}{3}$$

$$3+3r+3r^2 = 13r$$

$$3r^2-10r+3=0$$

$$r=3,\ r=\frac{1}{3}$$

If $a=7, r=3 \Rightarrow \frac{7}{3}, 7, 21$

If $a=7, r=\frac{1}{3} \Rightarrow 21, 7, \frac{7}{3}$

32) $10^3+11^3+12^3+\dots+20^3$

$$= (1^3+2^3+3^3+\dots+20^3)-(1^3+2^3+3^3+\dots+9^3)$$

$$= \left(\frac{n(n+1)}{2}\right)^2$$

$$= \left(\frac{20\times21}{2}\right)^2-\left(\frac{9\times10}{2}\right)^2$$

$$= 44100 - 2025$$

$$= 42075$$

33)

$$\boxed{a=49}$$

$$\boxed{b=-42}$$

34) $A(BC)=(AB)C$

$$BC = \begin{pmatrix}4&0\\1&5\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}$$

$$= \begin{pmatrix}8&0\\7&10\end{pmatrix}$$

$$A(BC) = \begin{pmatrix}1&2\\1&3\end{pmatrix}\begin{pmatrix}8&0\\7&10\end{pmatrix}$$

$$= \begin{pmatrix}8+14&0+20\\8+21&0+30\end{pmatrix}$$

$$= \begin{pmatrix}22&20\\29&30\end{pmatrix} - (1)$$

$$AB = \begin{pmatrix}1&2\\1&3\end{pmatrix}\begin{pmatrix}4&0\\1&5\end{pmatrix} = \begin{pmatrix}6&10\\7&15\end{pmatrix}$$

$$(AB)C = \begin{pmatrix}22&20\\29&30\end{pmatrix} - (2)$$

from (1) and (2) $A(BC)=(AB)C$

35) Statement, diagram $\}$ − (2)

Proof → (3)

$\}$ (5) marks

36) Area of the quadrilateral $= 28$ sq. units

$$\frac{1}{2}\begin{vmatrix}-4&-3&3&2&-4\\-2&k&-2&3&-2\end{vmatrix} = 28$$

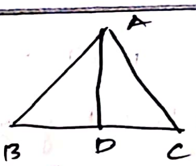
$$(-4k+6+9-4)-(6+3k-4-12)] = 56$$

$$-7k = 56-21$$

$$-7k = 35$$

$$\boxed{k=-5}$$

37) Slope of $AD = -\frac{2}{5}$




$$y-y_1 = m(x-x_1)$$

$$y-0 = -\frac{2}{5}(x-(-3))$$

$$5y = -2x-6$$

$$\Rightarrow 2x+5y+6=0$$

Now BE ⊥r to AC. To find equation of the altitude BE.

Slope of BE $= -5$

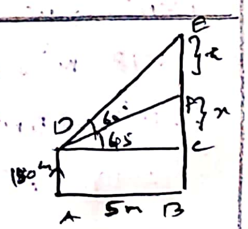
$$y-y_1 = m(x-x_1)$$

$$y-(-2) = -5(x-10)$$

$$y+2 = -5x+50$$

$$\boxed{5x+y-48=0}$$

38)




$\triangle AFD$

$$\tan 45° = \frac{DF}{AF}$$

$$1 = \frac{DF}{5}$$

$$DF = 5 \ -(1)$$

$\triangle AFC$

$$\tan 60° = \frac{CF}{AF}$$

$$\sqrt{3} = \frac{CF}{5}$$

$$CF = 5\sqrt{3} - (2)$$

Length of the window $= CD = 5\sqrt{3}-5$

$$= 5\times0.73$$

$$= 3.63\text{ m}$$

39) $r=8$ cm, $R=20$ cm, $h=16$ cm

Volume of the frustum $V = \frac{1}{3}\pi h (R^2+Rr+r^2)$ cu. units

$V = \frac{1}{3} \times \frac{22}{7} \times 16 \times (20^2 + 20\times 8 + 8^2)$

$= 10459.428 \, cm^3$ (1 litre)

The required cost $= 10.459 \times 40$

$= ₹418.36$

40)

| $x_i$ | $x_i^2$ |
|---|---|
| $\Sigma x$ | $\Sigma x^2$ |
| $=275$ | $=9497$ |

$n = 8$

$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$= \sqrt{\frac{9497}{8} - \left(\frac{275}{8}\right)^2}$

$= \sqrt{5.485}$

$\boxed{\sigma = 2.34}$

41) $n(S) = 36.$

Probability of getting a doublet $P(A) = 6/36$

face sum 4) $P(B) = 3/36$

$P(A \cap B) = \frac{1}{36}$

$P(A\cup B) = \frac{6}{36} + \frac{3}{36} - \frac{1}{36}$

$= \frac{8}{36}$

$= 2/9$

∴ the required probability $= 2/9$

42) $13m^3 + 13m^2 - 13m + 26$

$= 13(m^3 + m^2 - m + 2)$

$22m^3 - 55m^2 + 55m - 33$

$= 11(2m^3 - 5m^2 + 5m - 3)$

$(m^3 + m^2 - m + 2) \div 2m^3 - 5m^2 + 5m - 3$

The remainder $= 11m^2 - 11m + 11$

$(2m^3 - 5m^2 + 5m - 3) \div (11m^2 - 11m + 11)$

The remainder $= 0$

$GCD = 11m^2 - 11m + 11$

PART - IV

43) (a) Rough diagram — 2

Fair diagram — 7

Length of the tangent $= PA = PB$

$= 8.7 cm$

— 1

(b) Rough diagram — 2

Fair diagram — 8

44) (a) solution → 4

Graph → 4

8 Marks

$y = 8, \; x = 4$

(b) Solution → 4

Graph → 4

$\boxed{y = -1}$

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