NOTES ON TRIGONOMETRIC EQUATIONS AND IDENTITIES

A function $f(x)$ is said to be periodic if there exists some $T>0$ such that $f(x+T) = f(x)$ for all x in the domain of $f(x)$

In case of, T in the definition of period of $f(x)$ is the smallest positive real number then this T is called the period of $f(x)$

Periods have various trigonometric functions are listed below:

1) $\sin x$ has period 2π
2) $\cos x$ has period 2π
3) $\tan x$ has period π
4) $\sin(ax+b)$, $\cos(ax+b)$, $\sec(ax+b)$, $\operatorname{cosec}(ax+b)$ all are of period $2\pi/a$
5) $\tan(ax+b)$ and $\cot(ax+b)$ have π/a as their period
6) $|\sin(ax+b)|$, $|\cos(ax+b)|$, $|\sec(ax+b)|$, $|\operatorname{cosec}(ax+b)|$ all are of period π/a
7) $|\tan(ax+b)|$ and $|\cot(ax+b)|$ have $\pi/2a$ as their period

Sum and Difference formulae of trigonometry ratios

1) $\sin(a+B) = \sin(a)\cos(B) + \cos(a)\sin(B)$
2) $\sin(a-B) = \sin(a)\cos(B) - \cos(a)\sin(B)$
3) $\cos(a+B) = \cos(a)\cos(B) - \sin(a)\sin(B)$
4) $\cos(a-B) = \cos(a)\cos(B) + \sin(a)\sin(B)$
5) $\tan(a+B) = \dfrac{\tan(a)+\tan(B)}{1-\tan(a)\tan(B)}$

6) $\tan(a-B) = \dfrac{\tan(a)-\tan(B)}{1+\tan(a)\tan(B)}$

7) $\tan(\pi/4+\theta) = (1+\tan\theta)/(1-\tan\theta)$

8) $\tan(\pi/4-\theta) = (1-\tan\theta)/(1+\tan\theta)$

9) $\cot(a-B) = [\cot(a).\cot(B)+1]/[\cot(B)-\cot(a)]$

10) $\cot(a+B) = [\cot(a).\cot(B)-1]/[\cot(B)+\cot(a)]$

Double or Triple Angle Identities:

1) $\sin 2x = 2\sin x \cos x$

2) $\cos 2x = \cos 2x - \sin 2x$
$= 1-2\sin 2x = 2\cos 2x - 1$

3) $\tan 2x = 2\tan x / (1-\tan 2x)$

4) $\sin 3x = 3\sin x - 4\sin 3x$

5) $\cos 3x = 4\cos 3x - 3\cos x$

6) $\tan 3x = (3\tan x - \tan 3x)/(1-3\tan 2x)$

For angles A, B and C, we have

1) $\sin(A+B+C) = \sin A\cos B\cos C + \cos A\sin B\cos C + \cos A\cos B\sin C - \sin A\sin B\sin C$

2) $\cos(A+B+C) = \cos A\cos B\cos C - \cos A\sin B\sin C - \sin A\cos B\sin C - \sin A\sin B\cos C$

3) $\tan(A+B+C) = \dfrac{[\tan A+\tan B+\tan C-\tan A\tan B\tan C]}{[1-\tan A\tan B-\tan B\tan C-\tan A\tan C]}$

4) $\cot(A+B+C) = \dfrac{[\cot A\cot B\cot C-\cot A-\cot B-\cot C]}{[\cot A\cot B+\cot B\cot C+\cot A\cot C-1]}$

List of some other trigonometric formulas:

1) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
2) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
3) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$
4) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$
5) $\sin A + \sin B = 2\sin[(A+B)/2]\cos[(A-B)/2]$
6) $\sin A - \sin B = 2\sin[(A-B)/2]\cos[(A+B)/2]$
7) $\cos A + \cos B = 2\cos[(A+B)/2]\cos[(A-B/2]$
8) $\cos A - \cos B = 2\sin[(A+B)/2]\sin[(B-A)/2]$
9) $\tan A \pm \tan B = \sin(A\pm B)/\cos A\cos B$
10) $\cot A \pm \cot B = \sin(B\pm A)/\sin A\sin B$

Method of solving a trigonometric equation:

* If possible, reduce the equation in terms of any one variable, preferably x. Then solve the equation as you used to in case of single variable

* Try to derive the linear / algebric simultaneous equations from the equations (trigonometric equations) and solve them as algebric simultaneous equations

* At times, you might be required to make certain substitutions. It would be benefical when the system has only two trigonometric functions

Some results which are useful for solving trigonometric equations :

1) $\sin\theta = \sin a$ and $\cos\theta = \cos a \Rightarrow \theta = 2n\pi + a$
2) $\sin\theta = 0 \Rightarrow \theta = n\pi$
3) $\cos\theta = 0 \Rightarrow \theta = (2n+1)\pi/2$
4) $\tan\theta = 0 \Rightarrow \theta = n\pi$
5) $\sin\theta = \sin a \Rightarrow \theta = n\pi + (-1)^n a$ where $a \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
6) $\cos\theta = \cos a \Rightarrow \theta = 2n\pi \pm a$, where $a \in [0, \pi]$
7) $\tan\theta = \tan a \Rightarrow \theta = 2n\pi + a$, where $a \in [-\pi/2, \pi/2]$
8) $\sin\theta = 1 \Rightarrow \theta = (4n+1)\pi/2$
9) $\sin\theta = -1 \Rightarrow \theta = (4n-1)\pi/2$
10) $\sin\theta = -1 \Rightarrow \theta = (2n+1)\pi/2$
11) $|\sin\theta| = 1 \Rightarrow \theta = 2n\pi$
12) $\cos\theta = 1 \Rightarrow \theta = (2n+1)$
13) $|\cos\theta| = 1 \Rightarrow \theta = n\pi$

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