EDUCATION DEPARTMENT

VILLUPURAM DISTRICT

MATHEMATICS

10

Material for Students

2024-25

Achieved State Level 10th Place in SSLC Public Examination - March 2024

BEST WISHES

ரெ. அறிவழகன், M.A., M.A., B.Ed., M.Phill.,

Chief Educational Officer, Villupuram District.

தன்னம்பிக்கை + விடாமுயற்சி + கடின உழைப்பு = வெற்றி

"The Struggle you're in Today will definitely develop the strength you need for Tomorrow.

A. SIVAMOORTHY, Government High School, Perumbakkam, **Villupuram District.**

Mobile: 9750827997

Details of the Public Questions which took place in our Maths Material.

GOVT. QUESTION PAPER - MARCH 2024

Pub. Q.No.	Mat. P.No.	Mark	Pub. Q.No.	Mat. P.No.	Mark	Pub. Q.No.	Mat. P.No.	Mark	Total Marks
1	58-1	1	15	14-1	2	29	15-2(i)	5	8
2	58-11	1	16	67-1	2	30	19-13	5	8
3	59-5	1	17	20-1	2	31	26-9(ii)	5	8
4	59-10	1	18	27-2i	2	32	32-1	5	8
5	60-8	1	19	29-10	2	33	35-1	5	8
6	60-12	1	20	105-2	2	34	38-1	5	8
7	61-6	1	21	119-2	2	35	43-4i	5	8
8	61-12	1	22	117-24	2	36	125-17	5	8
9	62-1	1	23	44-3	2	37	130-4	5	8
10	63-7	1	24	-	-	38	49-4	5	6
11	64-3	-	25	138-8	2	39	-	-	2
12	64-15	1	26	49-1	2	40	146-1	5	8
13	-	-	27	51-17	2	41	56-16	5	7
14	65-12	1	28	71-2	2	42	25-4	5	8
		12			26	43 (a)	66-4	8	8
						43 (b)	6-9	8	8
						44 (a)	66-12	8	8
						44 (b)	11 - 6	8	8
								97	135

GOVT. QUESTION PAPER - JUNE 2024

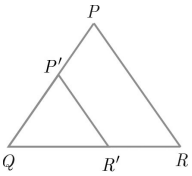
Pub. Q.No.	Mat. P.No.	Mark	Pub. Q.No.	Mat. P.No.	Mark	Pub. Q.No.	Mat. P.No.	Mark	Total Marks
1	58-10	1	15	14-4	2	29	17-6(i)	5	8
2	58-8	1	16	-	-	30	68-3	5	6
3	59-6	1	17	-	-	31	25-5(ii)	5	6
4	60-6	1	18	27-1(ii)	2	32	32-2	5	8
5	60-17	1	19	85-13	2	33	26-11	5	8
6	61-9	1	20	104-6	2	34	34-10	5	8
7	61-11	1	21	113-3	2	35	38-1	5	8
8	62-3	1	22	118-27	2	36	43-4(ii)	5	8
9	62-10	1	23	128-6	2	37	125-18	5	8
10	63-2	1	24	139-1	2	38	134-16	5	8
11	63-10	1	25	47-3	2	39	144-20	-	8
12	64-3	1	26	50-7	2	40	-	-	3
13	65-9	1	27	51-16	2	41	52-2	5	8
14	65-2	1	28	21-9	2	42	-	-	3
		14			24	43 (a)	7-12	8	8
						43 (b)	66-5	8	8
						44 (a)	12-8	8	8
						44 (b)	66 - 4	8	8
								87	125

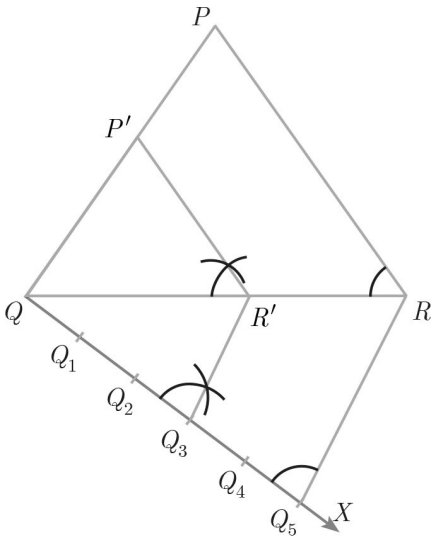
4 GEOMETRY

1. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR (scale factor $\frac{3}{5} < 1$)

Solution:

Rough diagram

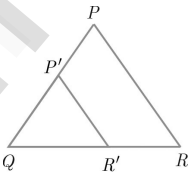


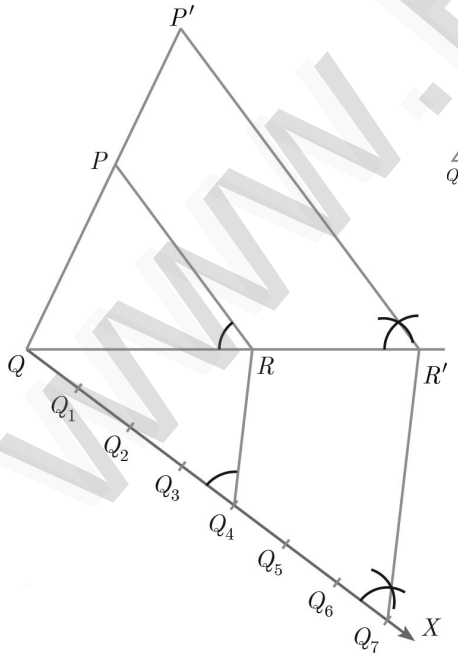


2. Cons Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{4}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{4} > 1$)

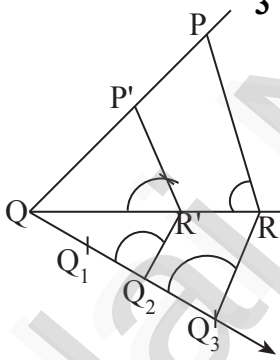
Solution:

Rough diagram

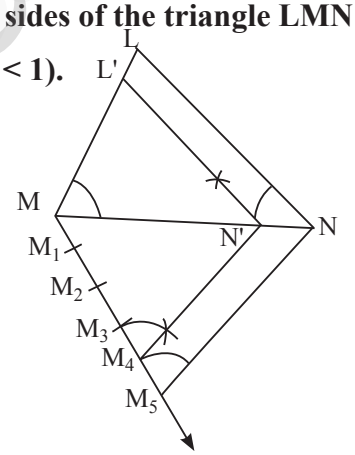




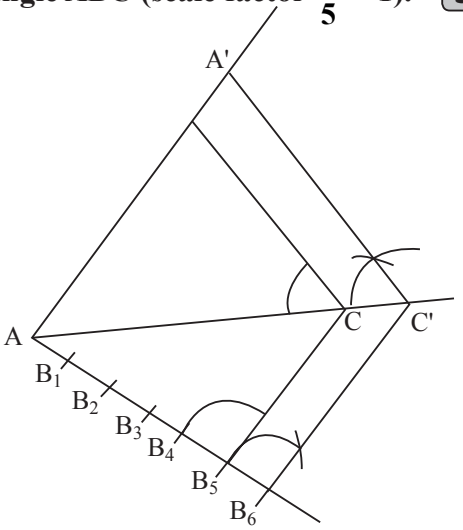
3. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{2}{3} < 1$).



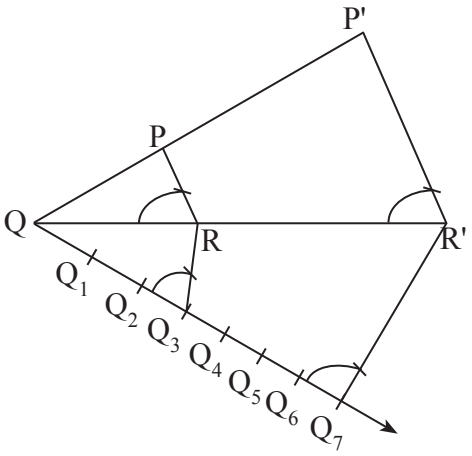
4. Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle LMN (scale factor $\frac{4}{5} < 1$).



5. Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC (scale factor $\frac{6}{5} > 1$). SEP-20



6. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{3} > 1$).

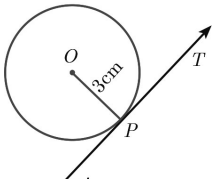


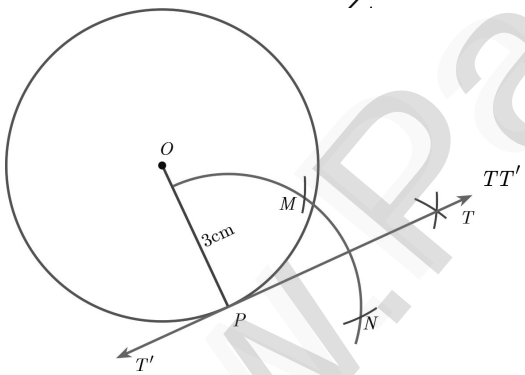
7. Draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P.

Solution:

Given, radius r = 3 cm

Rough diagram

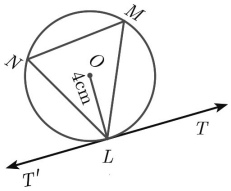


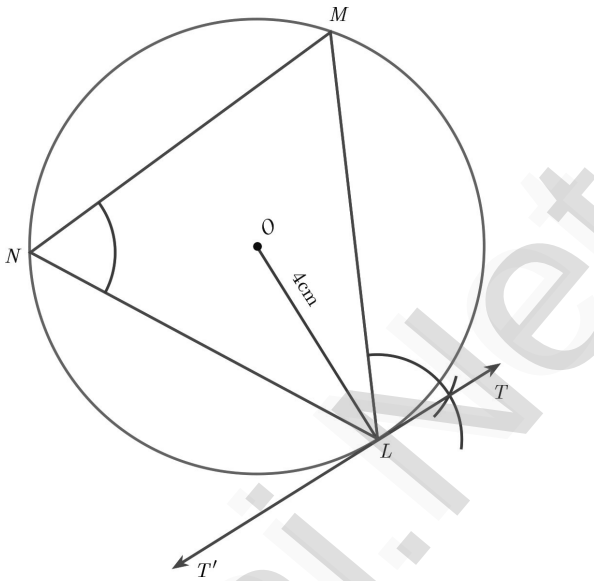


8. Draw a circle of radius 4 cm. At a point L on it draw a tangent to the circle using the alternate segment.

Solution:

Rough diagram





9. Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.

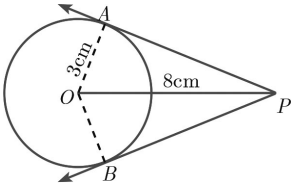
Verification: In the right angle triangle OAP.

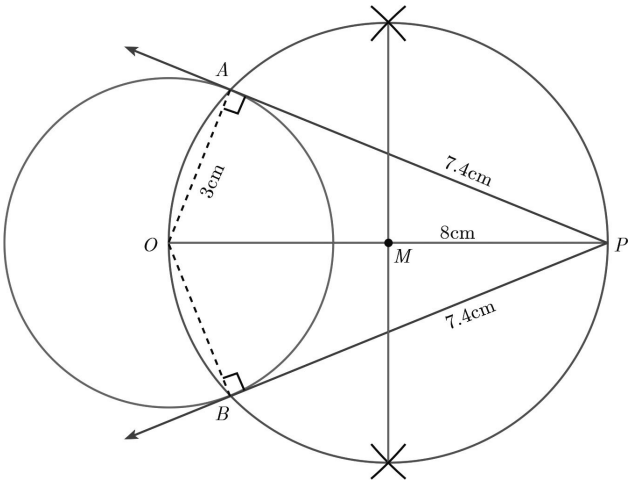
$PA^2 - OA^2 = 64 - 9 = 55$

$PA = \sqrt{55} = 7.4$ cm

Solution:

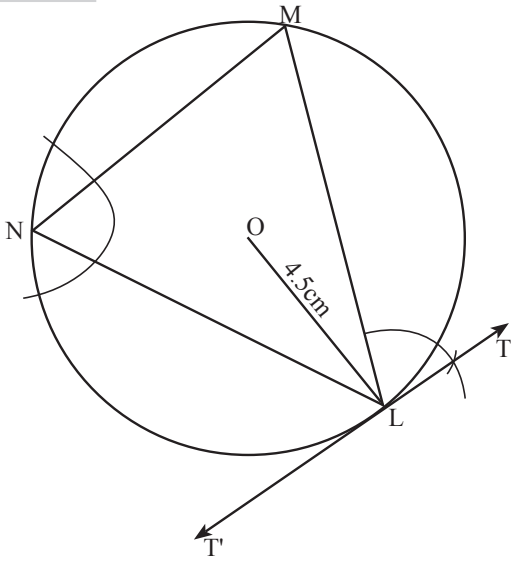
Rough diagram





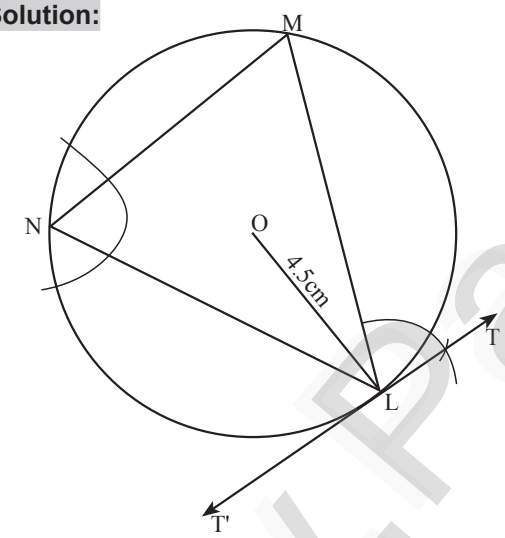
10. Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.

Solution:



11. Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.

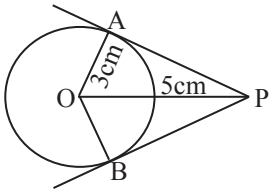
Solution:

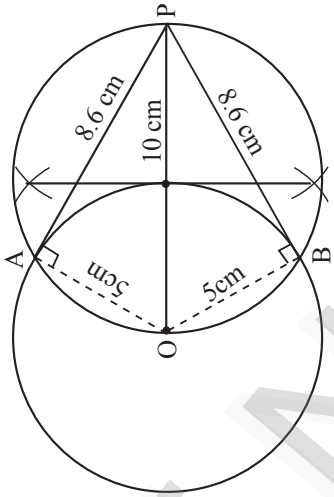


12. Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents. SEP-20

Solution:

Rough Diagram





Proof:

In ΔOPA

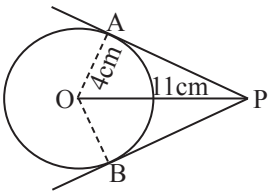
$PA^2 = OP^2 - OA^2$

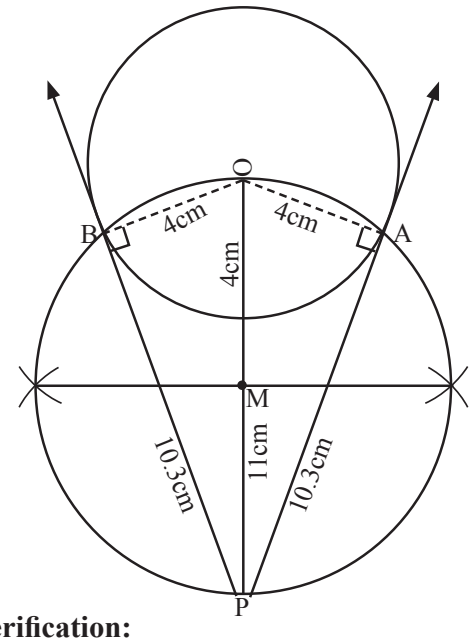
$= 10^2 - 5^2 = 100 - 25 = 75$

$PA = \sqrt{75} = 8.6$ cm (approx)

13. Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.

Rough Diagram





Verification:

In ΔOPA $AP^2 = OP^2 - OA^2$

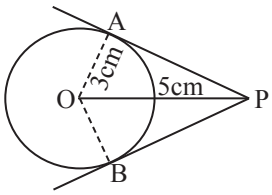
$= 11^2 - 4^2 = 121 - 16 = 105$

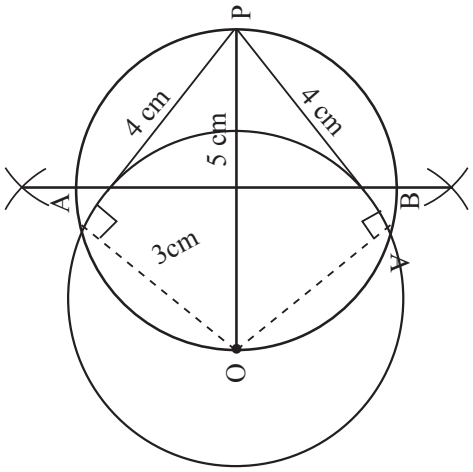
$AP = \sqrt{105} = 10.2$ cm

14. Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents. SEP-21

Solution:

Rough Diagram





Verification

In ΔOPA $AP^2 = OP^2 - OA^2$

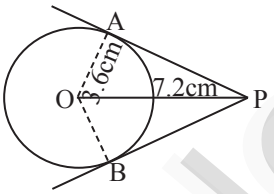
$= 5^2 - 3^2 = 25 - 9 = 16$

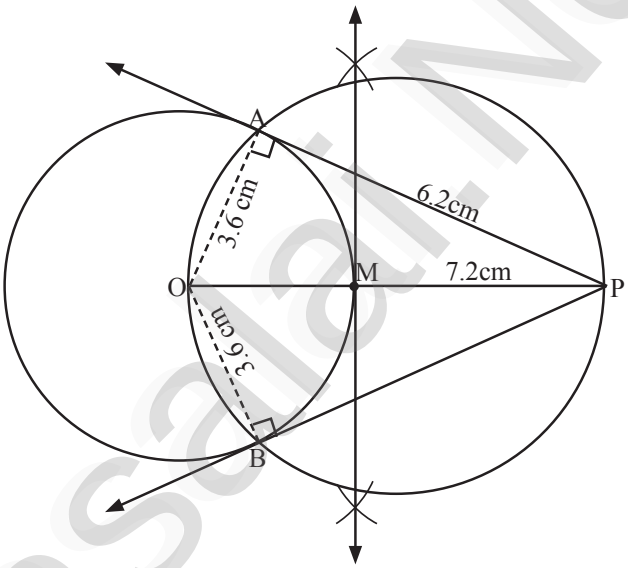
$AP = \sqrt{16} = 4$ cm

15. Draw a tangent to the circle from the point P having radius 3.6cm, and centre at O. Point P is at a distance 7.2 cm from the centre.

Solution:

Rough Diagram





Verification:

In ΔOPA, $PA^2 = OP^2 - OA^2$

$= 7.2^2 - 3.6^2$

$= 51.84 - 12.96$

$= 38.88$

$PA = \sqrt{38.88} = 6.2$ cm (approx)

***

GRAPH

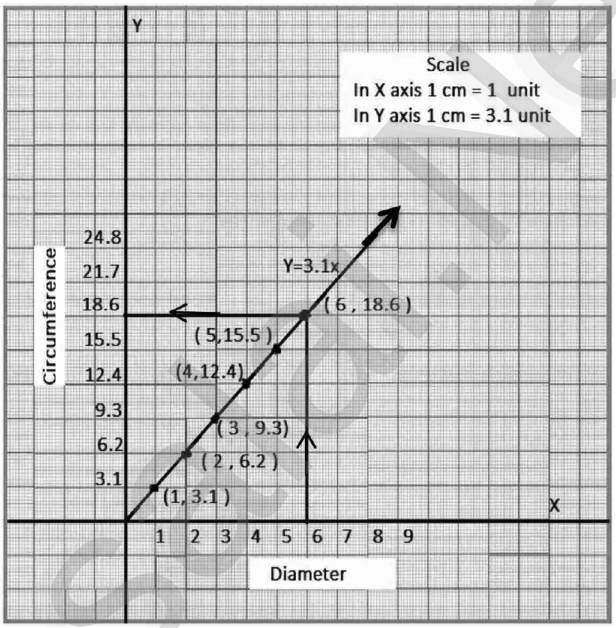
1. Varshika drew 6 circles with different sizes. Draw a graph for the relationship between the diameter and circumference of each circle as shown in the table and use it to find the circumference of a circle when its diameter is 6 cm.

Diameter (x) cm	1	2	3	4	5
Circumference (y) cm	3.1	6.2	9.3	12.4	15.5

Solution:

1. Table:

Diameter(x) cm	1	2	3	4	5	6
Circumference (y) cm	3.1	6.2	9.3	12.4	15.5	18.6



2. Variation:

Direct Variation

3. Equation:

$y = kx$

$k = \frac{y}{x} = \frac{3.1}{1} = 3.1$

$\therefore y = 3.1x$

4. Points:

(1, 3.1) (2, 6.2) (3, 9.3), (4, 12.4), (5, 15.5), (6, 18.6)

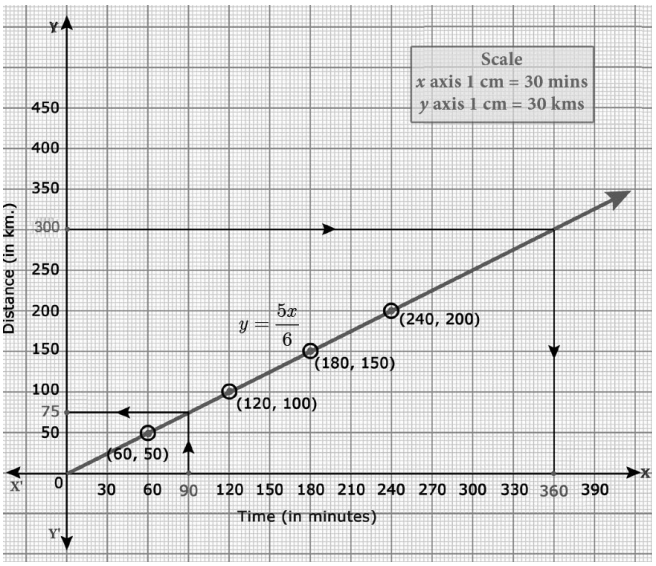
5. Solution:

From the graph, when diameter is 6 cm, its circumference is **18.6 cm**.

2. A bus is travelling at a uniform speed of 50 km/hr. Draw the distance-time graph and hence find (i) the constant of variation (ii) how far will it travel in 1½ hr (iii) the time required to cover a distance of 300 km from the graph.

Solution: 1. Table

Time taken x (in minutes)	60	120	180	240	300	360
Distance y (in km)	50	100	150	200	250	300



2. Variation:

Direct Variation.

3. Equation:

$y = kx$

$k = \frac{y}{x} = \frac{50}{60} = \frac{5}{6} \quad \therefore y = \frac{5}{6}x$

III. Points:

(60, 50), (120, 100), (180, 150), (240, 200), (300, 250)

IV. Solution:

From the graph,

(i) Constant of variation $k = \frac{5}{6}$

(ii) The distance travelled in 90 mins = **75 km**

(iii) The time taken to cover 300 km = **360 minutes = 6 hours.**

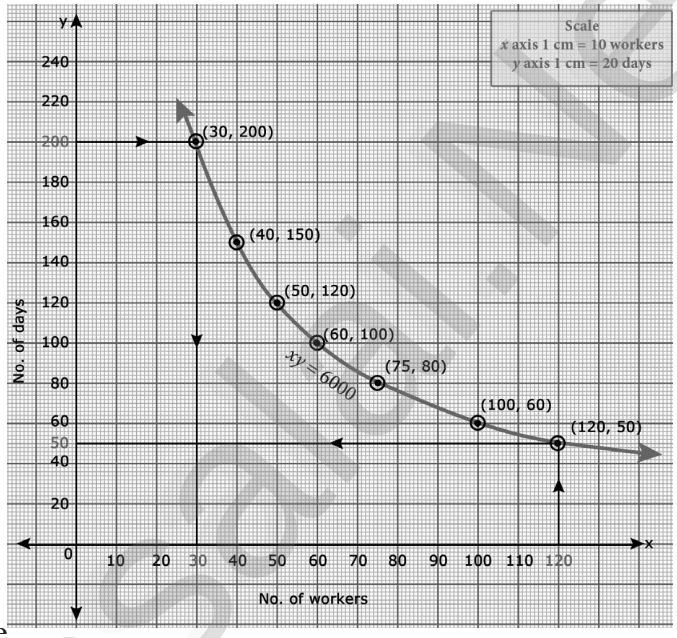
3. A company initially started with 40 workers to complete the work by 150 days. Later, it decided to fasten up the work increasing the number of workers as shown below.

Number of workers (x)	40	50	60	75
Number of days (y)	150	120	100	80

(i) Graph the above data and identify the type of variation. (ii) From the graph, find the number of days required to complete the work if the company decides to opt for 120 workers? (iii) If the work has to be completed by 30 days, how many workers are required?

Solution: **1. Table**

Number of workers (x)	40	50	60	75	100	120
Number of days (y)	150	120	100	80	60	50



2. Variation:
Indirect Variation.

3. Equation:
$xy = k$
$xy = 40 \times 150 = 6000$
$xy = 6000$

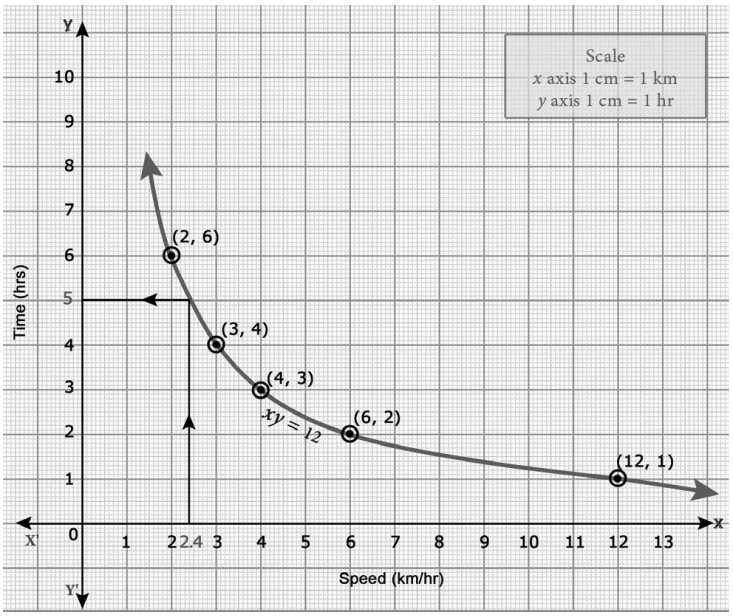
4. Points:
(30, 200), (40, 150) (50, 120) (60, 100), (75, 80)

5. Solution:
From the graph,
(i) Type of variation = **Indirect variation**
(ii) The required number of days to complete the work when the company decides to work with 120 workers = **50 days.**
(iii) If the work has to be completed by 200 days, the number of workers required = **30 workers**

4. Nishanth is the winner in a Marathon race of 12 km distance. He ran at the uniform speed of 12 km/hr and reached the destination in 1 hour. He was followed by Aradhana, Ponmozhi, Jeyanth, Sathya and Swetha with their respective speed of 6 km/hr, 4 km/hr, 3 km/hr and 2 km/hr. And, they covered the distance in 2 hrs, 3 hrs, 4 hrs and 6 hours respectively. Draw the speed-time graph and use it to find the time taken to Kaushik with his speed of 2.4 km/hr.

Solution: **1. Table:**

Speed x (km/hr)	12	6	4	3	2
Time y (hours)	1	2	3	4	6



2. Variation:
Indirect Variation

3. Equation
$xy = k$
$xy = 12 \times 1 = 12$
$xy = 12$

4. Points:
(12, 1), (6, 2), (4, 3), (3, 4), (2, 6)

5. Solution:
From the graph, The time taken by Kaushik to go at a speed of 2.4 km/hr = **5 hours.**

5. A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount. Hence find
(i) the marked price when a customer gets a discount of ₹ 3250 (from graph)
(ii) the discount when the marked price is ₹ 2500

Solution: 1. Table (Given)

Marked Price ₹ (x)	1000	2000	3000	4000	5000	6000	7000
Discounted Price ₹ (y)	500	1000	1500	2000	2500	3000	3500

2. Variation:
Direct variation.

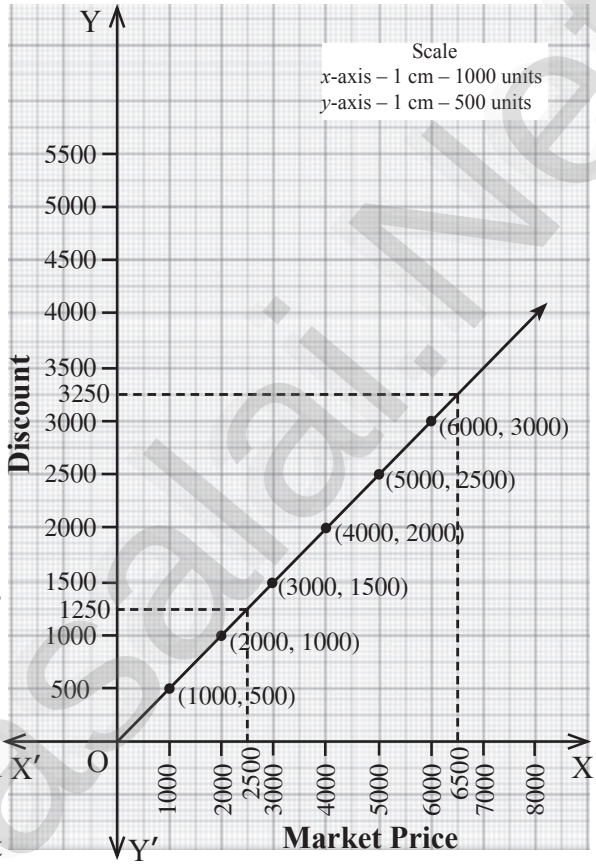
3. Equation:
$y = kx$

$k = \frac{y}{x} = \frac{500}{1000} = \frac{1}{2}$

$y = \frac{1}{2}x$

4. Points:
(1000, 500), (2000, 1000), (3000, 1500), (4000, 2000), (5000, 2500), (6000, 3000), (7000, 3500)

5. Solution:
From the graph,
(i) If the customer gets a discount of ₹ 3250, then the Marked price = ₹ 6500
(ii) If the marked price is ₹ 2500, then the discount = **₹ 1250**



6. Draw the graph of $xy = 24$, x, $y > 0$. Using the graph find,
(i) y when $x = 3$ and (ii) x when $y = 6$.

Solution: 1. Table:

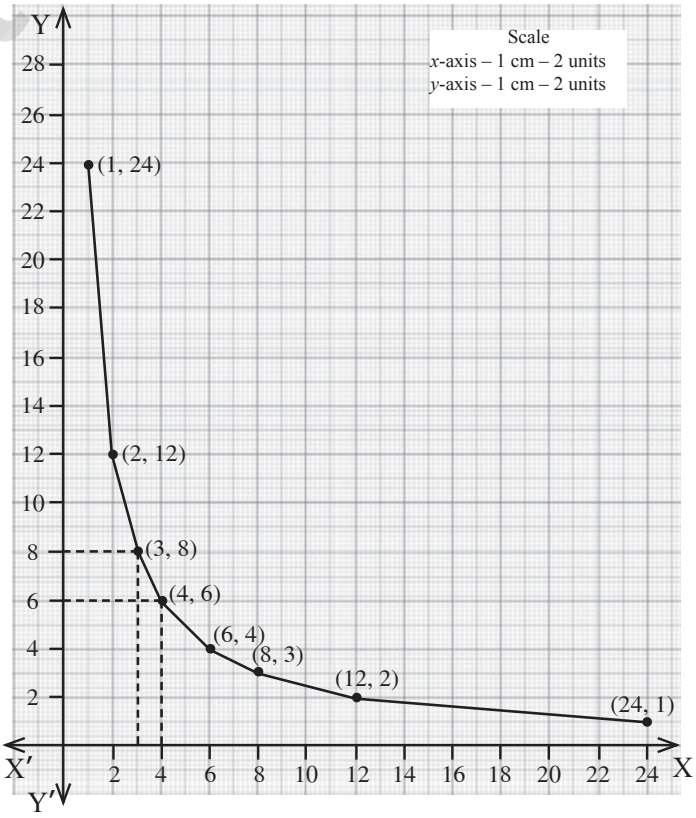
x	12	8	6	4	3	2
y	2	3	4	6	8	12

2. Variation:
Indirect variation.

3. Equation:
$xy = k$
$xy = 12 \times 2 = 24$
$xy = 24$

4. Points:
(12, 2), (8, 3), (6, 4), (4, 6), (3, 8), (2, 12)

5. Solution:
From the graph,
(i) If $x = 3$ then $y = 8$
(ii) If $y = 6$ then, $x = 4$



7. Graph the following linear function y =1/2 *x*. Identify the constant of variation and verify it with the graph. Also (i) find y when *x* = 9 (ii) find *x* when y = 7.5.

Solution: 1. Table:

x	2	4	6	8	10	12	14	16
y	1	2	3	4	5	6	7	8

2. Variation:
Direct Variation

3. Equation:

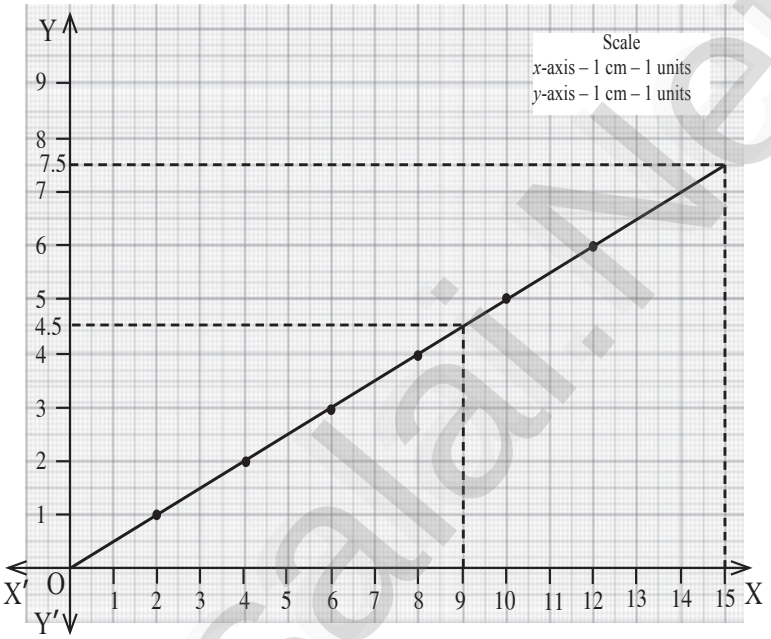
$y = kx$

$k = \frac{y}{x} = \frac{1}{2}$

$y = \frac{1}{2}x$

4. Points:
(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7), (16, 8)

5. Solution:
From the graph,
(i) If x = 9 then **y = 4.5**
(ii) If y = 7.5 then **x = 15**



8. The following table shows the data about the number of pipes and the time taken to till the same tank.

No. of pipes (*x*)	**2**	**3**	**6**	**9**
Time Taken (in min) (y)	**45**	**30**	**15**	**10**

Draw the graph for the above data and hence
(i) find the time taken to fill the tank when five pipes are used
(ii) find the number of pipes when the time is 9 minutes.

Solution:

1. Table:

No. of pipes (*x*)	2	3	5	6	9	10
Time Taken (y) (mins)	45	30	18	15	10	9

2. Variation:
Indirect variation

3. Equation:

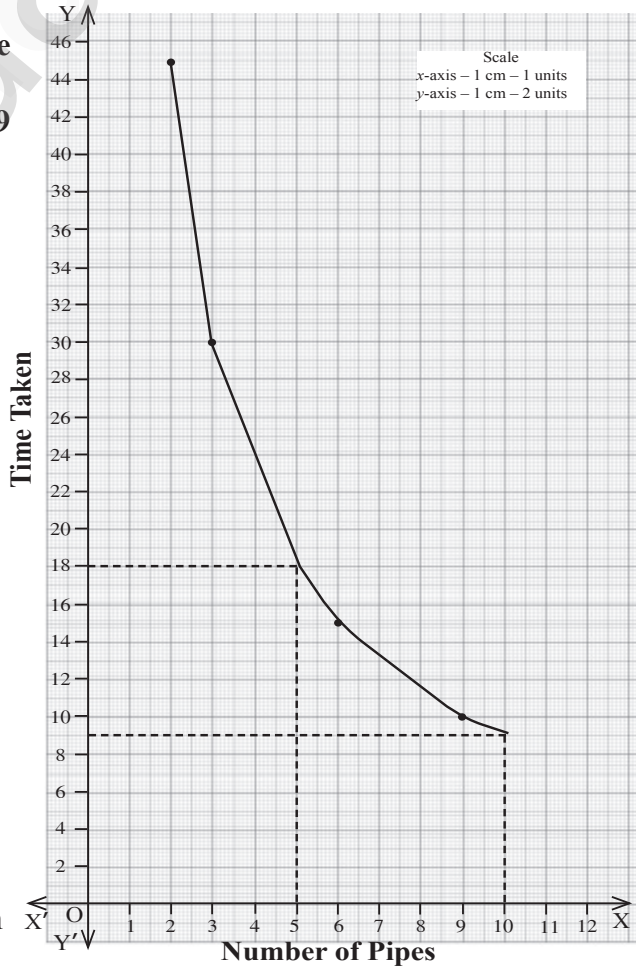
$xy = k$

$xy = 2 \times 45 = 90$

$xy = 90$

3. Points:
(2, 45), (3, 30), (5, 18), (6, 15), (9, 10), (10, 9)

4. Solution:
From the graph,
(i) Time taken to fill the tank if using 5 pipes = **18 minutes**
(ii) Number of pipes used if the tank fills up in 9 minutes = **10 pipes**



9. A school announces that for a certain competitions, the cash price will be distributed for all the participants equally as show below

No. of participants (x)	2	4	6	8	10
Amount for each participant in ₹ (y)	180	90	60	45	36

(i) Find the constant of variation.

(ii) Graph the above data and hence, find how much will each participant get if the number of participants are 12.

Solution: **1. Table:**

No. of participants (x)	2	4	6	8	10	12
Amount for each participant in ₹ (y)	180	90	60	45	36	30

2. Variation:

Indirect variation.

3. Equation:

$xy = k$

$xy = 2 \times 180 = 360$

$xy = 360$

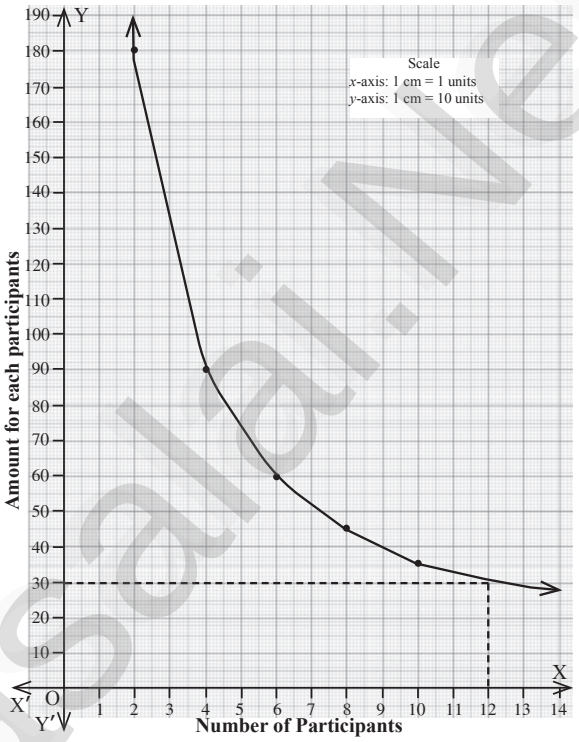
4. Points:

(2, 180), (4, 90), (6, 60), (8, 45), (10, 36), (12, 30)

5. Solution:

(i) Constant of Variation: **k = 360**

(ii) Cash Price each participant will get if 12 participants participate = **Rs. 30**



10. A two wheeler parking zone near bus stand charges as below.

Time (in hours) (x)	4	8	12	24
Amount ₹ (y)	60	120	180	360

Check if the amount charged are in direct variation or in inverse variation to the parking time. Graph the data. Also (i) find the amount to be paid when parking time is 6 hr; (ii) find the parking duration when the amount paid is ₹150.

Solution: **I. Table:**

Time (in hours) (x)	4	6	8	10	12	24
Amount in ₹ (y)	60	90	120	150	180	360

2. Variation:

Direct variation.

3. Equation:

$y = kx$,

$k = \frac{y}{x} = \frac{60}{4} = 15$

$y = 15x$

4. Points:

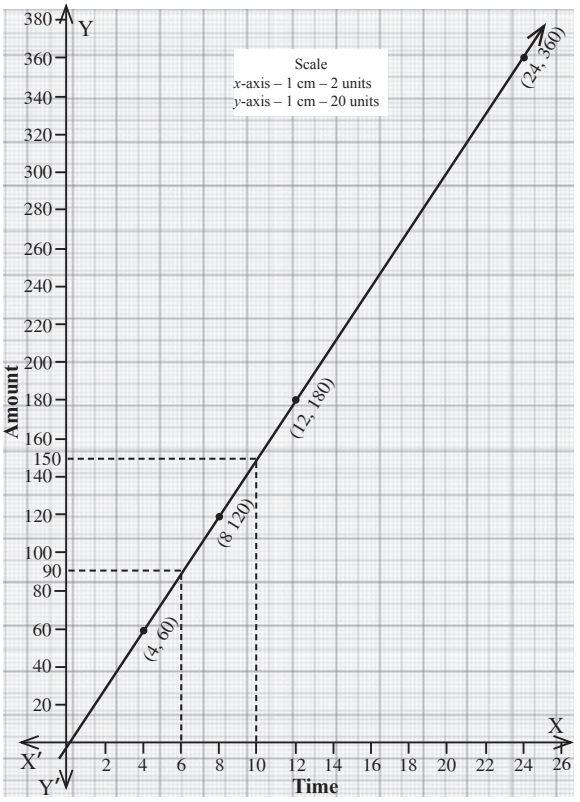
(4, 60), (6, 90), (8, 120), (10, 150), (12, 180), (24, 320)

5. Solution:

From the graph,

(i) If the parking time is 6 hours, then the parking charge = **₹ 90.**

(ii) If the amount ₹ 150 is paid, then the Parking time = **10 hours.**



1. Relations and Functions

2 Marks

1. If A×B = {(3, 2), (3, 4), (5, 2), (5, 4)} then find A and B. SEP-20

Solution:

A×B = {(3, 2), (3, 4), (5, 2), (5, 4)} then

A = {Set of all first coordinates of elements of A × B} ∴ A = {3, 5}

B = {Set of all second coordinates of elements of A × B} ∴ B = {2, 4}

Thus A = {3, 5} and B = {2, 4}

2. Find A × B, A × A and B × A

i) A={2, −2, 3} and B = {1,−4}

ii) A = B = {p, q}

iii) A = {m, n} ; B = f

Solution:

i. A × B = {2, –2, 3} × {1, –4}
= {(2, 1), (2, –4), (–2, 1), (–2, –4), (3, 1), (3, –4)}

A × A = {2, –2, 3} × {2, –2, 3}
= {(2, 2), (2, –2), (2, 3), (–2, 2), (–2, –2), (–2, 3), (3, 2), (3, –2), (3, 3)}

B × A = {1, –4} ×{2, –2, 3}
= {(1, 2), (1, –2), (1, 3), (–4, 2), (–4, –2), (–4, 3)}

ii. Given A = B = {p, q}

A × B = {p, q} × {p, q}
= {(p, p), (p, q), (q, p),(q ,q)}

A × A = {p, q} × p, q}
= {(p, p), (p, q), (q, p),(q ,q)}

B × A = {p, q} × {p, q}
= {(p, p), (p, q), (q, p),(q ,q)}

iii. A = {m, n}, B = φ

A × B = {(m, n) × { } = { }

A × A = {(m, n) } × {m, n}
= {(m, m), (m, n), (n, m), (n,n)}

B × A = { } × {m, n} = { }

3. Let A = {1, 2, 3} and B = {x | x is a prime number less than 10}. Find A × B and B × A. MAY-22

Solution:

A = {1, 2, 3} B = {2, 3, 5, 7}

A × B = {1, 2, 3} × {2, 3, 5, 7}
= {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)}

B × A = {2, 3, 5, 7} × {1, 2, 3}
= {(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)}

4. If B × A = {(−2, 3), (−2, 4), (0, 3), (0, 4), (3, 3), (3, 4)} find A and B.

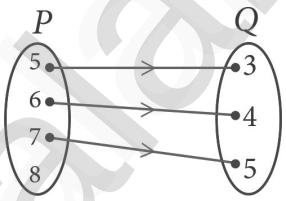
Solution:

A = {Set of all second coordinates of elements of B × A} ∴ A = {3, 4}

B = {Set of all first coordinates of elements of B × A} ∴ B = {–2, 0, 3}

Thus, A = {3, 4} B = {–2, 0, 3}

5. The arrow diagram shows a relationship between the sets P and Q. Write the relation in



(i) Set builder form (ii) Roster form

(iii) What is the domain and range of R.

Solution:

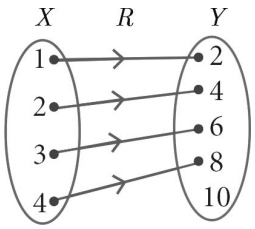
i. Set builder form of R = {$(x, y) \mid y = x - 2, x \in P, y \in Q$}

ii. Roster form R = {(5, 3), (6, 4), (7, 5)}

iii. Domain of R = {5, 6, 7} and range of R = {3, 4, 5}

6. Let X = {1, 2, 3, 4} and Y = {2, 4, 6, 8, 10} and R = {(1, 2), (2, 4), (3, 6), (4, 8)}. Show that R is a function and find its domain, co-domain and range?

Solution:



Pictorial representation of R is given diagram,

From the diagram, we see that for each $x \in X$, there exists only one $y \in Y$.

Thus all elements in X have only one image in Y.

Therefor R is a function.

Domain X = {1, 2, 3, 4}

Co-domain Y = { 2, 4, 6, 8, 10}

Range of f ={2, 4, 6, 8}

7. Let A = {1, 2, 3, 4, ..., 45} and R be the relation defined as "is square of a number" on A. Write R as a subset of A×A. Also, find the domain and range of R. SEP-21

Solution:

A = {1, 2, 3, …., 45}

R = {(1, 1), (2, 4), (3, 9), (4, 16), (5, 25), (6, 36)}

$R \subset (A \times A)$

∴ Domain of R = {1, 2, 3, 4, 5, 6}

Range of R = {1, 4, 9, 16, 25, 36}

8. A Relation R is given by the set $\{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range.

Solution:

$x = \{0, 1, 2, 3, 4, 5\}$

$f(x) = y = x + 3$

$f(0) = 3; \quad f(1) = 4; \quad f(2) = 5;$

$f(3) = 6; \quad f(4) = 7; \quad f(5) = 8$

∴ R = {(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)}

Domain of R = {0, 1, 2, 3, 4, 5}

Range of R = {3, 4, 5, 6, 7, 8}

9. Given the function f: $x \to x^2 - 5x + 6$, evaluate (i) $f(-1)$ (ii) $f(2a)$ (iii) $f(2)$ (iv) $f(x-1)$

Solution:

Given: f: $x \to x^2 - 5x + 6$

$\Rightarrow f(x) = x^2 - 5x + 6$

i. $f(-1) = (-1)^2 - 5(-1) + 6$

$= 1 + 5 + 6$

$= 12$

ii. $f(2a) = (2a)^2 - 5(2a) + 6$

$= 4a^2 - 10a + 6$

iii. $f(2) = (2)^2 - 5(2) + 6$

$= 4 - 10 + 6$

$= 0$

iv. $f(x-1) = (x-1)^2 - 5(x-1) + 6$

$= x^2 - 2x + 1 - 5x + 5 + 6$

$= x^2 - 7x + 12$

10. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.

Solution:

$f(x) = 3 - 2x$

$f(x^2) = [f(x)]^2$

$3 - 2x^2 = [3 - 2x]^2$

$\Rightarrow \quad 3 - 2x^2 = 9 + 4x^2 - 12x$

$3 - 2x^2 - 9 - 4x^2 + 12x = 0$

$\Rightarrow \quad -6x^2 + 12x - 6 = 0 \div -6$

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0 \quad x = 1, 1$

11. Let A = {1, 2, 3, 4} and B = N. Let f : A→ B be defined by $f(x) = x^3$ then, (i) find the range of f (ii) identify the type of function.

Solution:

$A = \{1, 2, 3, 4\}, B = N$

$f : A \to B, f(x) = x^3$

$f(1) = (1)^3 = 1; \quad f(2) = (2)^3 = 8;$

$f(3) = (3)^3 = 27; \quad f(4) = (4)^3 = 64$

i) Range of f = {1, 8, 27, 64}

ii) It is one-one and into function.

5 Marks

1. If A = {1, 3, 5} and B = {2, 3} then

(i) find A×B and B×A.

(ii) Is A×B = B×A? If not why?

(iii) Show that n(A×B) = n(B×A) = n(A)× n(B) SEP-21

Solution:

Given that A = {1, 3, 5} and B = {2, 3}

i. $A \times B = \{1, 3, 5\} \times \{2 \times 3\}$

= {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)} .....(1)

$B \times A = \{2 \times 3\} \times \{1, 3, 5\}$

= {(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)} .....(2)

ii. From (1) and (2) we conclude that $A \times B \neq B \times A$ as (1, 2) ≠ (2, 1) and (1, 3) ≠ (3, 1) etc

iii. n(A) = 3; n(B) = 2

From (1) and (2) we observe that,

$n(A \times B) = n(B \times A) = 6;$

We see that, $n(A) \times n(B) = 3 \times 2 = 6$

Thus, $n(A \times B) = n(B \times A) = n(A) \times n(B)$.

2. Let A = {$x \in N \mid 1 < x < 4$}, B = {$x \in W \mid 0 \le x < 2$} and C = {$x \in N \mid x < 3$}. Then verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Solution:

Given $A = \{x \in N \mid 1 < x < 4\} = \{2, 3\}$,

$B = \{x \in W \mid 0 \le x < 2\} = \{0, 1\}$,

$C = \{x \in N \mid x < 3\} = \{1, 2\}$

i. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$B \cup C = \{0,1\} \cup \{1, 2\} = \{0, 1, 2\}$

$A \times (B \cup C)$

$= \{2, 3\} \times \{0, 1, 2\}$

$= \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$ ..... (1)

$A \times B = \{2, 3\} \times \{0, 1\}$

$= (2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\}$

$= \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cup (A \times C)$

$= \{(2,0), (2,1), (3, 0), (3,1)\} \cup \{(2,1), (2, 2), (3, 1), (3, 2)\}$

$= \{(2, 0), (2, 1), (2, 2), (3,0), (3, 1), (3, 2)\}$ ..... (2)

From (1) = (2).

$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$ is verified.

ii. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$B \cap C = \{0, 1\} \cap \{1, 2\} = \{1\}$

$A \times (B \cap C) = \{2, 3\} \times \{1\}$

$= \{(2, 1), (3, 1)\}$ ..... (1)

$A \times B = \{2, 3\} \times \{0, 1\}$

$= \{(2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\}$

$= \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cap (A \times C)$

$= \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cap \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$= \{(2, 1), (3, 1)\}$ ..... (2)

(1) = (2)

$\therefore A \times (B \cap C) = (A \times B) \cap (A \times C)$

Hence it is Verified

3. If A = {5, 6}, B = {4, 5, 6}, C = {5, 6, 7}, Show that $A \times A = (B \times B) \cap (C \times C)$.

Solution:

Given A = {5, 6}, B = {4, 5, 6}, C = {5, 6, 7}

LHS:

$A \times A = \{5, 6\} \times \{5, 6\}$

$= \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ..... (1)

RHS = $(B \times B) \cap (C \times C)$.

$B \times B = \{4, 5, 6\} \times \{4, 5, 6\}$

$= \{(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$

$C \times C = \{5, 6, 7\} \times \{5, 6, 7\}$

$= \{(5, 5), (5, 6), (5, 7), (6, 5), (6, 6), (6, 7), (7, 5), (7, 6), (7, 7)\}$

$\therefore (B \times B) \cap (C \times C)$

$= \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ..... (2)

$\therefore$ From (1) and (2). LHS = RHS

4. Given A = {1, 2, 3}, B = {2, 3, 5}, C = {3, 4} and D = {1, 3, 5}, check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?

Solution:

$A \cap C = \{1, 2, 3\} \cap \{3, 4\}$

$A \cap C = \{3\}$,

$B \cap D = \{2, 3, 5\} \cap \{1, 3, 5\}$

$B \cap D = \{3, 5\}$

$(A \cap C) \times (B \cap D)$

$= \{3\} \times \{3, 5\} = \{(3, 3), (3, 5)\}$ ..... (1)

$A \times B = \{1, 2, 3\} \times \{2, 3, 5\}$

$= \{(1, 2), (1, 3), (1,5), (2, 2), (2, 3), (2, 5),(3, 2), (3, 3), (3, 5)\}$

$C \times D = \{3, 4\} \times \{1, 3, 5\}$

$= \{(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)\}$

$(A \times B) \cap (C \times D) = \{(3, 3), (3, 5)\}$ ..... (2)

(1), (2) are equal.

$\therefore (A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$

Hence it is verified.

5. Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \leq 4\}$ and C = {3, 5}. Verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ SEP-21

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Solution:

Given:

$A = \{x \in W / x < 2\} \Rightarrow A = \{0, 1\}$

$B = \{x \in N / 1 < x \leq 4\}$

$\Rightarrow B = \{2, 3, 4\}$; $C = \{3, 5\}$

i. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$B \cup C = \{2, 3, 4\} \cup \{3, 5\}$

$B \cup C = \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\}$

$= \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ..... (1)

$A \times B = \{0, 1\} \times \{2, 3, 4\}$

$= \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$A \times C = \{0, 1\} \times \{3, 5\}$

$= \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$\therefore (A \times B) \cup (A \times C)$

$= \{(0, 2), (0, 3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\}$ ..... (2)

$\therefore$(1) = (2) Hence Verified.

ii. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$B \cap C = \{2, 3, 4\} \cap \{3, 5\} = \{3\}$

$A \times (B \cap C) = \{(0, 3), (1, 3)\}$ ..... (1)

$A \times B = \{0, 1\} \times \{2, 3, 4\}$

$= \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\}$

$A \times C = \{0,1\} \times \{3, 5\}$

$= \{(0,3), (0,5), (1,3), (1,5)\}$

$\therefore (A \times B) \cap (A \times C) = \{(0, 3), (1, 3)\}$ ..... (2)

$\therefore$ (1) = (2). Hence Proved.

iii. $(A \cup B) \times C = (A \times C) \cup (B \times C)$

$A \cup B = \{0, 1\} \cup \{2, 3, 4\}$

$= \{0, 1, 2, 3, 4\}$

$\therefore (A \cup B) \times C = \{0, 1, 2, 3, 4\} \times \{3, 5\}$

$= \{(0,3), (0,5), (1,3), (1,5), (2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\}$ ..... (1)

$A \times C = \{0, 1\} \times \{3, 5\}$

$= \{(0,3), (0,5), (1,3), (1,5)\}$

$B \times C = \{2, 3, 4\} \times \{3, 5\}$

$= \{(2,3), (2,5), (3,3), (3,5), (4,3), (4,5)\}$

$\therefore (A \times C) \cup (B \times C)$

$= \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$ ..... (2)

$\therefore$ From (1) and (2) LHS = RHS.

6. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$ SEP-20

(ii) $A \times (B - C) = (A \times B) - (A \times C)$ MAY-22

Solution:

Given $A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 3, 5, 7\}$ $C = \{2\}$

To verify $(A \cap B) \times C = (A \times C) \cap (B \times C)$

$A \cap B = \{1, 2, 3, 4, 5, 6, 7\} \cap \{2, 3, 5, 7\}$

$= \{2, 3, 5, 7\}$

$(A \cap B) \times C = \{2, 3, 5, 7\} \times \{2\}$

$\therefore (A \cap B) \times C = \{(2,2), (3,2), (5,2), (7,2)\}$ .....(1)

$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\}$

$= \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$

$B \times C = \{2, 3, 5, 7\} \times \{2\}$

$= \{(2,2), (3,2), (5,2), (7,2)\}$

$(A \times C) \cap (B \times C)$

$= \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ..... (2)

$\therefore$ From (1) and (2), LHS = RHS

ii. To verify $A \times (B - C) = (A \times B) - (A \times C)$

$B - C = \{2, 3, 5, 7\} - \{2\} = \{3, 5, 7\}$

$A \times (B - C) = \{1, 2, 3, 4, 5, 6, 7\} \times \{3, 5, 7\}$

$= \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3, 7), (4,3), (4,5), (4,7), (5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)\}$ ..... (1)

$A \times B = \{1, 2, 3, 4, 5, 6, 7\} \times \{2, 3, 5, 7\}$

$= \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7), (4,2), (4,3), (4,5), (4,7), (5,2), (5,3), (5,5), (5,7), (6,2), (6,3), (6,5),(6,7), (7,2), (7,3), (7,5), (7,7)\}$

$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\}$

$= \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$

$(A \times B) - (A \times C)$

$= \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7), (4,3), (4,5), (4,7), (5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)\}$ ..... (2)

(1), (2) are equal.

$\therefore A \times (B - C) = (A \times B) - (A \times C)$.

Hence it is verified.

7. Let A = {3, 4, 7, 8} and B = {1, 7, 10}. Which of the following sets are relations from A to B?

(i) $R_1 = \{(3, 7), (4, 7), (7, 10), (8, 1)\}$

(ii) $R_2 = \{(3, 1), (4, 12)\}$

(iii) $R_3 = \{(3, 7), (4, 10), (7, 7), (7, 8), (8, 11), (8, 7), (8, 10)\}$

Solution:

$A \times B = \{(3,1), (3,7), (3,10), (4,1), (4,7), (4,10), (7,1), (7,7), (7,10), (8,1), (8,7), (8,10)\}$

i. We note that, $R_1 \subseteq A \times B$.

Thus R_1 is a relation from A and B.

ii. Here $(4, 12) \in R_2$, but $(4, 12) \notin A \times B$.

So R_2 is not a relation from A to B.

iii. Here $(7, 8) \in R_3$, but $(7, 8) \notin A \times B$.

So R_3 is not a relation from A to B.

8. Let A = {1, 2, 3, 7} and B = {3, 0, –1, 7}, which of the following are relation from A to B ?

(i) $R_1 = \{(2, 1), (7, 1)\}$ (ii) $R_2 = \{(–1, 1)\}$

(iii) $R_3 = \{(2, –1), (7, 7), (1, 3)\}$

(iv) $R_4 = \{(7, –1), (0, 3), (3, 3), (0, 7)\}$

Solution:

Given A = {1, 2, 3, 7} and B = {3, 0, –1, 7}

$\therefore A \times B$

= {1, 2, 3, 7} × {3, 0, –1, 7}

= {(1, 3),(1, 0), (1, –1), (1, 7), (2, 3), (2, 0), (2, –1), (2, 7), (3, 3), (3, 0), (3, –1), (3, 7), (7, 3), (7, 0), (7, –1), (7, 7)}

i. $R_1 = \{(2, 1), (7, 1)\}$, $(2, 1) \in R_1$
but $(2, 1) \notin A \times B$
$\therefore R_1$ is not a relation from A to B.

ii. $R_2 = \{(–1, 1)\}$, $(–1, 1) \in R_2$
but $(–1, 1) \notin A \times B$
$\therefore R_2$ is not a relation from A to B.

iii. $R_3 = \{(2, –1), (7, 7), (1, 3)\}$
We note that $R_3 \subseteq A \times B$
$\therefore R_3$ is a relation.

iv. $R_4 = \{(7, –1), (0, 3), (3, 3), (0, 7)\}$, (0, 3), (0, 7) $\in R_4$ but not in $A \times B$.
$\therefore R_4$ is not a relation from A to B.

9. Represent each of the given relations by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

(i) $\{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}$

(ii) $\{(x, y) | y = x + 3$, x, y are natural numbers < 10$\}$

Solution:

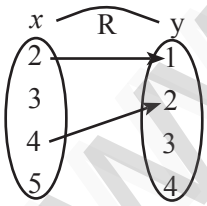
i. $\{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}$

$x = 2y$

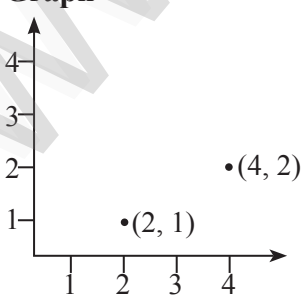
$f(x) = \frac{x}{2}$; $f(2) = \frac{2}{2} = 1$; $f(3) = \frac{3}{2}$;

$f(4) = \frac{4}{2} = 2$; $f(5) = \frac{5}{2}$

a) An Arrow diagram



b) Graph



c) Roster Form

{(2, 1), (4, 2)}

ii. **$\{(x, y) | y = x + 3$, x, y are natural numbers < 10$\}$**

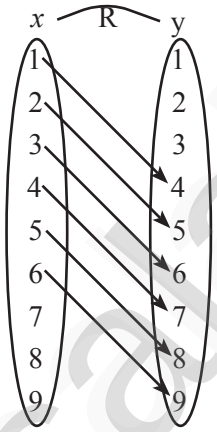
Solution:

$f(x) = x + 3$;

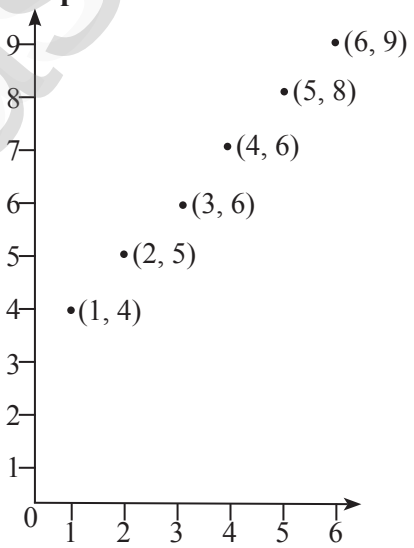
$f(1) = 4$; $f(2) = 5$; $f(3) = 6$;

$f(4) = 7$; $f(5) = 8$; $f(6) = 9$

a) An Arrow diagram



b) Graph



c) Roster Form

(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

10. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹ 10,000, ₹ 25,000, ₹ 50,000 and ₹ 1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation R is defined by xRy, where x is the salary given to

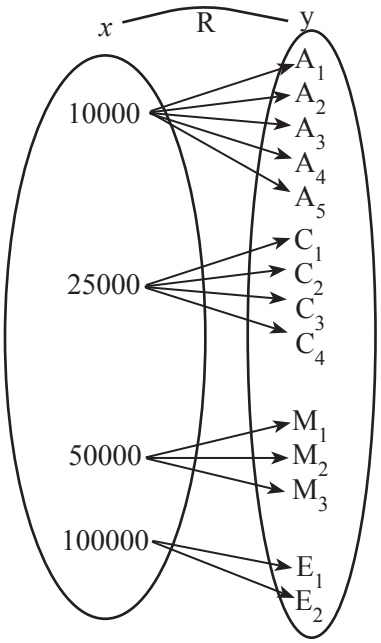
person y, express the relation R through an ordered pair and an arrow diagram.

Solution:

a) Ordered Pair:

$\{(10000, A_1), (10000, A_2), (10000, A_3), (10000, A_4), (10000, A_5)\ (25000, C_1), (25000, C_2), (25000, C_3), (25000, C_4), (50000, M_1), (50000, M_2), (50000, C_3)\ (100000, E_1), (100000, E_2)\}$

b) Arrow Diagram:



11. Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let f : A → B be a function given by f(x) = 3x − 1. Represent this function

(i) by arrow diagram

(ii) in a table form

(iii) as a set of ordered pairs

(iv) in a graphical form SEP-20

Solution:

A = {1, 2, 3, 4}, B = {2, 5, 8, 11, 14}

f(x) = 3x – 1

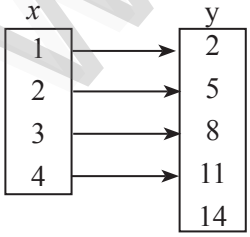
f(1) = 3(1) – 1 = 3 – 1 = 2;

f(2) = 3(2) – 1 = 6 – 1 = 5 f(3) = 3(3) – 1 = 9 – 1 = 8;

f(4) = 3(4) – 1 = 12 – 1 = 11.

R = {(1, 2), (2, 5), (3, 8), (4, 11)}

i) Arrow Diagram



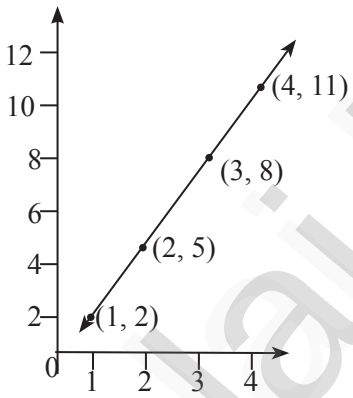
ii) Table

x	1	2	3	4
y	2	5	8	11

iii) Set of Ordered pairs

{(1, 2), (2, 5), (3, 8), (4, 11)}

iv) Graphical Form



12. Let f be a function f : N → N be defined by $f(x) = 3x + 2, x \in N$

(i) Find the images of 1, 2, 3

(ii) Find the pre-images of 29, 53

(iii) Identify the type of function

Solution:

The function f : N→ N be defined by

f(x) = 3x + 2

i. If x = 1, f(1) = 3(1) + 2 = 5

If x = 2, f(2) = 3(2) + 2 = 8; If x = 3, f(3) = 3(3) + 2 = 11

The images of 1, 2, 3 are 5, 8, 11 respectively.

ii. If x is the pre-image of 29, then f(x) = 29.

Hence 3x + 2 = 29 ; 3x = 27 ⇒ x = 9.

Similarly, if x is the pre-image of 53 then f(x) = 53. Hence 3x + 2 = 53

3x = 53 – 2 ⇒ 3x = 51 ⇒ x = 17.

Thus the pre-image of 29 and 53 are 9 and 17 respectively.

iii. Since different elements of N have different images in the co-domain, the function f is one-one function. The co-domain of f is N. But the range of f = {5, 8, 11, 14, 17,….}is a proper subset of N . Therefore f is not an onto function. That is, f is an into function. Thus f is one-one and into functions.

13. Let f : A → B be a function defined by f(x) = $\frac{x}{2}$ − 1 where A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9} . Represent f by

i) set of ordered pairs ii) a table
iii) an arrow diagram iv) a graph

Solution:

Given $f(x) = \frac{x}{2} - 1$

$x = 2 \Rightarrow f(2) = 1 - 1 = 0$

$x = 4 \Rightarrow f(4) = 2 - 1 = 1$

$x = 6 \Rightarrow f(6) = 3 - 1 = 2$

$x = 10 \Rightarrow f(10) = 5 - 1 = 4$

$x = 12 \Rightarrow f(12) = 6 - 1 = 5$

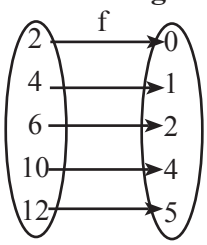
i) Set of Ordered Pairs:

$f = \{(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)\}$

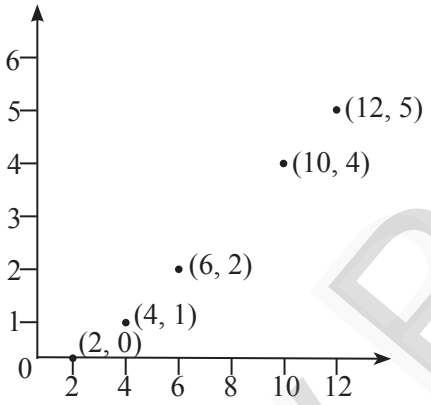
ii) Table

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

iii) Arrow Diagram



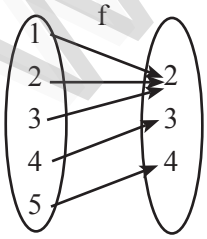
iv) Graph



14. Represent the function
f = {(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)} through
(i) an arrow diagram
(ii) a table form
(iii) a graph

Solution:

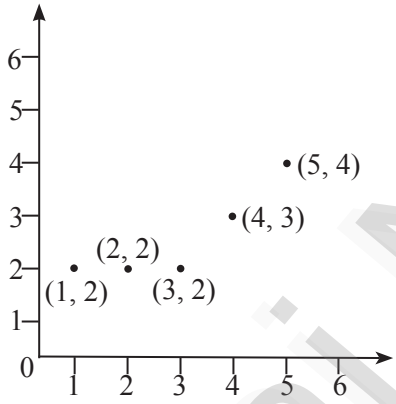
i) Arrow Diagram



ii) Table Form:

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

iii) Graph



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2. Numbers and Sequences

2 Marks

1. 'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

Solution:

$800 = a^b \times b^a$

2	800
2	400
2	200
2	100
2	50
5	25
	5

$800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$= 2^5 \times 5^2$

$\therefore a = 2, b = 5$ (or) $a = 5, b = 2$

2. Find the HCF of 252525 and 363636.

Solution:

2	363636
2	181818
3	90909
3	30303
3	10101
7	3367
13	481
37	37
	1

5	252525
5	50505
3	10101
7	3367
13	481
37	37
	1

$252525 = 3 \times 5^2 \times 7 \times 13 \times 37$

$363636 = 2^3 \times 3^3 \times 7 \times 13 \times 37$

H.C.F of 252525 and 363636

$= 3 \times 7 \times 13 \times 37$

$= 10101.$

3. If $13824 = 2^a \times 3^b$ then find a and b. MAY-22

Solution:

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

$\Rightarrow 13824 = 2^9 \times 3^3$

$\therefore a = 9, b = 3$

4. Find the LCM and HCF of 408 and 170 by applying the fundamental theorem of arithmetic.

Solution:

2	408	2	170
2	204	5	85
2	102		17
3	51		
	17		

$408 = 2^3 \times 3 \times 17$

$170 = 2 \times 5 \times 17$

H.C.F. of 408 & 170 $= 2 \times 17 = 34$

L.C.M. of 408 & 170 $= 2^3 \times 3 \times 5 \times 17$

$= 2040$

5. The general term of a sequence is defined as

$$a_n = \begin{cases} n(n+3);\ n \in N \text{ is odd} \\ n^2+1\ ;\ n \in N \text{ is even} \end{cases}$$

Find the eleventh and eighteenth terms.

Solution:

To find a_{11}, since 11 is odd,

we put n = 11 in

$a_n = n(n + 3)$

Thus,

the eleventh term $a_{11} = 11(11 + 3) = 154.$

To find a_{18}, since 18 is even,

we put n = 18 in

$a_n = n^2 + 1$

Thus, the eighteenth term $a_{18} = 18^2 + 1 = 325.$

6. Find the indicated terms of the sequences whose n^{th} terms are given by

(i) $a_n = \frac{5n}{n+2}$; a_6 and a_{13}

(ii) $a_n = -(n^2 - 4)$; a_4 and a_{11}

Solution:

i. $a_n = \frac{5n}{n+2}$

$a_6 = \frac{30}{8} = \frac{15}{4}$; $a_{13} = \frac{65}{15} = \frac{13}{3}$

ii. $a_n = -(n^2 - 4)$

$a_4 = -(16 - 4) = -12;$

$a_{11} = -(121 - 4) = -117$

7. Find a_8 and a_{15} whose nth term is

$$a_n = \begin{cases} \frac{n^2-1}{n+3} & ;n \text{ is even},\ n \in N \\ \frac{n^2}{2n+1} & ;n \text{ is odd},\ n \in N \end{cases}$$

Solution:

To find a_8 here n is even, so $a_n = \frac{n^2-1}{n+3}$

$a_8 = \frac{64-1}{11} = \frac{63}{11}$

To find a_{15}, here n is odd, so $a_n = \frac{n^2}{2n+1}$

$a_{15} = \frac{(15)^2}{30+1} = \frac{225}{31}$

8. Find the 19th term of an A.P. –11, –15, –19, ...

Solution:

General Form of an A.P. is $t_n = a + (n-1)d$

$a = -11$; $d = -15+11 = -4$; $n = 19$

$t_{19} = -11 + 18(-4)$

$= -11 - 72$

$t_{19} = -83$

9. Which term of an A.P. 16, 11, 6, 1,... is –54 ? MAY-22

Solution:

$n = \left(\frac{l-a}{d}\right) + 1$

$a = 16$; $d = 11 - 16 = -5$; $l = -54$

$n = \frac{-54-16}{-5} + 1 = \frac{-70}{-5} + 1$

$n = 14 + 1$

$n = 15$

10. Find the middle term(s) of an A.P. 9, 15, 21, 27, …, 183.

Solution:

$a = 9,\ d = 6,\ l = 183$

$$n = \left(\frac{l-a}{d}\right) + 1$$

$$= \frac{183-9}{6} + 1 = \frac{174}{6} + 1 = 29 + 1 = 30$$

$\therefore$ 15 and 16 are the middle terms.

$t_n = a + (n-1)d$

$\therefore t_{15} = a + 14d$ $\quad t_{16} = a + 15d$

$= 9 + 14(6)$ $\quad = 9 + 15(6)$

$= 9 + 84$ $\quad = 9 + 90$

$= 93$ $\quad = 99$

$\therefore$ 93, 99 are the middle terms of A.P.

11. If 3 + k, 18 – k, 5k + 1 are in A.P. then find k. SEP-21

Solution:

3 + k, 18 – k, 5k + 1 is a A.P

$$t_2 - t_1 = t_3 - t_2$$

$$(18-k) - (3+k) = (5k+1) - (18-k)$$

$$15 - 2k = 6k - 17$$

$$-2k - 6k = -17 - 15$$

$$-8k = -32$$

$$k = 4$$

12. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?

Solution:

First Term, a = 20

Common Difference, d = 2

$\therefore$ Number of seats in the last row

$= t_n = a + (n-1)d$

$t_{30} = a + 29d = 20 + 29(2) = 20 + 58 = 78$

13. Write an A.P. whose first term is 20 and common difference is 8.

Solution:

First Term, a = 20;

Common Difference, d = 8

Arithmetic Progression is a, a+d, a+3d, …

In this case,

we get 20, 20 + 8, 20 + 2(8), 20 + 3(8), ...

So, the required A.P. is 20, 28, 36, 44, ...

14. Find the number of terms in the A.P. 3, 6, 9, 12, …, 111. SEP-21

Solution:

First term a = 3,

Common difference d = 6 – 3 = 3,

Last term, $l = 111$

We know that, $n = \left(\frac{l-a}{d}\right) + 1$

$$n = \left(\frac{111-3}{3}\right) + 1 = 37$$

Thus the A.P. contains 37 terms.

15. Write the first three terms of the G.P. whose first term and the common ratio are given below.

(i) a = 6, r = 3 **(ii) $a = \sqrt{2}$, $r = \sqrt{2}$**

(iii) $a = 1000$, $r = \frac{2}{5}$

Solution:

i. General Form of an G.P. $\Rightarrow$ a, ar, ar^2, …..

a = 6, r = 3 G.P. $\Rightarrow$ 6, 6(3), $6(3)^2$…

$\Rightarrow$ 6, 18, 54, ….

ii. G.P. $\Rightarrow$ a, ar, ar^2, …

$a = \sqrt{2}$, $r = \sqrt{2}$

G.P. $\Rightarrow \sqrt{2}, \sqrt{2}\ \sqrt{2}, \sqrt{2}\ (\sqrt{2})^2$

$\Rightarrow \sqrt{2}, 2, 2\sqrt{2}$

iii. G.P. $\Rightarrow$ a, ar, ar^2, …..

$a = 1000,\ r = \frac{2}{5}$

G.P. $\Rightarrow 1000,\ 1000 \times \frac{2}{5},\ 1000 \times \left(\frac{2}{5}\right)^2$ …

G.P. $\Rightarrow$ 1000, 400, 160,….

16. In a G.P. 729, 243, 81, … find t_7.

Solution:

$t_n = ar^{n-1}$

$a = 729,\ r = \frac{243}{729} = \frac{1}{3},\ n = 7$

$$t_7 = 729 \times \left(\frac{1}{3}\right)^{7-1}$$

$$t_7 = 729 \times \left(\frac{1}{3}\right)^{6}$$

$$t_7 = 729 \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$$

17. Find x so that $x + 6$, $x + 12$ and $x + 15$ are consecutive terms of a Geometric Progression.

Solution:

Given $x + 6$, $x + 12$ and $x + 15$ are consecutive terms of a G.P.

$$\frac{t_2}{t_1} = \frac{t_3}{t_2}$$
$$\frac{x+12}{x+6} = \frac{x+15}{x+12}$$
$$(x+12)^2 = (x+6)(x+15)$$
$$x^2+24x+144 = x^2+21x+90$$
$$24x-21x = 90-144$$
$$3x = -54$$
$$x = -\frac{54}{3} = -18$$

18. Find the number of terms in the following G.P.

(i) 4, 8, 16, …, 8192?

(ii) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{2187}$

Solution:

i. G.P. $\Rightarrow$ 4, 8, 16, …, 8192.

Here $a = 4, r = 2, t_n = 8192$

$ar^{n-1} = t_n \Rightarrow 4(2)^{n-1} = 8192;$

$2^{n-1} = \frac{8192}{4} = 2048$

$2^{n-1} = 2^{11}$; $n-1 = 11$

$\Rightarrow n = 12$

ii. G.P. $\Rightarrow \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{2187}$.

Here $a = \frac{1}{3}, r = \frac{1}{3}, t_n = \frac{1}{2187}$

$$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} = \frac{1}{2187}$$
$$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{2187} \times 3$$
$$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{729} = \left(\frac{1}{3}\right)^6;$$
$$n-1 = 6 \Rightarrow n = 7$$

19. In a G.P. the 9^{th} term is 32805 and 6^{th} term is 1215. Find the 12^{th} term.

Solution:

From the given

$t_9 = 32805 \Rightarrow ar^8 = 32805$ ..... (1)

$t_6 = 1215 \Rightarrow ar^5 = 1215$ ..... (2)

(1) ÷ (2) $\Rightarrow$ $r^3 = 27 \Rightarrow r = 3$

(2) $\Rightarrow a(3)^5 = 1215 \Rightarrow a = 5$

To find t_{12},

$t_n = ar^{n-1}$

$t_{12} = (5)(3)^{11}$

20. Find the first term of a G.P. in which $S_6 = 4095$ and r = 4.

Solution:

Common ratio, = 4 > 1,

Sum of first 6 terms $S_6 = 4095$

Hence, $S_n = \frac{a(r^n-1)}{r-1} = 4095$

r = 4, $\frac{a(4^6-1)}{4-1} = 4095$

$\Rightarrow a \times \frac{4095}{3} = 4095$

First term, a = 3.

21. Find the value of

1 + 2 + 3 + ... + 50

Solution:

$1 + 2 + 3 + \ldots + 50$

Using $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

$1 + 2 + 3 + \ldots + 50 = \frac{50 \times (50+1)}{2} = 1275$

22. Find the sum of the following series

1 + 2 + 3 + ... + 60

Solution:

$$1 + 2 + 3 + \ldots + 60 = \frac{n(n+1)}{2}$$
$$= \frac{60 \times 61}{2}$$
$$= 30 \times 61 = 1830$$

23. Find the sum of

(i) 1 + 3 + 5 + ... to 40 terms

(ii) 2 + 4 + 6 + ... 80

(iii) 1 + 3 + 5 + ... + 55

Solution:

i. $1 + 3 + 5 + \ldots +$ n terms $= n^2$

$1 + 3 + 5 + \ldots + 40$ terms $= (40)^2 = 1640$

ii. $2 + 4 + 6 + \ldots + 80$

$= 2[1 + 2 + 3 + \ldots + 40]$

$= 2\left[\frac{n(n+1)}{2}\right] = 40 \times 41 = 1640$

iii. $1 + 3 + 5 + \ldots + 55$

Here the number of terms is not given.

Now, we have to find the number of terms using the formula.

$n = \frac{(55-1)}{2} + 1 = 28$

Therefore,

$1 + 3 + 5 + \ldots + 55 = (28)^2 = 784$

24. Find the sum of

(i) $1^2 + 2^2 + ... + 19^2$

(ii) $5^2 + 10^2 + 15^2 + ... + 105^2$

Solution:

i. $1^2 + 2^2 + ... + 19^2$

$= \frac{n(n+1)(2n+1)}{6}$

$= \frac{19 \times (19+1)(2 \times 19+1)}{6}$

$= \frac{19 \times 20 \times 39}{6} = 2170$

ii. $5^2 + 10^2 + 15^2 + ... + 105^2$

$= 5^2(1^2 + 2^2 + 3^2 + ... + 21^2)$

$= 25 \times \frac{21 \times (21+1) \times (2 \times 21+1)}{6}$

$= 25 \times \frac{21 \times 22 \times 43}{6} = 82775$

25. Find the sum of $1^3 + 2^3 + 3^3 + ... + 16^3$

Solution:

$1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$1^3 + 2^3 + 3^3 + ... + 16^3 = \left[\frac{16 \times 17}{2}\right]^2$

$= [136]^2 = 18496$

26. If $1 + 2 + 3 + ... + n = 666$ then find n.

Solution:

$1 + 2 + 3 + ... + n = 666$

$\frac{n(n+1)}{2} = 666$

$n^2 + n = 1332$

$n^2 + n - 1332 = 0$

$(n - 36)(n + 37) = 0$

$n = -37$ or $n = 36$

But $n \neq -37$ (Since n is a natural number)

Hence $n = 36$.

27. If $1 + 2 + 3 + ... + k = 325$, then find $1^3 + 2^3 + 3^3 + ... + k^3$.

Solution:

$1 + 2 + 3 + ... + k = \frac{k(k+1)}{2} = 325$

$1^3 + 2^3 + 3^3 + ... + k^3$

$= \left[\frac{k(k+1)}{2}\right]^2 = (325)^2 = 105625$

28. If $1^3 + 2^3 + 3^3 + ... + k^3 = 44100$ then find $1 + 2 + 3 + ... + k$.

Solution:

$1^3 + 2^3 + 3^3 + ... + k^3 = 44100 = \left[\frac{k(k+1)}{2}\right]^2$

$1 + 2 + 3 + ... + k = \frac{k(k+1)}{2} = 210$

29. How many terms of the series $1^3 + 2^3 + 3^3 + ...$ should be taken to get the sum 14400?

Solution:

$1^3 + 2^3 + 3^3 + ... + k^3 = \left[\frac{k(k+1)}{2}\right]^2 = 14400$

$\Rightarrow \frac{k(k+1)}{2} = \sqrt{14400} = 120$

$k(k + 1) = 240$

$k^2 + k - 240 = 0$

$(k - 15)(k + 16) = 0$

$k = +15$ or $k = -16$

k can't be negative

$\therefore k = 15$

5 Marks

1. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4

Solution:

2	113400
2	56700
2	28350
3	14175
3	4725
3	1575
3	525
5	175
5	35
7	7
	1

$113400 = 2^3 \times 3^4 \times 5^2 \times 7^1$

$\therefore P_1 = 2, P_2 = 3, P_3 = 5, P_4 = 7$

$x_1 = 3, x_2 = 4, x_3 = 2, x_4 = 1$

2. If $a_1 = 1, a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$ $n \geq 3, n \in N$, then find the first six terms of the sequence.

Solution:

Given $a_1 = a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$

$a_3 = 2a_2 + a_1 = 2(1) + 1 = 3;$

$a_4 = 2a_3 + a_2 = 2(3) + 1 = 7$

$a_5 = 2a_4 + a_3 = 2(7) + 3 = 17;$

$a_6 = 2a_5 + a_4 = 2(17) + 7 = 41$

3. Find *x*, y and z, given that the numbers *x*, 10, y, 24, z are in A.P.

Solution:

A.P. $\Rightarrow x$, 10, y, 24, z

That is $y = \frac{10+24}{2} = \frac{34}{2} = 17$

$\therefore$ A.P = x, 10, 17, 24, z

Here we know that d = 17 – 10 = 7

$\therefore x = 10 - 7 = 3$

$z = 24 + 7 = 31$

$\therefore x = 3$, y = 17, z = 31.

4. Find the sum to n terms of the series 5 + 55 + 555 + ...

Solution:

$S_n = 5 + 55 + 555 + \ldots + n$ terms

$= 5\,[1 + 11 + 111 + \ldots + n \text{ terms}]$

$= \frac{5}{9}\,[9 + 99 + 999 + \ldots + n \text{ terms}]$

$= \frac{5}{9}\,[10 - 1 + 100 - 1 + 1000 - 1 + \ldots n \text{ terms}]$

$= \frac{5}{9}\,[(10 + 100 + 1000 + \ldots.) - (1 + 1 + 1 + \ldots.)]$

$= \frac{5}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$

$= \frac{50}{81}\left[(10^n - 1) - \frac{5}{9}n\right]$

5. Find the sum to n terms of the series
(i) 0.4 + 0.44 + 0.444 + ... to n terms
(ii) 3 + 33 + 333 + ... to n terms

Solution:

i. 0.4 + 0.44 + 0.444 + ... n terms

$= \frac{4}{10} + \frac{44}{100} + \frac{444}{1000} + \ldots n terms$

$= 4\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ldots n terms\right]$

$= \frac{4}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots n terms\right]$

$= \frac{4}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \ldots n terms\right]$

$= \frac{4}{9}[(1+1+1+\ldots \text{ n terms}) - \left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ldots n terms\right)]$

$= \frac{4}{9}\left[n - \frac{1}{10}\left[\frac{1 - \left(\frac{1}{10}\right)^n}{1 - \frac{1}{10}}\right]\right] = \frac{4}{9}\left[n - \frac{1}{9}\left(1 - \left(\frac{1}{10}\right)^n\right)\right]$

ii. 3 + 33 + 333 + ... n

$= 3(1 + 11 + 111 + \ldots. + n \text{ terms})$

$= \frac{3}{9}\,(9 + 99 + 999 + \ldots + n \text{ terms})$

$= \frac{3}{9}\,((10-1) + (100-1) + (1000-1) + \ldots + n \text{ terms})$

$= \frac{3}{9}\,(10 + 100 + 1000 + \ldots + n \text{ terms}) - (1 + 11 + 111 + \ldots. + n \text{ terms})$

$= \frac{3}{9}\left(10\left(\frac{10^n - 1}{9}\right) - n\right)$

$= \frac{30}{81}\,(10n - 1) - \frac{3n}{9}$

6. Find the sum of the Geometric series 3 + 6 + 12 + ... + 1536

Solution:

3 + 6 + 12 + ... + 1536

a = 3, r = 2

$t_n = 1536$

$ar^{n-1} = 1536$

$3(2)^{n-1} = 1536$

$3(2)^{n-1} = 3(2)^9$

$2^{n-1} = 2^9$

$n - 1 = 9$

$\therefore \quad n = 10$

To find S_n,

$S_n = \frac{a(r^n - 1)}{r - 1} \quad \Rightarrow \quad S_{10} = \frac{3(2^{10} - 1)}{2 - 1}$

$= 3\,(1023) = 3069$

7. Find the value of 16 + 17 + 18 + ... + 75

Solution:

$16 + 17 + 18 + \ldots + 75$

$= (1 + 2 + 3 + \ldots. + 75) - (1 + 2 + 3 + \ldots + 15)$

$= \frac{75(75+1)}{2} - \frac{15(15+1)}{2}$

$= 2850 - 120$

$= 2730$

8. Find the sum of $9^3 + 10^3 + ... + 21^3$

Solution:

$9^3 + 10^3 + ... + 21^3$

$= (1^3 + 2^3 + 3^3 ... + 21^3) - (1^3 + 2^3 + 3^3 ... + 8^3)$

$= \left[\frac{21\times(21+1)}{2}\right]^2 - \left[\frac{8\times(8+1)}{2}\right]^2$

$= (231)^2 - (36)^2$

$= 52065$

9. Find the sum of the following series

(i) $6^2 + 7^2 + 8^2 + ... + 21^2$

(ii) $10^3 + 11^3 + 12^3 + ... + 20^3$

Solution:

i. $6^2 + 7^2 + 8^2 + ... + 21^2$

$= (1^2 + 2^2 + 3^2 ... + 21^2) - (1^2 + 2^2 + 3^2 + ... + 5^2)$

$= \frac{21\times(21+1)(42+1)}{6} - \frac{5\times(5+1)(10+1)}{6}$

$= \frac{21\times22\times43}{6} - \frac{5\times6\times11}{6}$

$= 3311 - 55 = 3256$

ii. $10^3 + 11^3 + 12^3 + ... + 20^3$

$= 1^3 + 2^3 + 3^3 + ... + 20^3 - 1^3 + 2^3 + 3^3 + ... + 9^3$

$= \left[\frac{20\times21}{6}\right]^2 - \left[\frac{9\times10}{3}\right]^2$

$= [210]^2 - (45)^2$

$= 44100 - 2025 = 42075$

10. The sum of the cubes of the first *n* natural numbers is 2025, then find the value of *n*.

Solution:

$1^2 + 2^2 + 3^2 + ... + n^2 = 285$

$\frac{n(n+1)(2n+1)}{2\times3} = 285$

$\frac{n(n+1)(2n+1)}{6} = 285$

$n(n+1)(2n+1) = 285 \times 6$ ..... (1)

$1^3 + 2^3 + 3^3 + ... + n^3 = 2025$

$\left[\frac{n(n+1)}{2}\right]^2 = 2025$

$\frac{n(n+1)}{2} = \sqrt{2025} = 45$

$n(n+1) = 45 \times 2$ ..... (2)

$\frac{(1)}{(2)} \Rightarrow \frac{n(n+1)(2n+1)}{n(n+1)} = \frac{285\times6}{45\times2}$

$2n + 1 = 19$

$2n = 19 - 1$

$\Rightarrow 2n = 18$

$\therefore n = 9$

11. Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm, ..., 24 cm. How much area can be decorated with these colour papers?

Solution:

The Required Area

$= 10^2 + 11^2 + 12^2 + ... + 24^2$

Area $= (1^2 + 2^2 + 3^2 + ... + 24^2)$

$- (1^2 + 2^2 + ... + 9^2)$

$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$

$= \frac{24\times25\times49}{6} - \frac{9\times10\times19}{6}$

$= 4900 - 285 = 4615$ cm^2

Therefore Rekha has 4615 cm^2 colour paper. She can decorate 4615cm^2 area with these colour papers.

12. Find the sum of $15^2 + 16^2 + 17^2 + ... + 28^2$

$15^2 + 16^2 + 17^2 + ... + 28^2$

$= (1^2 + 2^2 + 3^2 ... + 28^2)$

$- (1^2 + 2^2 + 3^2 ... + 14^2)$

$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$

$= \frac{28\times29\times57}{2\times3} - \frac{14\times15\times29}{2\times3}$

$= 14 \times 29 \times 19 - 7 \times 5 \times 29$

$= 7714 - 1015 = 6699$

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3. Algebra

2 Marks

1. Find the LCM of the given polynomials

(i) $4x^2y, 8x^3y^2$

(ii) $9a^3b^2, 12a^2b^2c$

(iii) $16m, 12m^2n^2, 8n^2$

(iv) $p^2 - 3p + 2, p^2 - 4$

(v) $2x^2 - 5x - 3, 4x^2 - 36$

(vi) $(2x^2 - 3xy)^2, (4x - 6y)^3, 8x^3 - 27y^3$

Solution:

i. $4x^2y, 8x^3y^2$

$4x^2y = 2^2x^2y$

$8x^3y^2 = 2^3x^3y^2$

$\therefore$ LCM$(4x^2y, 8x^3y^2) = 2^3x^3y^2 = 8x^3y^2$

ii. $9a^3b^2, 12a^2b^2c$

$9a^3b^2 = (1)(3)^2\, a^3b^2$

$12a^2b^2c = 2^2 \times 3 \times a^2 \times b^2 \times c$

$\therefore$ LCM$(9a^3b^2, 12a^2b^2c)$

$= (1) \times 2^2 \times 3^2 \times a^3 \times b^2 \times c = 36a^3b^2c$

iii. $16m, 12m^2n^2, 8n^2$

$16m = 2^4 \times m$

$12m^2n^2 = 2^2 \times 3 \times m^2 \times n^2$

$8n^2 = 2^3 \times n^2$

$\therefore$ LCM$(16m, 12m^2n^2, 8n^2)$

$= 2^4 \times 3 \times m^2 \times n^2 = 48m^2n^2$

iv. $p^2 - 3p + 2, p^2 - 4$

$p^2 - 3p + 2 = (p - 1)(p - 2)$

$p^2 - 4 = (p + 2)(p - 2)$

$\therefore$ LCM$(p^2 - 3p + 2, p^2 - 4)$

$= (p - 1)(p + 2)(p - 2)$

v. $2x^2 - 5x - 3, 4x^2 - 36$

$2x^2 - 5x - 3 = (x - 3)(2x + 1)$

$4x^2 - 36 = 4(x + 3)(x - 3)$

$\therefore$ LCM$(2x^2 - 5x - 3, 4x^2 - 36)$

$= 4(x - 3)(x + 3)(2x + 1)$

vi. $(2x^2 - 3xy)^2, (4x - 6y)^3, 8x^3 - 27y^3$

$(2x^2 - 3xy)^2 = x^2(2x - 3y)^2$

$(4x - 6y)^3 = 2^3(2x - 3y)^3$

$8x^3 - 27y^3 = (2x)^3 - (3y)^3$

$= (2x - 3y)(4x^2 + 6xy + 9y^2)$

$\therefore$ LCM$((2x^2 - 3xy)^2, (4x - 6y)^3, (8x^3 - 27y^3)$

$= 2^3 \times x^2 \times (2x - 3y)^3 (4x^2 + 6xy + 9y^2)$

$= 8x^2(2x - 3y)^3 (4x^2 + 6xy + 9y^2)$

2. Simplify:

i) $\dfrac{4x^2y}{2z^2} \times \dfrac{6xz^3}{20y^4}$

ii) $\dfrac{p^2 - 10p + 21}{p - 7} \times \dfrac{p^2 + p - 12}{(p - 3)^2}$

iii) $\dfrac{5t^3}{4t - 8} \times \dfrac{6t - 12}{10t}$

Solution:

i. $\dfrac{4x^2y}{2z^2} \times \dfrac{6xz^3}{20y^4} = \dfrac{3x^3z}{5y^3}$

ii. $\dfrac{p^2 - 10p + 21}{p - 7} \times \dfrac{p^2 + p - 12}{(p - 3)^2}$

$= \dfrac{(p - 7)(p - 3)}{(p - 7)} = \dfrac{(p + 4)(p - 3)}{(p - 3)^2} = (p + 4)$

iii. $\dfrac{5t^3}{4t - 8} \times \dfrac{6t - 12}{10t}$

$= \dfrac{5t^3}{4(t - 2)} \times \dfrac{6(t - 2)}{10t} = \dfrac{3t^2}{4}$

3. Simplify: $\dfrac{x^3}{x - y} + \dfrac{y^3}{y - x}$

Solution:

$\dfrac{x^3}{x - y} + \dfrac{y^3}{y - x} = \dfrac{x^3 - y^3}{x - y}$

$= \dfrac{(x^2 + xy + y^2)(x - y)}{(x - y)}$

$= x^2 + xy + y^2$

4. Find the excluded values of the following expressions (if any). MAY-22

i) $\dfrac{x + 10}{8x}$ **ii)** $\dfrac{7p + 2}{8p^2 + 13p + 5}$

Solution:

i. The expression $\dfrac{x + 10}{8x}$ is undefined when $8x = 0$ or $x = 0$.

When the excluded value is 0.

ii. The expression $\dfrac{7p + 2}{8p^2 + 13p + 5}$ is undefined when $8p^2 + 13p + 5 = 0$ that is

$(8p + 5)(p + 1) = 0$ $p = \dfrac{-5}{8}$, $p = -1$.

The excluded values are $\dfrac{-5}{8}$ and -1.

5. Find the excluded values, if any of the following expressions.

i) $\frac{y}{y^2-25}$ ii) $\frac{t}{t^2-5t+6}$

iii) $\frac{x^2+6x+8}{x^2+x-2}$ iv) $\frac{x^3-27}{x^3+x^2-6x}$

Solution:

i. The expression $\frac{y}{y^2-25}$ is undefined

when $y^2 - 5^2 = 0$

$y^2 - 5^2 = 0$

$(y + 5)(y - 5) = 0$

$y + 5 = 0, y - 5 = 0$

$y = -5, y = 5$

Hence the excluded values are –5 and 5.

ii. The expression $\frac{t}{t^2-5t+6}$ is undefined

when $t^2 - 5t + 6 = 0$

$t^2 - 5t + 6 = 0$

$(t - 2)(t - 3) = 0$

$t - 2 = 0, t - 3 = 0$

$t = 2, t = 3$

Hence the excluded values are 2 and 3.

iii. $\frac{x^2+6x+8}{x^2+x-2} = \frac{(x+4)(x+2)}{(x+2)(x-1)} = \frac{x+4}{x-1}$

The expression $\frac{x+4}{x-1}$ is undefined when

$x - 1 = 0$. Hence the excluded value is 1.

iv. $\frac{x^3-27}{x^3+x^2-6x} = \frac{(x-3)(x^2+3x+9)}{x(x^2+x-6)}$

$= \frac{(x-3)(x^2+3x+9)}{(x)(x+3)(x-2)}$

The expression $\frac{x^3-27}{x^3+x^2-6x}$ is undefined

when $x^3 + x^2 - 6x = 0$

$\Rightarrow (x)(x+3)(x-2) = 0$

$\Rightarrow x = 0$ or $x = -3$ or $x = 2$

Hence the excluded values are 0, –3, 2

6. Find the square root of the following rational expression.

$\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$

Solution:

$\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4} = \sqrt{\frac{4y^8z^{12}}{x^4}} = 2\left|\frac{y^4z^6}{x^2}\right|$

7. Find the square root of the following expressions

i) $256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}$

ii) $\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$

Solution:

i. $\sqrt{\left(256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}\right)}$

$= 16\,|(x-a)^4(x-b)^2(x-c)^8(x-d)^{10}|$

ii. $\sqrt{\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}} = \frac{4}{3}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$

8. Find the square root of the following rational expression.

$\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$

Solution:

$\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4} =$

$\sqrt{\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}}$

$= \frac{11}{9}\left|\frac{(a+b)^4(x+y)^4}{(a-b)^6}\right|$

9. Determine the quadratic equations, whose sum and product of roots are

(i) –9, 20 (ii) $\frac{5}{3}$, 4 SEP-21

Solution:

i. –9, 20

$x^2 - [\alpha + \beta]x + \alpha\beta = 0$

$x^2 - [-9]x + 20 = 0 \Rightarrow x^2 + 9x + 20 = 0$

ii. $\frac{5}{3}$, 4

Required Quadratic Equations

x^2 – (Sum of the roots)x + product of the roots = 0

$x^2 - \frac{5}{3}x + 4 = 0$

Multiply 3 on both sides

$3x^2 - 5x + 12 = 0$

10. Find the sum and product of the roots for each of the following quadratic equations

(i) $x^2 + 3x - 28 = 0$ (ii) $x^2 + 3x = 0$

Solution:

i. $x^2 + 3x - 28 = 0$

$a = 1, b = 3, c = -28$

Sum of the roots $= \alpha+\beta = -\frac{b}{a} = -\frac{3}{1} = -3$

Product of the roots $= \alpha\beta = \frac{c}{a}$

$= -\frac{28}{1} = -28$

ii. $x^2 + 3x = 0$

$a = 1, b = 3, c = 0$

Sum of the roots $= \alpha+\beta = -\frac{b}{a} = -\frac{3}{1} = -3$

Product of the roots $= \alpha\beta = \frac{c}{a} = \frac{0}{1} = 0$

11. In the matrix $\mathbf{A} = \begin{pmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{pmatrix}$, **write**

(i) The number of elements

(ii) The order of the matrix

(iii) Write the elements a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}.

Solution:

i) Number of elements $= 4 \times 4 = 16$

ii) Order of matrix $= 4 \times 4$

iii) $a_{22} = \sqrt{7}$; $a_{23} = \frac{\sqrt{3}}{2}$; $a_{24} = 5$;

$a_{34} = 0$; $a_{43} = -11$; $a_{44} = 1$

12. If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?

Solution:

Matrix having 18 elements 1×18 (or) 2×9 (or) 3×6 (or) 6×3 (or) 9×2 (or) 18×1

Matrix having 6 elements 1×6 (or) 2×3 (or) 3×2 (or) 6×1

13. Construct a 3×3 matrix whose elements are given by

(i) $a_{ij} = i - 2j$ **(ii) $a_{ij} = \frac{(i+j)^3}{3}$**

Solution:

i. $a_{ij} = |\, i - 2j \,|$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$= \begin{bmatrix} |1-2| & |1-4| & |1-6| \\ |2-2| & |2-4| & |2-6| \\ |3-2| & |3-4| & |3-6| \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$$

ii. $a_{ij} = \frac{(i+j)^3}{3}$

$$= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$= \begin{bmatrix} \frac{8}{3} & 9 & \frac{64}{3} \\ 9 & \frac{64}{3} & \frac{125}{3} \\ \frac{64}{3} & \frac{125}{3} & 72 \end{bmatrix}$$

14. Construct a 3×3 matrix whose elements are $a_{ij} = i^2j^2$

Solution:

The general 3×3 matrix is given by

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$a_{11} = 1^2 \times 1^2 = 1 \times 1 = 1$; $a_{12} = 1^2 \times 2^2 = 1 \times 4 = 4$;
$a_{13} = 1^2 \times 3^2 = 1 \times 9 = 9$; $a_{21} = 2^2 \times 1^2 = 4 \times 1 = 4$;
$a_{22} = 2^2 \times 2^2 = 4 \times 4 = 16$; $a_{23} = 2^2 \times 3^2 = 4 \times 9 = 36$
$a_{31} = 3^2 \times 1^2 = 9 \times 1 = 9$; $a_{32} = 3^2 \times 2^2 = 9 \times 4 = 36$;
$a_{33} = 3^2 \times 3^2 = 9 \times 9 = 81$

Hence the required matrix is $A = \begin{pmatrix} 1 & 4 & 9 \\ 4 & 16 & 36 \\ 9 & 36 & 81 \end{pmatrix}$

15. If $\mathbf{A} = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ **then find the transpose of A.**

Solution:

$A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & 7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ $A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$

16. If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of $-A$. **SEP-20**

Solution:

$$A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix} \quad -A = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}$$

$$(-A)^T = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$$

17. If $A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ then verify $(A^T)^T = A$

Solution:

$$A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{pmatrix}$$

$$(A^T)^T = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$$

$\therefore (A^T)^T = A$

18. Find the values of x, y and z from the following equations

(i) $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

(ii) $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

(iii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

Solution:

i. $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

$\Rightarrow 12 = y;\ 3 = z;\ x = 3$

ii. $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

$\Rightarrow 5 + z = 5 \quad x + y = 6;$

$z = 5 - 5 \quad y = 6 - x;$

$z = 0$

$$xy = 8$$
$$x(6-x) = 8$$
$$6x - x^2 - 8 = 0$$
$$\Rightarrow \quad x^2 - 6x + 8 = 0$$
$$(x-2)(x-4) = 0$$
$$x - 2 = 0 \quad \text{(or)} \quad x - 4 = 0$$
$$x = 2 \text{ (or) } x = 4$$

If $x = 2$ then $y = \frac{8}{x} = \frac{8}{2} = 4;$

If $x = 4$ then $y = \frac{8}{4} = 2$

iii. $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

$x + y + z = 9$ ..... (1)

$x + z = 5$ ..... (2)

$y + z = 7$ ..... (3)

Substitute (3) in (1)

$x + 7 = 9 \Rightarrow x = 9 - 7 = 2$

Substitute x =2 in (2)

$2 + z = 5 \Rightarrow z = 5 - 2 = 3$

Substitute z = 3 in (3)

$y + 3 = 7 \Rightarrow y = 7 - 3 \Rightarrow y = 4$

19. If $A = \begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$ then Find 2A+B.

Solution:

$$2A + B = 2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14+4 & 16+11 & 12-3 \\ 2-1 & 6+2 & 18+4 \\ -8+7 & 6+5 & -2+0 \end{pmatrix}$$

$$= \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$$

20. If $A=\begin{pmatrix}5 & 4 & -2\\ \frac{1}{2} & \frac{3}{4} & \sqrt{2}\\ 1 & 9 & 4\end{pmatrix}$, $B=\begin{pmatrix}-7 & 4 & -3\\ \frac{1}{4} & \frac{7}{2} & 3\\ 5 & -6 & 9\end{pmatrix}$, find 4A–3B.

Solution:

$$4A-3B=4\begin{pmatrix}5 & 4 & -2\\ \frac{1}{2} & \frac{3}{4} & \sqrt{2}\\ 1 & 9 & 4\end{pmatrix}-3\begin{pmatrix}-7 & 4 & -3\\ \frac{1}{4} & \frac{7}{2} & 3\\ 5 & -6 & 9\end{pmatrix}$$

$$=\begin{pmatrix}20 & 16 & -8\\ 2 & 3 & 4\sqrt{2}\\ 4 & 36 & 16\end{pmatrix}+\begin{pmatrix}21 & -12 & 9\\ -\frac{3}{4} & -\frac{21}{2} & -9\\ -15 & 18 & -27\end{pmatrix}$$

$$=\begin{pmatrix}20+21 & 16-12 & -8+9\\ 2-\frac{3}{4} & 3-\frac{21}{2} & 4\sqrt{2}-9\\ 4-15 & 36+18 & 16-27\end{pmatrix}$$

$$=\begin{pmatrix}41 & 4 & 1\\ \frac{5}{4} & -\frac{15}{2} & 4\sqrt{2}-9\\ -11 & 54 & -11\end{pmatrix}$$

21. If $A=\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}$, $B=\begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix}$ then verify that (i) A+B = B +A

(ii) A+(−A) = (−A)+A =O.

Solution:

i. A + B = B + A

L.H.S.

$$A+B=\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}+\begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix}$$

$$=\begin{pmatrix}6 & 16\\ 6 & 7\\ 9 & -3\end{pmatrix} \quad \ldots\ldots (1)$$

R.H.S.

$$B+A=\begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix}+\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}$$

$$=\begin{pmatrix}6 & 16\\ 6 & 7\\ 9 & -3\end{pmatrix} \quad \ldots\ldots (2)$$

(1), (2) $\Rightarrow$ A + B = B + A

ii. A + (–A) = (–A) + A = 0

$$A+(-A)=\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}+\begin{pmatrix}-1 & -9\\ -3 & -4\\ -8 & 3\end{pmatrix}$$

$$=\begin{pmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix} \quad \ldots\ldots (1)$$

$$(-A)+A=\begin{pmatrix}-1 & -9\\ -3 & -4\\ -8 & 3\end{pmatrix}+\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}$$

$$=\begin{pmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix} \quad \ldots\ldots (2)$$

(1), (2) $\Rightarrow$ A+ (–A) = (–A) + A = 0

22. If $A=\begin{pmatrix}0 & 4 & 9\\ 8 & 3 & 7\end{pmatrix}$, $B=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}$ find the value of (i) B – 5A (ii) 3A – 9B

Solution:

$$A=\begin{pmatrix}0 & 4 & 9\\ 8 & 3 & 7\end{pmatrix}, B=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}$$

i. B – 5A

$$=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}-5\begin{pmatrix}0 & 4 & 9\\ 8 & 3 & 7\end{pmatrix}$$

$$=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}+\begin{pmatrix}0 & -20 & -45\\ -40 & -15 & -35\end{pmatrix}$$

$$=\begin{pmatrix}7 & -17 & -37\\ -39 & -11 & -26\end{pmatrix}$$

ii. 3A – 9B

$$=3\begin{pmatrix}0 & 4 & 9\\ 8 & 3 & 7\end{pmatrix}-9\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}$$

$$=\begin{pmatrix}0 & 12 & 27\\ 24 & 9 & 21\end{pmatrix}+\begin{pmatrix}-63 & -27 & -72\\ -9 & -36 & -81\end{pmatrix}$$

$$=\begin{pmatrix}-63 & -15 & -45\\ 15 & -27 & -60\end{pmatrix}$$

5 Marks

1. Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$

Solution:

		8	–1	1		
	8	64	–16	17	–2	1
		(–) 64				
16	–1		–16	17		
		(+)	–16	(–) 1		
16	–2			16	–2	1
	1			(–) 16	(+) –2	(–) 1
					0	

Required Square root = $|8x^2 - x + 1|$

2. If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, find the values of a and b.

Solution:

		3	2	4		
	3	9	12	28	a	b
		(–) 9				
6	2		12	28		
		(–)	12	(–) 4		
6	4	4		24	a	b
				(–) 24	(–) 16	(–) 16
					a = 16,	b = 16

3. Find the square root of the following polynomials by division method

Solution:

i. $x^4 - 12x^3 + 42x^2 - 36x + 9$

		1	–6	3		
	1	1	–12	42	–36	9
		(–) 1				
2	–6		–12	42		
		(+)	–12	(–) 36		
2	–12			6	–36	9
	3			(–) 6	(+) –36	(–) 9
					0	

Required Square root = $|x^2 - 6x + 3|$

ii. $37x^2 - 28x^3 + 4x^4 + 42x + 9$

		2	–7	–3		
	2	4	–28	37	42	9
		(–) 4				
4	–7		–28	37		
		(+)	–28	(–) 49		
4	–14			–12	42	9
	–3			(+)	–12 (–) 42	(–) 9
					0	

Required Square root = $|2x^2 - 7x - 3|$

iii. $16x^4 + 8x^2 + 1$

		4	0	1		
	4	16	0	8	0	1
		(–) 16				
8	0		0	8		
			0	0		
8	0	1		8	0	1
				(–) 8	(–)0	(–)1
					0	

Required Square root = $|4x^2 + 1|$

iv. $121x^4 - 198x^3 - 183x^2 + 216x + 144$

		11	–9	–12		
	11	121	–198	–183	216	144
		(–)121				
22	–9		– 198	–183		
			(+) –198	(–)81		
22	–18	–12		– 264	216	144
				(+) –264	(–)216	(–)144
					0	

Required Square root = $|11x^2 - 9x - 12|$

4. Find the values of a and b if the following polynomials are perfect squares

i. $4x^4 - 12x^3 + 37x^2 + bx + a$

Solution:

		2	–3	7		
	2	4	–12	37	b	a
		(–) 4				
4	–3		–12	37		
			(+) –12	(–) 9		
4	–6	7		28	b	a
				(–) 28	(+)– 42	(–) 49
					a = 49 ,	b = –42

ii. $ax^4 + bx^3 + 361x^2 + 220x + 100$

Solution:

		10	11	12		
	10	100	220	361	b	a
		(–)100				
	20	11	220	361		
			(–) 220	(–)121		
20	22	12		240	b	a
				(–) 240	(–)264	(–)144

a = 144 , b = 264

5. Find the values of m and n if the following polynomials are perfect sqaures

i. $36x^4 - 60x^3 + 61x^2 - mx + n$ **MAY-22**

Solution:

		6	–5	3		
	6	36	–60	61	–m	n
		(–) 36				
	12	–5	–60	61		
			(+) –60	(–)25		
	12	–10	36	–m	n	
		3	(–) 36	(+)– 30	(–) 9	

–m = –30, m = 30

n = 9

ii. $x^4 - 8x^3 + mx^2 + nx + 16$

Solution:

		1	–4	4		
	1	1	–8	m	n	16
		(–) 1				
	2	–4	–8	m		
			(+) –8	(–)16		
2	–8	4	m–16	n	16	
			(–) 8	(+)– 32	(–) 16	
			0			

$\frac{m-16}{2} = 4$

m – 16 = 8, n = –32

m = 8 + 16

m = 24

6. If $A = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix}$ **and** $C = \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$ **then verify that A + (B + C) = (A + B) + C.**

Solution:

$$A + (B + C) = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \left(\begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix} + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix} \right)$$

$$= \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 10 & 6 & 8 \\ 2 & 7 & 5 \\ -5 & 5 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix} \quad \ldots\ldots (1)$$

$$(A + B) + C = \left(\begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix} \right) + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix} \quad \ldots\ldots (2)$$

From (1) & (2) LHS = RHS

7. If $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$, $C = \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$ **verify that A(B + C) = AB + AC.**

Solution:

$$B + C = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1-7 & 2+6 \\ -4+3 & 2+2 \end{pmatrix} = \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

$$LHS = A(B + C) = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -6-1 & 8+4 \\ 6-3 & -8+12 \end{pmatrix} = \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1-4 & 2+2 \\ -1-12 & -2+6 \end{pmatrix} = \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix}$$

$$AC = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -7+3 & 6+2 \\ 7+9 & -6+6 \end{pmatrix} = \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

RHS = AB + AC

$$= \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix} + \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -3-4 & 4+8 \\ -13+16 & 4+0 \end{pmatrix} = \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix}$$

$\therefore$ LHS = RHS

8. **If** $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$ **and** $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$

show that $(AB)^T = B^T A^T$ **SEP-20**

Solution:

$$AB = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2-2+0 & -1+8+2 \\ 4+1+0 & -2-4+2 \end{pmatrix} = \begin{pmatrix} 0 & 9 \\ 5 & -4 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix} \quad A^T = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 2-2+0 & 4+1+0 \\ -1+8+2 & -2-4+2 \end{pmatrix} = \begin{pmatrix} 0 & 9 \\ 5 & -4 \end{pmatrix}$$

$\therefore$ LHS = RHS

9. **Given that** $A = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix}$,

$B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$

verify that A(B +C) = AB + AC.

Solution:

$A = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$

To verify that A (B + C) = AB + AC

LHS

$$B + C = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix}$$

$$A(B + C) = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \times \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 2-3 & 2+18 & 4+15 \\ 10+1 & 10-6 & 20-5 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix} \quad \ldots\ldots(1)$$

$$AB = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1+9 & -1+15 & 2+6 \\ 5-3 & -5-5 & 10-2 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 14 & 8 \\ 2 & -10 & 8 \end{pmatrix}$$

$$AB + AC = \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix} \quad \ldots\ldots(2)$$

(1), (2) $\Rightarrow$ A (B + C) = AB + AC.

10. **If** $A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$

verify that $(AB)^T = B^T A^T$

Solution:

$A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$

$$AB = \begin{pmatrix} 5+2+45 & 35+4-9 \\ 1+2+40 & 7+4-8 \end{pmatrix} = \begin{pmatrix} 52 & 30 \\ 43 & 3 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix} \quad A^T = \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix} \times \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 5+2+45 & 1+2+40 \\ 35+4-9 & 7+4-8 \end{pmatrix}$$

$$= \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$$

(1), (2) $\Rightarrow (AB)^T = B^T A^T$

11. If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ show that $A^2 - 5A + 7I_2 = 0$

Solution:

$$A^2 = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix}$$

$$A^2 - 5A + 7I_2$$

$$= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} -15 & -5 \\ 5 & -10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Hence, $A^2 - 5A + 7I_2 = 0$

***

4. Geometry

2 Marks

1. If ΔABC is similar to ΔDEF such that BC = 3 cm, EF= 4 cm and area of ΔABC = 54 cm². Find the area of ΔDEF.

Solution:

Since the ratio of area of two similar triangles is equal to the ratio of the squares of any two corresponding sides, we have

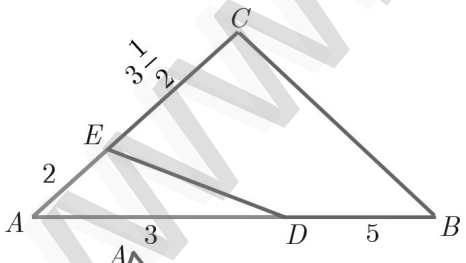
$$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{BC^2}{EF^2}$$

gives $\frac{54}{Area(\Delta DEF)} = \frac{3^2}{4^2}$

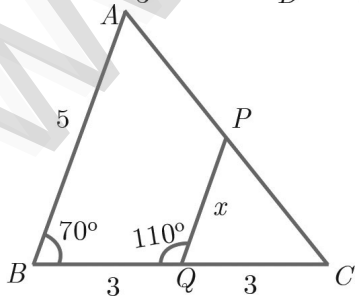
$$\text{Area } (\Delta DEF) = \frac{16 \times 54}{9} = 96 \text{ cm}^2$$

2. Check whether the which triangles are similar and find the value of *x*.

i)



ii)



Solution:

i. From the figure, in ΔABC and ADE

$$\frac{AC}{AE} = \frac{3\frac{1}{2}+2}{2} = \frac{\frac{7}{2}+2}{2} = \frac{\frac{7+4}{2}}{2}$$

$$= \frac{11}{2} \times \frac{1}{2} = \frac{11}{4} \quad \ldots\ldots (1)$$

$$\frac{AB}{AD} = \frac{3+5}{3} = \frac{8}{3} \quad \ldots\ldots (2)$$

From (1), (2) $\Rightarrow \frac{AC}{AE} \neq \frac{AB}{AD}$

$\therefore \Delta ABC$ and ΔADE are not similar

ii. From the figure, in ΔABC and ΔPQC

$\angle ABC = \angle PQC = 70°$ ..... (1)
(Corresponding angles are equal)

$\angle C = \angle C$ (Common Angles) ..... (2)

$\therefore \angle A = \angle QPC$ ($\because$ AAA criterian)

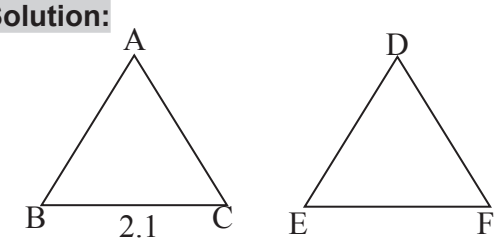
Hence, ΔABC and ΔPQC are similar triangles

Then, $\frac{AB}{PQ} = \frac{BC}{QC} \Rightarrow \frac{5}{x} = \frac{6}{3} = 2$

$\therefore x = \frac{5}{2} = 2.5$

3. If ΔABC ~ ΔDEF such that area of ΔABC is 9 cm² and the area of ΔDEF is 16 cm² and BC = 2.1 cm. Find the length of EF.

Solution:



Given ΔABC ~ ΔDEF

$$\frac{Area\ of\ (\Delta ABC)}{Area\ of\ (\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$= \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$$

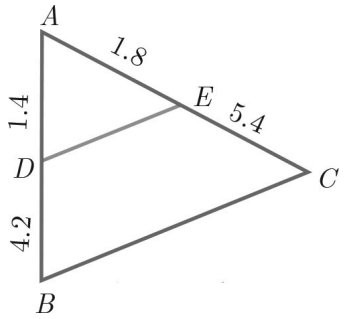
$$\Rightarrow \quad \frac{9}{16} = \frac{(2.1)^2}{EF^2}$$

$$\Rightarrow \quad EF^2 = (2.1)^2 \times \frac{16}{9}$$

$$\Rightarrow \quad EF = 2.1 \times \frac{4}{3} = 2.8 \text{ cm}$$

4. D and E are respectively the points on the sides AB and AC of a ΔABC such that AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm, show that DE || BC .

Solution:



AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm

BD = AB – AD = 5.6 – 1.4 = 4.2 cm and

EC = AC – AE = 7.2 – 1.8 = 5.4 cm

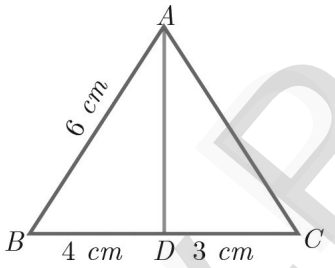
$\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3}$ and $\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Therefore, by converse of Basic Proportionality Theorem, we have DE is parallel to BC.

Hence Proved.

5. In the Figure, AD is the bisector of ∠A. If BD = 4 cm, DC = 3 cm and AB = 6 cm, find AC. MAY-22



Solution:

In ΔABC, AD is the bisector of ∠A.

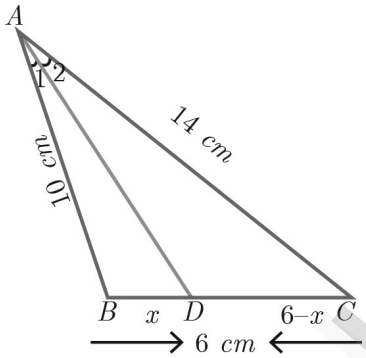
Therefore by Angle Bisector Theorem

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$\frac{4}{3} = \frac{6}{AC}$ gives 4AC = 18

Hence AC $= \frac{9}{2} = 4.5$ cm

6. In the Figure, AD is the bisector of ∠BAC, if AB = 10 cm, AC = 14 cm and BC = 6 cm. Find BD and DC.



Solution:

AD is the bisector of ∠BAC

AB =10 cm, AC = 14 cm, BC = 6 cm

By Angle Bisector Theorem

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{x}{6-x} = \frac{10}{14}$$

$$\frac{x}{6-x} = \frac{5}{7}$$

$$7x = 30 - 5x$$

$$12x = 30$$

$$x = \frac{30}{12} = 2.5 \text{ cm}$$

∴ BD = 2.5 cm DC = 3.5 cm

7. In ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC SEP-21

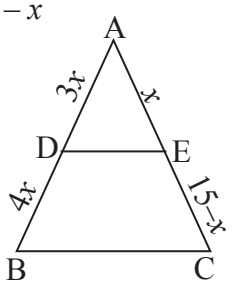
(i) If $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm find AE.

(ii) If AD = 8x − 7, DB = 5x − 3, AE = 4x − 3 and EC = 3x − 1, find the value of x.

Solution:

i. If $\frac{AD}{DB} = \frac{3}{4}$, AC = 15 cm , AE = x,

EC = 15 – x



DE || BC then by basic proportionality theorem.

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{x}{15-x}$$

$3(15-x) = 4x$

$45 - 3x = 4x$

$45 = 7x$

$x = \frac{45}{7} = 6.43$ cm

ii. Given AD = $8x - 7$, DB = $5x - 3$, AE = $4x - 3$ and EC = $3x - 1$

By basic proportionality theorem

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$$

$\Rightarrow (8x-7)(3x-1) = (5x-3)(4x-3)$

$\Rightarrow 24x^2 - 29x + 7 = 20x^2 - 27x + 9$

$24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0$

$\Rightarrow 4x^2 - 2x - 2 = 0$

$\Rightarrow 2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$x = 1, x = -\frac{1}{2}$ (Not Admissible).

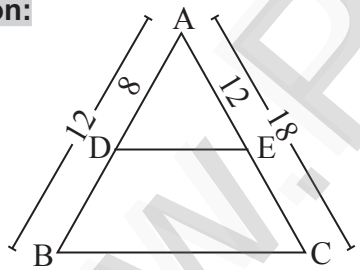
$\therefore x = 1$

8. In ΔABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that DE || BC.

(i) AB = 12 cm, AD = 8 cm, AE = 12 cm and AC = 18 cm.

(ii) AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm.

Solution:



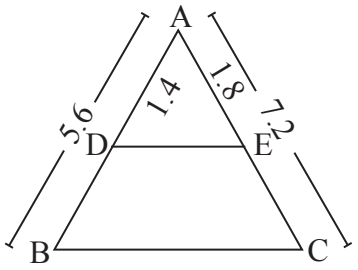
i. AB = 12 cm, AD = 8 cm AE = 12 cm and AC = 18cm

$$\frac{AD}{AB} = \frac{8}{12} = \frac{2}{3} \quad \text{..... (1)}$$

$$\frac{AE}{AC} = \frac{12}{18} = \frac{2}{3} \quad \text{..... (2)}$$

From (1) & (2) $\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$

$\therefore$ DE || BC



ii. AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm

$$\frac{AD}{AB} = \frac{1.4}{5.6} = \frac{1}{4} \quad \text{..... (1)}$$

$$\frac{AE}{AC} = \frac{1.8}{7.2} = \frac{1}{4} \quad \text{..... (2)}$$

(1), (2) $\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$

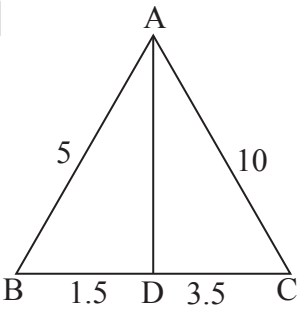
$\therefore$ DE || BC

9. Check whether AD is bisector of ∠A of ΔABC in each of the following

(i) AB = 5 cm, AC = 10 cm, BD= 1.5 cm and CD= 3.5 cm. SEP-20

(ii) AB= 4 cm, AC = 6 cm, BD = 1.6 cm and CD= 2.4 cm.

Solution:



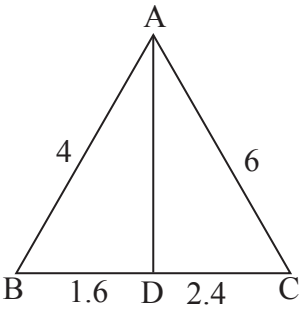
i. AB = 5cm , AC =10 cm, BD =1.5 cm and CD = 3.5 cm

$$\frac{AB}{AC} = \frac{5}{10} = \frac{1}{2} \quad \text{..... (1)}$$

$$\frac{BD}{CD} = \frac{1.5}{3.5} = \frac{3}{7} \quad \text{..... (2)}$$

(1), (2) $\Rightarrow \frac{AB}{AC} \neq \frac{BD}{CD}$ ($\because$ By ABT)

AD is not a bisector of ∠A in ΔABC



ii. AB = 4 cm, AC = 6 cm, BD = 1.6 cm and CD = 2.4 cm

$\frac{AB}{AC} = \frac{4}{6} = \frac{2}{3}$ ..... (1)

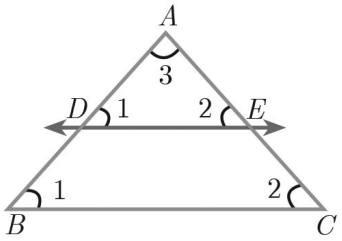
$\frac{BD}{CD} = \frac{1.6}{2.4} = \frac{2}{3}$ ..... (2)

(1), (2) $\Rightarrow \frac{AB}{AC} = \frac{BD}{CD}$ ($\because$ By ABT)

AD is a bisector of $\angle A$ in ΔABC

5 Marks

1. State and Prove Basic Proportionality Theorem (BPT) or Thales Theorem. MAY-22



Statement:

A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.

Proof

Given:

In ΔABC, D is a point on AB and E is a point on AC

To Prove:

$\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Draw a line DE || BC

No.	Statement	Reason
1.	$\angle ABC = \angle ADE \longrightarrow 1$	Corresponding angles are equal because DE || BC
2.	$\angle ACB = \angle AED \longrightarrow 2$	Corresponding angles are equal because DE || BC
3.	$\angle DAE = \angle BAC \longrightarrow 3$	Both triangles have a common angle.
	$\Delta ABC \sim \Delta ADE$	By AAA similarity
	$\frac{AB}{AD} = \frac{AC}{AE}$	Corresponding sides are proportional
	$\frac{AD + DB}{AD} = \frac{AE + EC}{AE}$	Split AB and AC using the points D and E
	$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$	On Simplication
	$\frac{DB}{AD} = \frac{EC}{AE}$	Cancelling 1 on both sides
	$\frac{AD}{DB} = \frac{AE}{EC}$	Taking reciprocals
	Hence Proved	

2. State and Prove Angle Bisector Theorem.

Statement: SEP-20

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle

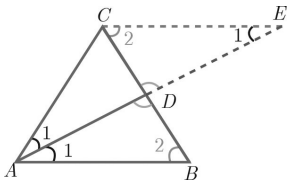
Proof

Given:

In ΔABC, AD is the internal bisector

To Prove:

$\frac{AB}{AC} = \frac{BD}{CD}$.



Construction:

Draw a line through C parallel to AB.

Extend AD to meet line through C at E.

No.	Statement	Reason
1.	$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate angles equal.
2.	ΔACE is isosceles AC = CE .... (1)	In ΔACE $\angle CAE = \angle CEA$.
3.	$\Delta ABD \sim \Delta ECD$ $\frac{AB}{CE} = \frac{BD}{CD}$	By AA Similarity
4.	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) AC = CE. Hence Proved.

3. State and Prove Pythagoras Theorem.

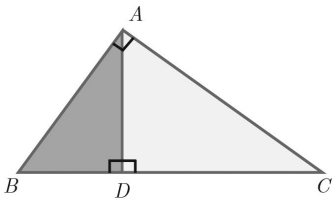
Statement:

In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Proof

Given:

In ΔABC, $\angle A = 90°$



To Prove:

$AB^2 + AC^2 = BC^2$

Construction: Draw $AD \perp BC$

No.	Statement	Reason
1.	Compare ΔABC and ΔABD $\angle B$ is common $\angle BAC = \angle BDA = 90°$ Therefore, $\Delta ABC \sim \Delta ABD$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ ..... (1)	Given $\angle BAC = 90°$ and by construction $\angle BDA = 90°$ By AA similarity
2.	Compare ΔABC and ΔADC $\angle C$ is common $\angle BAC = \angle ADC = 90°$ Therefore, $\Delta ABC \sim \Delta ADC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ ... (2)	Given $\angle BAC = 90°$ and by construction $\angle CDA = 90°$ By AA similarity

Adding (1) and (2) we get

$AB^2 + AC^2 = BC \times BD + BC \times DC$

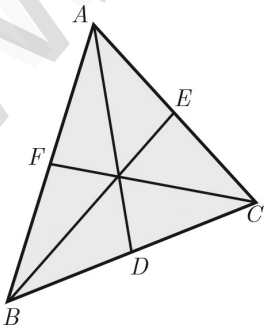
$= BC\,(BD + DC)$

$AB^2 + AC^2 = BC \times BC = BC^2$

Hence the theorem is proved.

4. Show that in a triangle, the medians are concurrent. SEP-21

Solution:



Medians are line segments joining each vertex to the midpoint of the corresponding opposite sides.

Thus medians are the cevians where D, E, F are midpoints of BC, CA and AB respectively.

Since D is midpoint of BC,

$BD = DC$. So $\frac{BD}{DC} = 1$ ..... (1)

Since E is midpoint of CA,

$CE = EA$. So $\frac{CE}{EA} = 1$ ..... (2)

Since F is midpoint of AB,

$AF = FB$. So $\frac{AF}{FB} = 1$ ..... (3)

Thus, multiplying (1), (2), (3) we get

$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB}$

$= 1 \times 1 \times 1 = 1$

And so, Ceva's theorem is satisfied.

Hence the Medians are concurrent.

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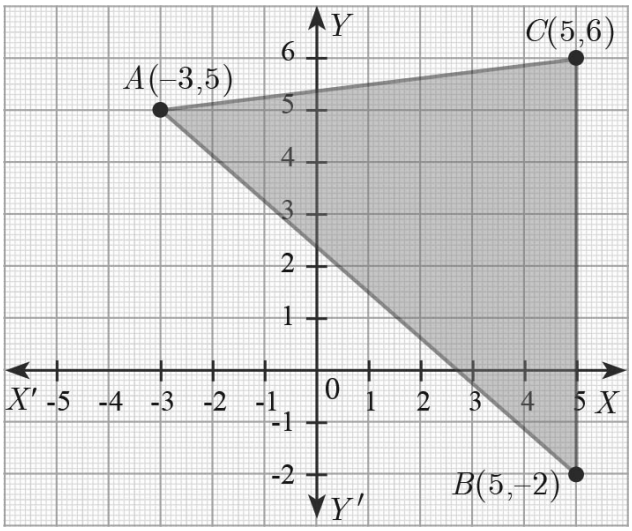
5. Coordinate Geometry

2 Marks

1. Find the area of the triangle whose vertices are (–3, 5), (5, 6) and (5, –2)

Solution:



A(–3, 5), B(5, –2), C(5,6)

$x_1 y_1$ $x_2 y_2$ $x_3 y_3$

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -3 & 5 \\ 5 & -2 \\ 5 & 6 \\ -3 & 5 \end{vmatrix}$$

$$= \frac{1}{2}[(6+30+25) - (25-10-18)]$$

$$= \frac{1}{2}[61 + 3]$$

$$= \left|\frac{64}{2}\right| = 32 \text{ sq. units.}$$

2. Show that the points P (–1.5, 3), Q (6, –2), R (–3, 4) are collinear. MAY-22

Solution:

Area of $\Delta PQR = 0$

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 0 \Rightarrow \frac{1}{2}\begin{vmatrix} -1.5 & 3 \\ 6 & -2 \\ -3 & 4 \\ -1.5 & 3 \end{vmatrix} = 0$$

$$\frac{1}{2}[(3+24-9) - (18+6-6)] = 0$$

$$\frac{1}{2}[18 - 18] = 0$$

$\therefore$ Therefore, the given points are collinear.

3. If the area of the triangle formed by the vertices A (–1, 2), B (*k*, –2) and C (7, 4) (taken in order) is 22 sq. units, find the value of *k*.

Solution:

The vertices are A (–1, 2), B (k, –2) and C (7, 4)

Area of ΔABC is 22 sq.units

$$\frac{1}{2}\begin{vmatrix} -1 & 2 \\ k & -2 \\ 7 & 4 \\ -1 & 2 \end{vmatrix} = 22$$

$$\begin{vmatrix} -1 & 2 \\ k & -2 \\ 7 & 4 \\ -1 & 2 \end{vmatrix} = 44$$

$$\{(2 + 4k + 14) - (2k - 14 - 4)\} = 44$$

$$4k + 16 - 2k + 18 = 44$$

$$2k + 34 = 44$$

$$2k = 10$$

$$\text{Therefore } k = 5$$

4. Find the area of the triangle formed by the points (i) (1, –1), (–4, 6) and (–3, –5)
(ii) (ii) (–10, –4), (–8, –1) and (–3, –5)

Solution:

i. $$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & -1 \\ -4 & 6 \\ -3 & -5 \\ 1 & -1 \end{vmatrix}$$

$$= \frac{1}{2}[(6+20+3) - (4-18-5)]$$

$$= \frac{1}{2}[6+20+3-4+18+5]$$

$$= \frac{1}{2}[(6+20+3+18+5)-4]$$

$$= \frac{1}{2}[52-4]$$

$$= \frac{1}{2}[48] = 24 \text{ sq.units.}$$

ii. $$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -10 & -4 \\ -8 & -1 \\ -3 & -5 \\ -10 & -4 \end{vmatrix}$$

$$= \frac{1}{2}[(10+40+12)-(32+3+50)$$

$$= \frac{1}{2}[62 - 85]$$

$$= \frac{1}{2}[-23] = -11.5 \text{ sq.units.}$$

$\therefore$ Area of the Triangle = 11.5 sq.units

5. Determine whether the sets of points are collinear?

(i) $\left(-\frac{1}{2}, 3\right)$ (–5, 6) and (–8, 8)

Solution:

$\left(-\frac{1}{2}, 3\right)$ (–5, 6) and (–8, 8)

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -\frac{1}{2} & 3 \\ -5 & 6 \\ -8 & 8 \\ -\frac{1}{2} & 3 \end{vmatrix}$$

$$= \frac{1}{2}[(-3-40-24) - (-15-48-4)]$$

$$= \frac{1}{2}[(-67) - (-67)] = 0$$

$\therefore$ The given points are collinear.

(ii) (a, b+c), (b, c+a) and (c, a+b)

Solution:

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a & b+c \\ b & c+a \\ c & a+b \\ a & b+c \end{vmatrix}$$

$$= \frac{1}{2}[(ac + a^2 + ab + b^2 + bc + c^2) - (b^2 + bc + c^2 + ca + a^2 + ab]$$

$$= \frac{1}{2}[ac + a^2 + ab + b^2 + bc + c^2 - b^2 - bc - c^2 - ca - a^2 - ab]$$

$$= \frac{1}{2}[0] = 0 \text{ sq.units.}$$

Aliter:

(a, b+c), (b, c+a), (c, a+b)

x_1, y_1 x_2, y_2 x_3, y_3

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a-b & a-c \\ b+c-c-a & b+c-a-b \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a-b & a-c \\ -(a-b) & -(a-c) \end{vmatrix}$$

$$= \frac{1}{2}[(a-b)(a-c) + (a-b)(a-c)]$$

$$= \frac{1}{2}[0] = 0$$

$\therefore$ The given points are collinear.

6. Vertices of given triangles are taken in order and their areas are provided aside. In each case, find the value of 'p'.

S.No.	Vertices	Area (sq.units)
(i)	(0, 0), (p, 8), (6, 2)	20
(ii)	(p, p), (5, 6), (5, –2)	32

Solution:

i. A (0, 0), B (p, 8), C (6, 2)

Area of ΔABC = 20 sq.units.

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \text{Area of } \Delta ABC$$

$$\frac{1}{2}\begin{vmatrix} 0 & 0 \\ p & 8 \\ 6 & 2 \\ 0 & 0 \end{vmatrix} = 20$$

$$(0+2p+0) - (0+48+0) = 40$$

$$2p - 48 = 40$$

$$2p = 88$$

$$p = 44$$

ii. A (p, p), B (5, 6), C (5, –2)

Area of Δ = 32 sq.units

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 32$$

$$\frac{1}{2}\begin{vmatrix} p & p \\ 5 & 6 \\ 5 & -2 \\ p & p \end{vmatrix} = 32$$

$$\begin{vmatrix} p & p \\ 5 & 6 \\ 5 & -2 \\ p & p \end{vmatrix} = 64$$

$$(6p-10+5p) - (5p+30-2p) = 64$$

$$6p - 10 + 5p - 5p - 30 + 2p = 64$$

$$8p - 40 = 64$$

$$\Rightarrow 8p = 64 + 40$$

$$8p = 104$$

$$\Rightarrow p = \frac{104}{8}$$

$$\Rightarrow p = 13$$

7. In each of the following, find the value of '*a*' for which the given points are collinear.

(i) (2, 3), (4, a) and (6, –3)

(ii) (a, 2–2a), (–a+1, 2a) and (–4–a, 6–2a)

Solution:

i. (2, 3), (4, a) and (6, –3)

$\Delta = 0$

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 0 \quad \Rightarrow \begin{vmatrix} 2 & 3 \\ 4 & a \\ 6 & -3 \\ 2 & 3 \end{vmatrix} = 0$$

$$[(2a-12+18) - (12+6a-6)] = 0$$

$$2a - 12 + 18 - 12 - 6a + 6 = 0$$

$$-4a = 0$$

$$\therefore a = 0$$

ii. (a, 2–2a), (–a+1, 2a) and (–4–a, 6–2a)

$\Delta = 0$ sq.units.

$(2a^2–6a+2a^2+6–2a–8+8a–2a+2a^2) –$
$(–2a+2a^2+2–2a–8a–2a^2+6a–2a^2) = 0$

$\Rightarrow (6a^2–2a–2) – (–2a^2–6a+2) = 0$

$\Rightarrow 8a^2 + 4a – 4 = 0 \div 4$

$2a^2 + a – 1 = 0$

$(a+1)(2a–1) = 0$

$\Rightarrow \therefore a = +\frac{1}{2}$ and $a = –1$

Aliter:

$(a, a–2a), \quad (–a+1, 2a), \quad (–4–a, 6–2a)$

$x_1, y_1 \quad x_2, y_2 \quad x_3, y_3$

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} a+a-1 & a+4+a \\ 2-2a-2a & 2-2a-6+2a \end{vmatrix} = 0$$

$$\begin{vmatrix} 2a-1 & 2a+4 \\ 2-4a & -4 \end{vmatrix} = 0$$

$–4(2a–1) – (2–4a)(2a+4) = 0$

$–8a+4 – [4a+8–8a^2–16a] = 0$

$–8a+4–4a–8+8a^2+16a = 0$

$8a^2+4a–4 = 0$

$2a^2+a–1 = 0$

$(a+1)(2a–1) = 0$

$a = –1$ (or) $a = \frac{1}{2}$

5 Marks

1. The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such triangle has the vertices at (–3, 2), (–1, –1) and (1, 2). If the floor of the hall is completely covered by 110 tiles, find the area of the floor.

Solution:

Vertices of one triangular tile are at
(–3, 2), (–1, –1) (1, 2)

$$\text{Area of this tile} = \frac{1}{2}\begin{vmatrix} -3 & 2 \\ -1 & -1 \\ 1 & 2 \\ -3 & 2 \end{vmatrix}$$

$= \frac{1}{2}\{(3–2+2)–(–2–1–6)\}$

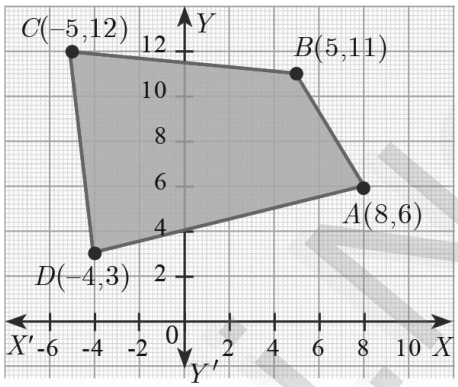
$= \frac{1}{2}(12) = 6$ sq.units

Since the floor is covered by 110 triangle shaped identical tiles,

Area of the floor = $110 \times 6 = 660$ sq. units

2. Find the area of the quadrilateral formed by the points (8, 6), (5, 11), (–5, 12) and (–4, 3).

Solution:



Before determining the area of the quadrilateral, plot the vertices in a graph A (8, 6), B (5, 11), C (–5, 12) and D (–4, 3).

Therefore, area of the quadrilateral ABCD

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 8 & 6 \\ 5 & 11 \\ -5 & 12 \\ -4 & 3 \\ 8 & 6 \end{vmatrix}$$

$= \frac{1}{2}[(88 + 60 – 15 – 24) – (30 – 55 – 48 + 24)]$

$= \frac{1}{2}[88 + 60 – 15 – 24 – 30 + 55 + 48 – 24]$

$= \frac{1}{2}[88 + 60 + 55 + 48 – 15 – 24 – 30 – 24]$

$= \frac{1}{2}[251 – 93]$

$= \frac{1}{2}[158] = 79$ sq.units.

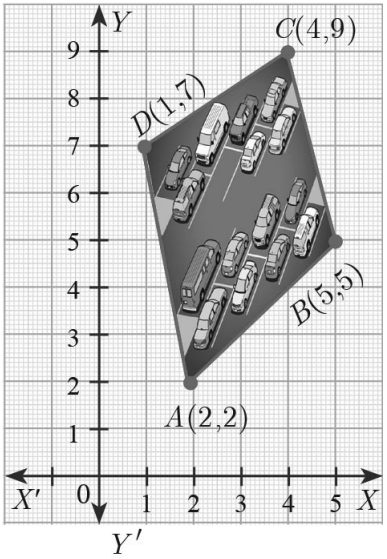
3. The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹ 1300 per square feet. What will be the total cost for making the parking lot?

Solution:

The parking lot is a quadrilateral whose vertices A (2, 2), B (5, 5), C (4, 9) and D (1, 7).

Therefore, Area of parking lot is

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & 2 \\ 5 & 5 \\ 4 & 9 \\ 1 & 7 \\ 2 & 2 \end{vmatrix}$$



$= \frac{1}{2}[(10 + 45 + 28 + 2) - (10 + 20 + 9 + 14)]$

$= \frac{1}{2}[85 - 53]$

$= \frac{1}{2}[32] = 16$ sq.units

So, Area of parking lot = 16 sq.feets.

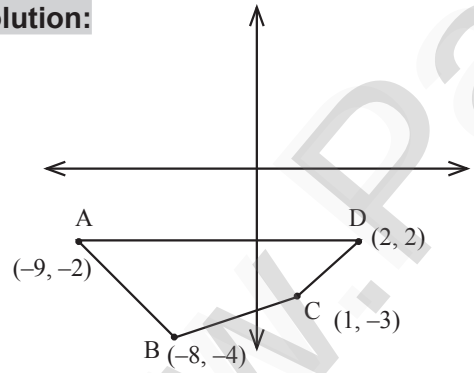
Construction rate per square fee = ₹ 1300

Therefore, total cost for constructing the parking lot = 16×1300 = ₹ 20,800

4. Find the area of the quadrilateral whose vertices are at

(i) (–9, –2), (–8, –4), (2, 2) and (1, –3)

Solution:



Let A (–9, –2), B (–8, –4), C (1, –3), D (2, 2)

Area of the quadrilateral $= \frac{1}{2}\begin{vmatrix} -9 & -2 \\ -8 & -4 \\ 1 & -3 \\ 2 & 2 \\ -9 & -2 \end{vmatrix}$

$= \frac{1}{2}[(36+24+2-4) - (16-4-6-18)]$

$= \frac{1}{2}[(36+24+2-4-16+4+6+18)]$

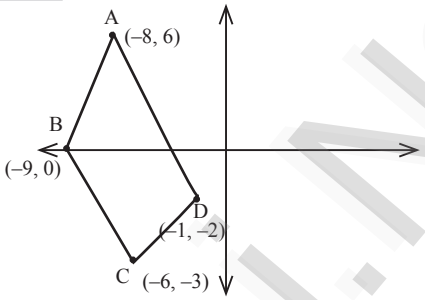
$= \frac{1}{2}[(36+24+2+4+6+18) - (4+16)]$

$= \frac{1}{2}[90-(20)]$

$= \frac{1}{2}[70] = 35$ sq. units

(ii) (–9, 0), (–8, 6), (–1, –2) and (–6, –3)

Solution:



A (–8, 6), B (–9, 0), C (–6, –3), D (–1, –2)

Area of the quadrilateral $= \frac{1}{2}\begin{vmatrix} -8 & 6 \\ -9 & 0 \\ -6 & -3 \\ -1 & -2 \\ -8 & 6 \end{vmatrix}$

$= \frac{1}{2}[(0+27+12-6) - (-54+0+3+16)]$

$= \frac{1}{2}[27+12-6+54-3-16]$

$= \frac{1}{2}[(27+12+54) - (6+3+16)]$

$= \frac{1}{2}[93-25] = \frac{1}{2}[68] = 34$ sq. units

Aliter:

A(–8, 6), B(–9, 0), C(–6, –3), D(–1, –2)

x_1, y_1 x_2, y_2 x_3, y_3 x_4, y_4

Area of the quadrilateral

$= \frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -8-(-6) & -9-(-1) \\ 6-(-3) & 0-(-2) \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -8+6 & -9+1 \\ 6+3 & 0+2 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & -8 \\ 9 & 2 \end{vmatrix}$

$= \frac{1}{2}[-4 + 72] = \frac{1}{2}[68] = 34$ sq. units

5. Find the value of k, if the area of a quadrilateral is 28 sq.units, whose vertices are (–4, –2), (–3, k), (3, –2) and (2, 3) SEP-20

Solution:

$$\frac{1}{2}\begin{vmatrix} -4 & -2 \\ -3 & k \\ 3 & -2 \\ 2 & 3 \\ -4 & -2 \end{vmatrix} = 28$$

$\Rightarrow (-4k + 6 + 9 - 4) - (6 + 3k - 4 - 12) = 56$

$\Rightarrow (11 - 4k) - (3k - 10) = 56$

$\Rightarrow 11 - 4k - 3k + 10 = 56$

$\Rightarrow 21 - 7k = 56$

$\Rightarrow 7k = -35$

$\Rightarrow k = -5$

6. If the points A (–3, 9), B (a, b) and C (4, –5) are collinear and if a + b = 1, then find a and b.

Solution:

Given A (–3, 9), B(a, b), C(4, –5) are collinear and a + b = 1 ..... (1)

Area of the triangle formed by 3 points = 0

$$\frac{1}{2}\begin{vmatrix} -3 & 9 \\ a & b \\ 4 & -5 \\ -3 & 9 \end{vmatrix} = 0$$

$\Rightarrow (-3b - 5a + 36) - (9a + 4b + 15) = 0$

$\Rightarrow -5a - 3b + 36 - 9a - 4b - 15 = 0$

$\Rightarrow -14a - 7b + 21 = 0$

$\Rightarrow -14a - 7b = -21$

$\Rightarrow 14a + 7b = 21 \ (\div 7)$

$\Rightarrow 2a + b = 3$ ..... (2)

Given $a + b = 1$ ..... (1)

(1) – (2) $\Rightarrow a = 2 \quad b = -1$

7. A triangular shaped glass with vertices at A (–5, –4) , B (1, 6) and C (7, –4) has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.

Solution:

The required number of buckets = $\frac{\text{Area of the } \Delta ABC}{\text{Area of the paint covered by one bucket}}$

Area of the $\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -5 & -4 \\ 1 & 6 \\ 7 & -4 \\ -5 & -4 \end{vmatrix}$

$= \frac{1}{2}[(-30 - 4 - 28) - (-4 + 42 + 20)]$

$= \frac{1}{2}[-62 - 58]$

$= \frac{1}{2}[-120]$

$= 60$ sq. units.

$\therefore$ The required number of buckets $= \frac{60}{6} = 10$

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6. Trigonometry

2 Marks

1. Prove that $\frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$

Solution:

$$\frac{\sin A}{1+\cos A} = \frac{\sin A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$$

$$= \frac{\sin A(1-\cos A)}{(1+\cos A)(1-\cos A)}$$

$$= \frac{\sin A(1-\cos A)}{1-\cos^2 A}$$

$$= \frac{\sin A(1-\cos A)}{\sin^2 A} = \frac{1-\cos A}{\sin A}$$

2. Prove that $1+\frac{\cot^2\theta}{1+cosec\theta} = \text{cosec}\theta$

Solution:

$$1 + \frac{\cot^2\theta}{1+cosec\theta} = 1+\frac{cosec^2\theta - 1}{cosec\theta + 1}$$

$[\because \text{cosec}^2\theta - 1 = \cot^2\theta]$

$$= 1+\frac{(cosec\theta+1)(cosec\theta-1)}{cosec\theta+1}$$

$= 1+(\text{cosec}\theta - 1) = \text{cosec}\theta$

3. Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \text{cosec}\theta + \cot\theta$

Solution:

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$$

$$= \sqrt{\left(\frac{1+\cos\theta}{\sin\theta}\right)^2} = \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

LHS = cosecθ + cotθ

∴ LHS = RHS

4. Prove the following identities.

(i) cotθ + tanθ = secθ cosecθ

Solution:

LHS = cotθ + tanθ

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

= secθ cosecθ

∴ LHS = RHS.

(ii) $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Solution:

LHS = $\tan^4\theta + \tan^2\theta = \tan^2\theta(\tan^2\theta + 1)$

= $\tan^2\theta(\sec^2\theta)$ ($\because 1 + \tan^2\theta = \sec^2\theta$)

= $(\sec^2\theta - 1)(\sec^2\theta)$ ($\because \tan^2\theta = \sec^2\theta - 1$)

= $\sec^4\theta - \sec^2\theta$

∴ LHS = RHS

5. Prove the following identities.

(i) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$ SEP-20

Solution:

$$\text{LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} \times \sqrt{\frac{1+\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

= secθ + tanθ = RHS

Hence Proved.

(ii) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

Solution:

$$\text{LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

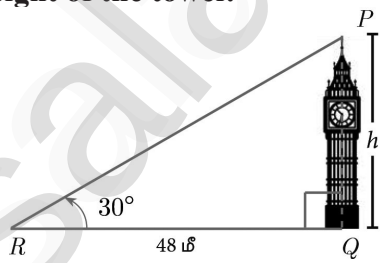
$$= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta+1-\sin\theta}{\cos\theta} = \frac{2}{\cos\theta}$$

= 2secθ

Hence Proved.

6. A tower stands vertically on the ground. From a point on the ground, which is 48 m away from the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.



Solution:

In ΔPQR $\tan\theta = \frac{PQ}{QR}$

$$\tan 30° = \frac{h}{48}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{48}$$

$$h = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{48\sqrt{3}}{3} = 16\sqrt{3}$$

Therefore the height of the tower is,

$h = 16\sqrt{3}$ m

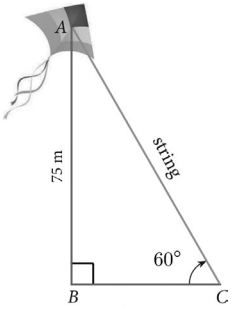
7. A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60° . Find the length of the string, assuming that there is no slack in the string.

Solution:

In ΔABC $\sin\theta = \frac{AB}{AC}$

$$\sin 60° = \frac{75}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{75}{AC}$$



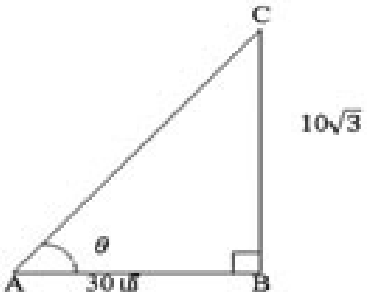
$$AC = \frac{75 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{150\sqrt{3}}{3}$$

$$AC = 50\sqrt{3} \text{ m}$$

$\therefore$ Hence, the length of the string is $50\sqrt{3}$ m.

8. Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m. SEP-21

Solution:



In ΔABC

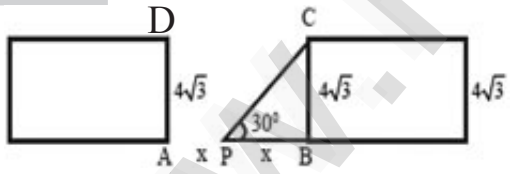
$$\tan\theta = \frac{opposite\ side}{Adjacent\ side} \Rightarrow \tan\theta = \frac{10\sqrt{3}}{30}$$

$$\tan\theta = \frac{\sqrt{3}}{3} \Rightarrow \tan\theta = \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} \quad \theta = 30°$$

9. A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$ m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road.

Solution:



In the figure, BC – House, AB – Width of Road, P – Median of Road

AP = PB = x

In ΔPBC, $\tan 30° = \frac{BC}{PB}$

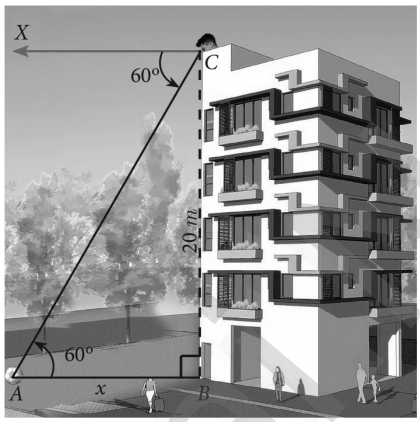
$$\Rightarrow \tan 30° = \frac{4\sqrt{3}}{PB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{PB}$$

$$PB = 4\sqrt{3} \times \sqrt{3} = 4 \times 3 = 12$$

Hence, Width of Road

$$= AP + PB = 12 + 12 = 24 \text{ m}$$

10. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. ($\sqrt{3}$ = 1.732)



Solution:

Let BC be the height of the tower and A be the position of the ball lying on the ground.

Then, BC = 20 m and

$\angle XCA = 60° = \angle CAB$

Let AB = x metres.

In the right angled triangle ABC,

$$\tan 60° = \frac{20}{AB}$$

$$\sqrt{3} = \frac{20}{AB}$$

$$AB = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

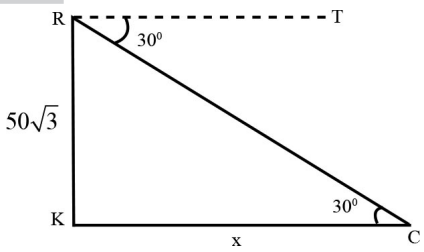
$$AB = \frac{20\sqrt{3}}{3} = \frac{20 \times 1.732}{3}$$

$$= \frac{34.640}{3} = 11.54 \text{ m}$$

Hence, the distance between the foot of the tower and the ball is 11.55 m.

11. From the top of a rock $50\sqrt{\ }$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock. MAY-22

Solution:



In ΔABC, $\tan\theta = \frac{opposite\ side}{Adjacent\ side}$

$$\tan 30° = \frac{50\sqrt{3}}{KC}$$

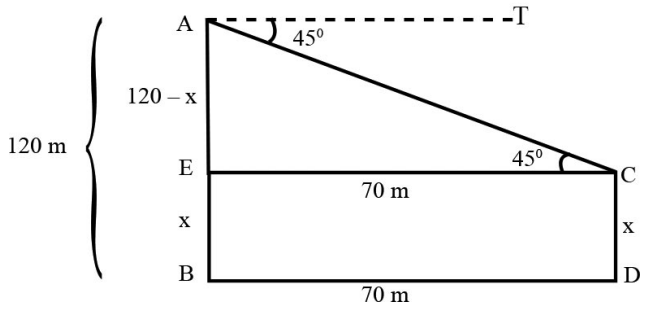
$$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{KC}$$

$$KC = 50\sqrt{3} \times \sqrt{3}$$

$$= 50\,(3) = 150 \text{ m}$$

12. The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is 45°. If the height of the second building is 120 m, find the height of the first building. ($\sqrt{3}$ = 1.732)

Solution:



CD – First Building,

AB – Second Building

From the figure AB = 120 m,

EB = CD = x, AE=120 – x,

EC = BD = 70 m

In ΔACE, $\tan 45° = \frac{AE}{EC}$

$$\Rightarrow \quad 1 = \frac{120 - x}{70}$$

$$\Rightarrow \quad 120 - x = 70 \text{ m}$$

$$\therefore x = 50 \text{ m}$$

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7. Mensuration

2 Marks

1. The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

Solution:

l = 5 cm, R = 4 cm, r = 1 cm

C.S.A of the frustum = π (R + r) l sq.units

$$= \frac{22}{7}(4+1) \times 5$$

$$= \frac{22 \times 5 \times 5}{7} = \frac{550}{7}$$

$$= 78.57 \text{ cm}^2$$

2. The radius and height of a cylinder are in the ratio 5:7 and its curved surface area is 5500 sq.cm. Find its radius and height.

Solution:

r : h = 5 : 7 $\Rightarrow$ r = 5x cm , h = 7x cm

CSA = 5500 sq.cm

$$2\pi rh = 5500 \Rightarrow 2 \times \frac{22}{7} \times 5x \times 7x = 5500$$

$$x^2 = \frac{5500}{2 \times 22 \times 5} = 25 \Rightarrow x = 5$$

Hence, Radius = 5 × 5 = 25 cm,

Height = 7 × 5 = 35 cm

3. The volumes of two cones of same base radius are 3600 cm^3 and 5040 cm^3. Find the ratio of heights. |May 22|

Solution:

Ratio of the volumes of two cones

$$= \frac{1}{3}\pi r^2 h_1 : \frac{1}{3}\pi r^2 h_2$$

$$= h_1 : h_2$$

$$= 3600 : 5040$$

$$= 360 : 504$$

$$= 40 : 56$$

$$= 5 : 7$$

4. If the ratio of radii of two spheres is 4:7, find the ratio of their volumes.

Solution:

The ratio of radii of two spheres = 4 : 7

Let radius of first sphere is 4x,

that is $r_1 = 4x$

Let radius of second sphere is 7x,

that is $r_2 = 7x$

The ratio of their volumes

$$= \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3} = \frac{(4x)^3}{(7x)^3} = \frac{4^3 \times x^3}{7^3 \times x^3}$$

$$= \frac{4^3}{7^3} = \frac{64}{343}$$

Hence the ratio of the volumes is 64 : 343

5. A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3}$:4.

Solution:

Given

Total Surface Area of a solid Sphere

= Total surface Area of a solid hemisphere

$\Rightarrow 4\pi R^2 = 3\pi r^2$

$\Rightarrow \therefore \frac{R^2}{r^2} = \frac{3}{4} \quad \Rightarrow \quad \therefore \frac{R}{r} = \frac{\sqrt{3}}{2}$

$\therefore$ Ratio of their volumes

$$= \frac{\frac{4}{3}\pi R^3}{\frac{2}{3}\pi r^3} = \frac{2R^3}{r^3} = 2\left[\frac{R}{r}\right]^3 = 2\left[\frac{\sqrt{3}}{2}\right]^3$$

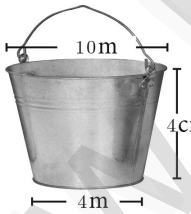
$$\Rightarrow 2 \times \frac{3\sqrt{3}}{8} = \frac{3\sqrt{3}}{4}$$

$\therefore$ Ratio of their volumes = $3\sqrt{3}$: 4

5 Marks

1. An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10 m and 4 m and whose height is 4 m. Find the curved and total surface area of the bucket.

Solution:



Let h, l, R and r be the height, slant height, outer radius and inner radius of the frustum.

Given that, diameter of the top =10 m;

radius of the top R = 5 m.

diameter of the bottom = 4 m;

radius of the bottom r = 2 m, height h = 4 m

Now, $l = \sqrt{h^2 + (R-r)^2}$

$= \sqrt{4^2 + (5-2)^2}$

$l = \sqrt{16+9} = \sqrt{25} = 5$ m

C.S.A. $= \pi(R + r)l$ sq. units

$= \frac{22}{7}(5 + 2) \times 5$

$= \frac{22}{7} \times 7 \times 5$

$= 110$ m^2

T.S.A. $= \pi(R + r)l + \pi R^2 + \pi r^2$ sq. units

$= \pi[(R+r)l + R^2 + r^2]$

$= \frac{22}{7}[(5+2)5 + 5^2 + 2^2]$

$= \frac{22}{7}(35+25+4) = \frac{1408}{7} = 201.14$m^2

Therefore, C.S.A. = 110 m^2 and

T.S.A. = 201.14 m^2

2. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is ₹ 2.

Solution:

From the given figure, r = 6m, R = 12m and h = 8m.

But, $l = \sqrt{h^2 + (R-r)^2}$

$= \sqrt{8^2 + 6^2} = \sqrt{10} = 10$

$l = 10$ m

The required total arc of table lamp

= CSA of frustrum + Area of the top

$= \pi(R + r)l + \pi r^2$

$= \frac{22}{7} \times 18 \times 10 + \frac{22}{7} \times 6 \times 6$

$= \frac{22}{7} \times 6[30+6] = \frac{22}{7} \times 6 \times 36$

$= 678.86$ m^2

Cost of painting for 1 sq.m. is ₹ 2.

$\therefore$ The total cost of painting

= 678.86 × 2 = ₹1357.72.

3. A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of ₹40 per litre. MAY-22

Solution:

h = 16 cm, r = 8 cm, R = 20 cm,

Volume of the frustum

$= \frac{1}{3}\ \pi h[R^2 + Rr + r^2]$ cu. units

$= \frac{1}{3} \times \frac{22}{7} \times 16\ [20^2 + 20(8) + 8^2]$

$= \frac{1}{3} \times \frac{22}{7} \times 16\ [400 + 160 + 64]$

$= \frac{1}{3} \times \frac{22}{7} \times 16 \times 624$

$= 10459\ cm^3$

= 10.459 litre

The cost of milk is ₹ 40 per litre

The cost of 10.459 litres milk = 10.459 × 40

= ₹ 418.36

4. If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum. SEP-21

Solution:

height of the frustum, h = 45 cm,

bottom radii, R = 28 cm,

top radii, r = 7 cm

Volume of the frustum

$= \frac{1}{3}\ \pi h\ [R^2 + Rr + r^2]$ cu.units

$= \frac{1}{3} \times \frac{22}{7} \times 45\ [28^2 + 28 \times 7 + 7^2]$

$= \frac{1}{3} \times \frac{22}{7} \times 45\ [784 + 196 + 49]$

$= \frac{1}{3} \times \frac{22}{7} \times 45 \times 1029$

$= 22 \times 15 \times 147 = 48510\ cm^3$

***

8. Statistics and Probability

2 Marks

1. Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution:

Largest value L = 67; Smallest value S = 18

Range R = L − S = 67 − 18 = 49

Coefficient of range $= \frac{L-S}{L+S}$

Coefficient of range $= \frac{67-18}{67+18} = \frac{49}{85} = 0.576$

2. Find the range of the following distribution.

Age (in years)	16-18	18-20	20-22	22-24	24-26	26-28
Number of students	0	4	6	8	2	2

Solution:

Here

Largest value, L = 28

Smallest Value, S = 18

Range R = L − S

R = 28 − 18 = 10 Years.

3. The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.

Solution:

Range R = 13.67

Largest value L = 70.08

Range R = L − S

13.67 = 70.08 −S

S = 70.08 −13.67 = 56.41

Therefore, the smallest value is 56.41

4. Find the range and coefficient of range of following data SEP-20

(i) 63, 89, 98, 125, 79, 108, 117, 68

(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Solution:

i. 63, 89, 98, 125, 79, 108, 117, 68

L = 125, S = 63

Range, R = L − S = 125 − 63 = 62

Coefficient of Range= $\frac{L-S}{L+S}$

$= \frac{125-63}{125+63} = \frac{62}{188} = 0.33$

ii. 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

L = 61.4, S = 13.6

Range, R = L – S = 61.4 – 13.6 = 47.8

Coefficient of Range = $\frac{L-S}{L+S}$

$= \frac{47.8}{61.4+13.6} = \frac{47.8}{75.0} = 0.64$

5. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.

Solution:

Range, R = 36.8

Smallest Value, S = 13.4

Largest Value, L = R + S

= 36.8 + 13.4 = 50.2

6. Calculate the range of the following data.

Income	400-450	450-500	500-550
Number of workers	8	12	30
Income	550-600	600-650	
Number of workers	21	6	

Solution:

Given: Largest Value, L = 650

Smallest Value,, S = 400

∴ Range = L – S = 650 – 400 = 250

7. Find the standard deviation of first 21 natural numbers.

Solution:

Standard Deviation of first 21 natural numbers,

$\sigma = \sqrt{\frac{n^2-1}{12}}$

$= \sqrt{\frac{(21)^2-1}{12}} = \sqrt{\frac{441-1}{12}} = \sqrt{\frac{440}{12}}$

$= \sqrt{36.66} = 6.05$

8. If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

Solution:

standard deviation of a data, σ = 4.5

each value of the data decreased by 5,

the new standard deviation does not change and it is also 4.5.

9. If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.

Solution:

The new standard deviation of a data is 3.6, and each of the data is divided by 3 then the new standard deviation is also divided by 3.

The new standard deviation = $\frac{3.6}{3}$ =1.2

The new variance = (Standard Deviation)^2

= $\sigma^2 = (1.2)^2 = 1.44$

10. The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.

Solution:

Mean $\bar{x}$ = 25.6

Coefficient of variation, C.V. = 18.75

$C.V = \frac{\sigma}{\bar{x}} \times 100$

$18.75 = \frac{\sigma}{25.6} \times 100$

$\sigma = \frac{18.75 \times 25.6}{100} = 4.8$

11. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.

Solution:

Co-efficient of variation C.V. = $\frac{\sigma}{\bar{x}} \times 100$.

σ = 6.5, $\bar{x}$ =12.5

$CV = \frac{\sigma}{\bar{x}} \times 100 = \frac{6.5}{12.5} \times 100$

$= \frac{6500}{125} = 52\ \%$

12. If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

Solution:

$\bar{x}$ = 15 , C.V. = 48,

$CV = \frac{\sigma}{\bar{x}} \times 100$

$\sigma = \frac{C.V \times \bar{x}}{100} = \frac{48 \times 15}{100} = \frac{720}{100} = 7.2$

13. If n = 5 , x = 6 , x^2 = 765, then calculate the coefficient of variation.

Solution:

n = 5, $\bar{x}$ = 6 , $\Sigma x^2 = 765$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{765}{5} - (6)^2}$$

$$= \sqrt{153 - 36} = \sqrt{117}$$

$$= 10.8$$

$$CV = \frac{\sigma}{\bar{x}} \times 100\%$$

$$= \frac{10.8}{6} \times 100 = \frac{1080}{6} = 180\%$$

14. A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Solution:

Total number of possible outcomes

$n(S) = 5 + 4 = 9$

i) Let A be the event of getting a blue ball.
Number of favourable outcomes for the event A. Therefore, $n(A) = 5$
Probability that the ball drawn is blue.
Therefore, $P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$

ii) A will be the event of not getting a blue ball.
So $P(\overline{A}) = 1 - P(A) = 1 - \frac{5}{9} = \frac{4}{9}$

15. Two coins are tossed together. What is the probability of getting different faces on the coins? **MAY-22**

Solution:

When two coins are tossed together, the sample space is

$S = \{HH, HT, TH, TT\}$; $n(S) = 4$

Let A be the event of getting different faces on the coins.

$A = \{HT, TH\}$; $n(A) = 2$

Probability of getting different faces on the coins is

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

16. A coin is tossed thrice. What is the probability of getting two consecutive tails?

Solution:

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$n(S) = 8$

Event A :

Two Consecutive tails = {HTT, TTH, TTT}

$n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

17. What is the probability that a leap year selected at random will contain 53 Saturdays.

Solution:

A leap year has 366 days.

So it has 52 full weeks and 2 days.

52 Saturdays must be in 52 full weeks.

S = {(Sun - Mon, Mon - Tue, Tue - Wed, Wed - Thu, Thu - Fri, Fri - Sat, Sat - Sun)}

$n(S) = 7$

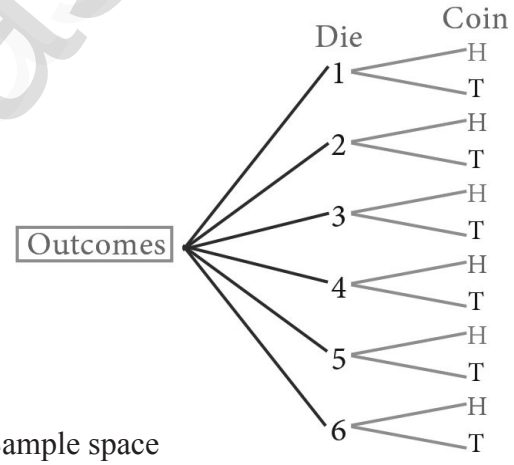
Let A be the event of getting 53rd Saturday.

Then A = {Fri - Sat, Sat - Sun} $n(A) = 2$

Probability of getting 53 Saturdays in a leap year is $P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$

18. A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head. **SEP-21**

Solution:



Sample space

S = {1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T};

$n(S) = 12$

Let A be the event of getting an odd number and a head.

$A = \{1H, 3H, 5H\}$; $n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

19. If P(A) = 0.37 , P(B) = 0.42 , P(A∩B) = 0.09 then find P(A∪B) .

Solution:

$P(A) = 0.37$, $P(B) = 0.42$, $P(A \cap B) = 0.09$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.37 + 0.42 - 0.09 = 0.7$

21. If P(A) = $\frac{2}{3}$, P(B) = $\frac{2}{5}$, P(A∪B) = $\frac{1}{3}$ then find P(A∩B).

Solution:

$$P(A) = \frac{2}{3}, P(B) = \frac{2}{5}, P(A\cup B) = \frac{1}{3}$$

$$P(A\cap B) = P(A) + P(B) - P(A\cup B)$$

$$= \frac{2}{3} + \frac{2}{5} - \frac{1}{3}$$

$$= \frac{10+6-5}{15}$$

$$P(A\cap B) = \frac{11}{15}$$

22. The probability that atleast one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find P($\overline{A}$)+P($\overline{B}$).

Solution:

Given $P(A\cup B) = 0.6,\ P(A\cap B) = 0.2$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(A) + P(B) = P(A\cup B) + P(A\cap B)$$

$$= 0.6 + 0.2$$

$$= 0.8$$

$$\therefore P(\overline{A}) + P(\overline{B}) = 1 - P(A) + 1 - P(B)$$

$$= 2 - [P(A) + P(B)]$$

$$= 2 - 0.8$$

$$= 1.2$$

5 Marks

1. Find the mean and variance of the first *n* natural numbers.

Solution:

$$\text{Mean } \overline{x} = \frac{\text{Sum of all the observations}}{\text{Number of observations}}$$

$$= \frac{\Sigma x_i}{n} = \frac{1+2+3+...+n}{n} = \frac{n(n+1)}{2\times n}$$

$$\overline{x} = \frac{n+1}{2}$$

Variance σ^2

$$= \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 \quad \left|\begin{array}{l}\Sigma x_i^2 = 1^2+2^2+3^2+...+n^2 \\ (\Sigma x_i)^2 = (1+2+3+...+n)^2\end{array}\right.$$

$$= \frac{n(n+1)(2n+1)}{6\times n} - \left|\frac{n(n+1)}{2\times n}\right|^2$$

$$= \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$$

$$= \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right]$$

$$\text{Variance } \sigma^2 = \frac{n+1}{2}\left[\frac{n-1}{6}\right] = \frac{n^2-1}{12}$$

2. Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13 SEP-21

Solution:

When we roll two dice, the sample space is given by

S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) };

n (S) = 36

i) Let A be the event of getting the sum of outcome values equal to 4.

Then A = {(1, 3), (2, 2), (3, 1)}; n(A)= 3.

Probability of getting the sum of outcomes equal to 4 is

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

ii) Let B be the event of getting the sum of outcome values greater than 10.

Then B = {(5, 6), (6, 5), (6, 6)}; n(B) = 3

Probability of getting the sum of outcomes greater than 10 is

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

iii) Let C be the event of getting the sum of outcomes less than 13. Here all the outcomes have the sum value less than 13. Hence C = S .

Therefore, n(C) = n(S) = 36

Probability of getting the total value less than 13 is

$$P(C) = \frac{n(C)}{n(S)} = \frac{36}{36} = 1$$

3. Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Solution:

n(S) = 36

When two dice are rolled together, there will be 6×6 = 36 outcomes.

Let S be the sample space. Then n(S) = 36

Let A be the event of getting a doublet and B be the event of getting face sum 4.

Then A = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

B = {(1,3), (2,2), (3,1)}

$\therefore A \cap B = \{(2,2)\}$

Then, $n(A) = 6$, $n(B) = 3$, $n(A \cap B) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

$\therefore$ P (getting a doublet or a total of 4) = $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$$

Hence, the required probability is $\frac{2}{9}$.

4. If A is an event of a random experiment such that P(A) : P($\overline{A}$)=17:15 and n(S) = 640 then find (i) P($\overline{A}$) (ii) n(A).

Solution:

Given n(S) = 640

$$\frac{P(A)}{P(\overline{A})} = \frac{17}{15}$$

$$\frac{1 - P(\overline{A})}{P(\overline{A})} = \frac{17}{15}$$

$$15[1 - P(\overline{A})] = 17P(\overline{A})$$

$$15 - 15P(\overline{A}) = 17P(\overline{A})$$

$$15 = 15P(\overline{A}) + 17P(\overline{A})$$

$$32P(\overline{A}) = 15$$

$$P(\overline{A}) = \frac{15}{32}$$

$$P(A) = 1 - P(\overline{A})$$

$$= 1 - \frac{15}{32}$$

$$= \frac{32 - 15}{32} = \frac{17}{32}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\frac{17}{32} = \frac{n(A)}{640}$$

$$n(A) = \frac{17 \times \overset{20}{\cancel{640}}}{\cancel{32}}$$

$$n(A) = 340$$

5. Two unbiased dice are rolled once. Find the probability of getting
(i) a doublet (equal numbers on both dice)
(ii) the product as a prime number
(iii) the sum as a prime number
(iv) the sum as 1 SEP-20

Solution:

n(S) = 36

i) A = Probability of getting Doublets (Equal numbers on both dice)

A = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

$$n(A) = 6;\ P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

B = Probability of getting the product of the prime number

ii) B = {(1,2), (1,3), (1,5), (2,1), (3,1), (5,1)}

$$n(B) = 6;\ P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

C = Probability of getting sum of the prime number.

iii) C = {(1,1), (2,1), (1,2), (1,4), (4,1), (1,6), (6,1), (2,3), (2,5), (3,2), (3,4), (4,3), (5,2), (5,6), (6,5)}

$$n(C) = 14;\ P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

iv) D = Probability of getting the sum as 1

$$n(D) = 0;\ P(D) = \frac{n(D)}{n(S)} = 0$$

6. Three fair coins are tossed together. Find the probability of getting
(i) all heads
(ii) atleast one tail
(iii) atmost one head
(iv) atmost two tails

Solution:

Possible Outcomes = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

No.of possible outcomes,

$n(S) = 2 \times 2 \times 2 = 8$

i) A = Probability of getting all heads

A = {HHH} n(A) = 1

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

ii) B = Probability of getting atleast one tail

B = {HHT, HTH, THH, TTT, TTH, THT, HTT}

$n(B) = 7 \quad P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$

iii) C = Probability of getting atmost one head.

C = {TTT, TTH, THT, HTT}

$n(C) = 4 \quad P(C) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

iv) D = Probability of getting atmost two tails.

D = {TTH, THT, HTT, HHT, HTH, THH, HHH}

$n(D) = 7 \quad P(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$

7. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is

(i) white

(ii) black or red

(iii) not white

(iv) neither white nor black

Solution:

S = {5 Red, 6 White, 7 Green, 8 Black}

n(S) = 26

i) A – probability of getting white balls

$n(A) = 6; \quad P(A) = \frac{6}{26} = \frac{3}{13}$

ii) B – Probability of getting black (or) red balls

$n(B) = 8 + 5 = 13; \quad P(B) = \frac{13}{26} = \frac{1}{2}$

iii) C – Probability of not getting white balls

$n(C) = 20; \quad P(C) = \frac{20}{26} = \frac{10}{13}$

iv) D – Probability of getting of neither white nor black

$n(D) = 12; \quad P(D) = \frac{12}{26} = \frac{6}{13}$

8. In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$ then, find the number of defective bulbs.

Solution:

In a box there are 20 non – defective and x defective bulbs

$n(S) = x + 20$

Let A – probability of getting Defective Bulbs

$n(A) = x$

$P(A) = \frac{n(A)}{n(S)} = \frac{x}{x+20}$

From Given data

$\frac{x}{x+20} = \frac{3}{8}$

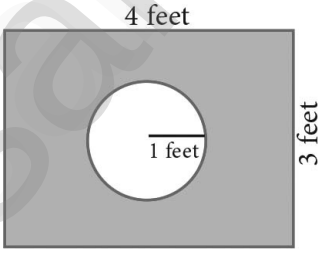
$8x = 3x + 60$

$5x = 60$

$x = 12$

∴ Number of defective bulbls = 12

9. Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game? ($\pi = 3.14$)



Solution:

Total Region = 4 × 3 = 12 sq.ft

∴ n(S) = 12

Winning Region = Area of circle

$= \pi r^2 = \pi(1)^2$

$= \pi = 3.14$ sq. unit

n(A) = 3.14

$P(\text{Winning the Game}) = \frac{n(A)}{n(S)}$

$= \frac{3.14}{12} = \frac{314}{1200}$

$= \frac{157}{600}$

10. The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.

Solution:

$\sigma = 1.2, \ CV = 25.6, \ \bar{x} = ?$

$CV = \frac{\sigma}{\bar{x}} \times 100$

$\bar{x} = \frac{\sigma}{CV} \times 100 = \frac{1.2}{25.6} \times 100 = \frac{1200}{256}$

$\bar{x} = 4.7$

11. Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on
(i) the same day
(ii) different days
(iii) consecutive days?

Solution:

n(S) = 36

i) A be the Probability of Priya and Amuthan to visit shop on same day

A = {(Mon, Mon), (Tue, Tue), (Wed, Wed), (Thurs, Thurs), (Fri, Fri), (Sat, Sat)}

n(A) = 6

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

ii) P (Priya and Amuthan Visit on Different Days)

$$= P(\overline{A}) = 1 - P(A) = 1 - \frac{1}{6} = \frac{5}{6}$$

iii) C be the Probability of Priya and Amuthan to visit on Consequent days

C = {(Mon, Tue), (Tue, Wed), (Wed, Thurs), (Thurs, Fri), (Fri, Sat) (Tue, Mon), (Wed, Tue), (Thurs, Wed), (Fri, Thurs), (Sat, Fri,)}

n(C) = 10

$$P(C) = \frac{n(C)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

12. In a game, the entry fee is ₹ 150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry . If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.

Solution:

S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

n(S) = 8

i) For Receiving double entry Fees have to get Three Heads

A = Probability of Getting three Heads

A = {HHH}

n(A) = 1

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

ii) For getting Entry Fess getting atleast one Head

B = Probability of Getting One or Two Heads

B = {TTH, THT, HTT, HHT, HTH, THH}

n(B) = 6

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{8} = \frac{3}{4}$$

iii) To loss the entry fees, she have to get no Heads

C = Probability of Getting No Heads

C = {TTT}

n(C) = 1

$$P(C) = \frac{n(C)}{n(S)} = \frac{1}{8}$$

13. If A and B are two events such that $P(A) = \frac{1}{4}$ $P(B) = \frac{1}{2}$ and $P(A \text{ and } B) = \frac{1}{8}$, find
(i) P (A or B)
(ii) P(not A and not B).

Solution:

i. $P(A \text{ or } B) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$$P(A \text{ or } B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$$

ii. $P(\text{not A and not B}) = P(\overline{A} \cap \overline{B})$

$= P\left(\overline{A \cup B}\right)$

$= 1 - P(A \cup B)$

$$P(\text{not A and not B}) = 1 - \frac{5}{8} = \frac{3}{8}$$

14. A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

Solution:

Total number of cards = 52 ; n(S) = 52.

Let A be the event of getting a king card.

$$n(A) = 4 ; \quad P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$$

Let B be the event of getting a heart card

$$n(B) = 13; \quad P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$$

Let C be the event of getting a red card

$n(C) = 26; \quad P(C) = \frac{n(C)}{n(S)} = \frac{26}{52}$

$P(A \cap B) = P(\text{getting heart king}) = \frac{1}{52}$

$P(B \cap C) = P(\text{getting red and heart}) = \frac{13}{52}$

$P(A \cap C) = P(\text{getting red king}) = \frac{2}{52}$

$P(A \cap B \cap C) = P(\text{getting heart, king which is red}) = \frac{1}{52}$

Therefore, required probability is

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52}$

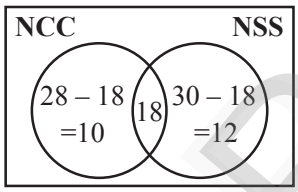
$= \frac{28}{52} = \frac{7}{13}$

15. In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that

(i) The student opted for NCC but not NSS.

(ii) The student opted for NSS but not NCC.

(iii) The student opted for exactly one of them. MAY-22



Solution:

Total number of students n(S) = 50

i. A : A : opted only NCC but not NSS

$P(A) = \frac{n(A)}{n(S)} = \frac{10}{50} = \frac{1}{5}$

ii. B : opted only NSS but not NCC

$P(B) = \frac{n(B)}{n(S)} = \frac{12}{50} = \frac{6}{25}$

iii. C : opted only one

$P(C) = \frac{n(C)}{n(S)} = \frac{(10+12)}{50} = \frac{22}{50} = \frac{11}{25}$

16. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Solution:

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

n(S) =36

A = Probability of getting an even number in the first die.

A = {(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6))}

$n(A) = 18; \quad P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$

B = Probability of getting a total face sum is 8

B = {(2,6), (3,5), (4,4), (5,3), (6,2)}

$n(B) = 5; \quad P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

$A \cap B = \{(2,6), (4, 4), (6, 2)\}$

$n(A \cap B) = 3$

$P(A \cap B) = \frac{3}{36}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$

$= \frac{20}{36} = \frac{5}{9}$

17. A box contains cards numbered 3, 5, 7, 9, … 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

Solution:

S = {3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37}

n(S) = 18

Let A = Multiple of 7

A= {7, 21, 35}, n(A) = 3

$P(A) = \frac{3}{18}$

Let B = a Prime number

B = {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}

$n(B) = 11$; $P(B) = \frac{11}{18}$

$A \cap B = \{7\} \Rightarrow n(A \cap B) = 1$

$P(A \cap B) = \frac{1}{18}$

P(Either A or B) = $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{3}{18} + \frac{11}{18} - \frac{1}{18} = \frac{13}{18}$

18. Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.

Solution:

S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

$n(S) = 8$

A = Probability of getting atmost 2 tails

A = {HHH, HHT, HTH, THH, TTH, THT, HTT}

$n(A) = 7 P(A) = \frac{7}{8}$

B = Probability of getting atmost 2 heads

B = {HHT, HTH, THH, HHH}

$n(B) = 4 P(B) = \frac{4}{8}$

$P(A \cap B) = \frac{4}{8}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{7}{8} + \frac{4}{8} - \frac{4}{8} = \frac{7}{8}$

19. A flower is selected at random from a basket containing 80 yellow, 70 red and 50 white flowers. Find the probability of selecting a yellow or red flower?

Solution:

Total number of flowers

$n(S) = 80 + 70 + 50 = 200$

No. of yellow flowers $n(Y) = 80$

$\therefore P(Y) = \frac{n(Y)}{n(S)} = \frac{80}{200}$

No. of red flowers $n(R) = 70$

$\therefore P(R) = \frac{n(R)}{n(S)} = \frac{70}{200}$

Y and R are mutually exclusive

$P(Y \cup R) = P(Y) + P(R)$

Probability of drawing either a yellow or red flower

$P(Y \cup R) = \frac{80}{200} + \frac{70}{200} = \frac{150}{200} = \frac{3}{4}$

20. In an apartment, in selecting a house from door numbers 1 to 100 randomly, find the probability of getting the door number of the house to be an even number or a perfect square number or a perfect cube number.

Solution:

Total number of houses $n(S) = 100$

Let A be the event of getting door number even.

A = {2, 4, 6, 8, ... 100}

$n(A) = 50$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{50}{100}$

Let B be the event of getting door number perfect square

A = {1,4,9,16,25,36,49,64,81,100}

$n(A) = 10$

$\therefore P(A) = \frac{n(B)}{n(S)} = \frac{10}{100}$

Let C be the event of getting door number perfect cube

A = {1, 8, 27, 64}

$n(A) = 4$

$\therefore P(A) = \frac{n(C)}{n(S)} = \frac{4}{100}$

$P(A \cap B) = P$ (getting even perfect square number) $= \frac{5}{100}$

$P(B \cap C) = P$ (getting even perfect square and perfect cube number) $= \frac{2}{100}$

$P(A \cap C) = P$ (getting even perfect cube number) $= \frac{2}{100}$

$P(A \cap B \cap C) = P$ (getting even perfect square and perfect cube number) $= \frac{1}{100}$

Required probability

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(AB \cap C)$

$= \frac{50}{100} + \frac{10}{100} + \frac{4}{100} - \frac{5}{100} - \frac{2}{100} - \frac{2}{100} + \frac{1}{100}$

$= \frac{65}{100} - \frac{9}{100} = \frac{56}{100} = \frac{14}{25}$

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1 MARK QUESTIONS

1. Relations and Functions

1. If n (A×B) = 6 and A = {1, 3} then n (B) is

A) 1 B) 2
C) 3 D) 6

2. A = {a, b, p}, B = {2, 3}, C = {p, q, r, s} then n[(A ∪ C) × B] is

A) 8 B) 20
C) 12 D) 16

3. If A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8} then state which of the following statement is true.

A) (A×C) ⊂ (B×D) B) (B×D) ⊂ (A×C)
C) (A×B) ⊂ (A×D) D) (D×A) ⊂ (B×A)

4. If there are 1024 relations from a set A = {1, 2, 3, 4, 5} to a set B, then the number of elements in B is

A) 3 B) 2
C) 4 D) 8

5. The range of the relation R = $\{(x, x^2) \mid x$ is a prime number less than 13$\}$ is

A) {2,3,5,7} B) {2,3,5,7,11}
C) {4,9,25,49,121} D) {1,4,9,25,49,121}

6. If the ordered pairs (a+2, 4) and (5, 2a+b) are equal then (a, b) is

A) (2, –2) B) (5, 1)
C) (2, 3) D) (3, –2)

7. Let n(A) = m and n(B) = n then the total number of non-empty relations that can be defined from A to B is

A) m^n B) n^m
C) $2^{mn} - 1$ D) 2^{mn}

8. If {(a, 8), (6, b)} represents an identity function, then the value of a and b are respectively

A) (8, 6) B) (8, 8)
C) (6, 8) D) (6, 6)

9. Let A = {1, 2, 3, 4} and B = {4, 8, 9, 10}. A function f : A → B given by f = {(1, 4), (2, 8), (3, 9), (4, 10)} is a

A) Many-one function B) Identity function
C) One-to-one function D) Into function

10. If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, then f o g is

A) $\frac{3}{2x^2}$ B) $\frac{2}{3x^2}$
C) $\frac{2}{9x^2}$ D) $\frac{1}{6x^2}$

11. If f : A → B is a bijective function and if n(B) = 7, then n (A) is equal to

A) 7 B) 49
C) 1 D) 14

12. Let f and g be two functions given by $f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 7)\}$, $g = \{(0, 2), (1, 0), (2, 4), (-4, 2), (7, 0)\}$ then the range of f o g is

A) {0, 2, 3, 4, 5} B) {–4, 1, 0, 2, 7}
C) {1, 2, 3, 4, 5} D) {0, 1, 2}

13. Let $f(x) = \sqrt{1 + x^2}$ then

A) $f(xy) = f(x).f(y)$ B) $f(xy)^3 \geq f(x).f(y)$
C) $f(xy) \leq f(x).f(y)$ D) None of these

14. If $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ is a function given by g $(x) = \alpha x + \beta$ then the values of a and b are

A) (–1, 2) B) (2, –1)
C) (–1, –2) D) (1, 2)

15. $f(x) = (x + 1)^3 - (x - 1)^3$ represents a function which is

A) linear B) cubic
C) reciprocal D) quadratic

2. Numbers and Sequences

1. Euclid's division lemma states that for positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must satisfy.

A) 1 < r < b B) 0 > r > b
C) 0 ≤ r < b D) 0 < r ≤ b

2. Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are

A) 0, 1, 8 B) 1, 4, 8
C) 0, 1, 3 D) 1, 3, 5

3. If the HCF of 65 and 117 is expressible in the form of $65m - 117$, then the value of m is

A) 4 B) 2
C) 1 D) 3

4. **The sum of the exponents of the prime factors in the prime factorization of 1729 is**
   A) 1 B) 2
   C) 3 D) 4

5. **The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is**
   A) 2025 B) 5220
   C) 5025 D) 2520

6. **$74k \equiv$ _____ (mod 100)**
   A) 1 B) 2
   C) 3 D) 4

7. **Given $F_1 = 1$, $F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is**
   A) 3 B) 5
   C) 8 D) 11

8. **The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P.**
   A) 4551 B) 10091
   C) 7881 D) 13531

9. **If 6 times of 6th term of an A.P. is equal to 7 times the 7th term, then the 13th term of the A.P. is**
   A) 0 B) 6
   C) 7 D) 13

10. **An A.P. consists of 31 terms. If its 16th term is m, then the sum of all the terms of this A.P. is**
    A) 16 m B) 62 m
    C) 31 m D) $\frac{31}{2}$ m

11. **In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P. must be taken for their sum to be equal to 120?**
    A) 6 B) 7
    C) 8 D) 9

12. **If $A = 2^{65}$ and $B = 2^{64} + 2^{63} + 2^{62} + \ldots\ldots + 2^0$ which of the following is true?**
    A) B is 264 more than A
    B) A and B are equal
    C) B is larger than A by 1
    D) A is larger than B by 1

13. **The next term of the sequence 3**
    A) 124 B) 127
    C) 23 D) 181

14. **If the sequence $t_1, t_2, t_3, \ldots\ldots$ are in A.P. then the sequence $t_6, t_{12}, t_{18}, \ldots\ldots$ is**
    A) a Geometric Progression
    B) an Arithmetic Progression
    C) neither an Arithmetic Progression nor a Geometric Progression
    D) a constant sequence

15. **The value of $(1^3 + 2^3 + 3^3 + \ldots\ldots + 15^3) - (1 + 2 + 3 + \ldots\ldots + 15)$ is**
    A) 14400 B) 14200
    C) 14280 D) 14520

3. Algebra

1. **A system of three linear equations in three variables is inconsistent if their planes**
   A) intersect only at a point
   B) intersect in a line
   C) coincides with each other
   D) do not intersect

2. **The solution of the system $x + y - 3z = -6$, $-7y + 7z = 7$, $3z = 9$ is**
   A) $x = 1, y = 2, z = 3$ B) $x = -1, y = 2, z = 3$
   C) $x = -1, y = -2, z = 3$ D) $x = 1, y = -2, z = 3$

3. **If $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$ then the value of k is**
   A) 3 B) 5
   C) 6 D) 8

4. $\frac{3y-3}{y} \div \frac{7y-7}{3y^2}$ **is**
   A) $\frac{9y}{7}$ B) $\frac{9y^3}{(21y-21)}$
   C) $\frac{21y^2-42y+21}{3y^3}$ D) $\frac{7(y^2-2y+1)}{y^2}$

5. $y^2 + \frac{1}{y^2}$ **is not equal to**
   A) $\frac{y^4+1}{y^2}$ B) $\left(y+\frac{1}{y}\right)^2$
   C) $\left(y-\frac{1}{y}\right)^2 + 2$ D) $\left(y+\frac{1}{y}\right)^2 - 2$

6. $\frac{x}{x^2-25}-\frac{8}{x^2-6x+5}$ gives

A) $\frac{x^2-7x+40}{(x-5)(x+5)}$ B) $\frac{x^2+7x+40}{(x-5)(x+5)(x+1)}$

C) $\frac{x^2-7x+40}{(x^2-25)(x+1)}$ D) $\frac{x^2+10}{(x^2-25)(x+1)}$

7. The square root of is $\frac{256x^8y^4z^{10}}{25x^6y^6z^6}$ equal to

A) $\frac{16}{5}\left|\frac{x^2z^4}{y^2}\right|$ B) $16\left|\frac{y^2}{x^2z^4}\right|$

C) $\frac{16}{5}\left|\frac{y}{xz^2}\right|$ D) $\frac{16}{5}\left|\frac{xz^2}{y}\right|$

8. Which of the following should be added to make $x^4 + 64$ a perfect square

A) $4x^2$ B) $16x^2$

C) $8x^2$ D) $-8x^2$

9. The solution of $(2x - 1)^2 = 9$ is equal to

A) -1 B) 2

C) $-1, 2$ D) None of these

10. The values of a and b if $4x^4 - 24x^3 + 76x^2 + ax + b$ is a perfect square are

A) 100, 120 B) 10, 12

C) -120, 100 D) 12, 10

11. If the roots of the equation $q^2x^2 + p^2x + r^2 = 0$ are the squares of the roots of the equation $qx^2 + px + r = 0$, then q, p, r are in _____

A) A.P B) G.P

C) Both A.P and G.P D) none of these

12. Graph of a linear equation is a _____.

A) straight line B) circle

C) parabola D) hyperbola

13. The number of points of intersection of the quadratic polynomial $x^2 + 4x + 4$ with the X-axis is

A) 0 B) 1

C) 0 or 1 D) 2

14. For the given matrix $A = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{pmatrix}$ the order of the matrix A^T is

A) 2×3 B) 3×2

C) 3×4 D) 4×3

15. If A is a 2×3 matrix and B is a 3×4 matrix, how many columns does AB have

A) 3 B) 4

C) 2 D) 5

16. If number of columns and rows are not equal in a matrix then it is said to be a

A) diagonal matrix B) rectangular matrix

C) square matrix D) identity matrix

17. Transpose of a column matrix is

A) unit matrix B) diagonal matrix

C) column matrix D) row matrix

18. Find the matrix X if $2X + \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 9 & 5 \end{pmatrix}$

A) $\begin{pmatrix} -2 & -2 \\ 2 & -1 \end{pmatrix}$ B) $\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$

C) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ D) $\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}$

19. Which of the following can be calculated from the given matrices $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

(i) A^2 (ii) B^2

(iii) AB (iv) BA

A) (i) and (ii) only B) (ii) and (iii) only

C) (ii) and (iv) only D) all of these

20. If $A = A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 1 \\ -2 & 5 \end{pmatrix}$. Which of the following statements are correct?

(i) $AB + C = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$ (ii) $BC = \begin{pmatrix} 0 & 1 \\ 2 & -3 \\ -4 & 10 \end{pmatrix}$

(iii) $BA + C = \begin{pmatrix} 2 & 5 \\ 3 & 0 \end{pmatrix}$ (iv) $(AB)C = \begin{pmatrix} -8 & 20 \\ -8 & 13 \end{pmatrix}$

A) (i) and (ii) only B) (ii) and (iii) only

C) (iii) and (iv) only D) all of these

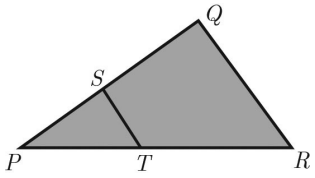
4. Geometry

1. If in triangles ABC and EDF, $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when
   A) $\angle B = \angle E$ B) $\angle A = \angle D$
   C) $\angle B = \angle D$ D) $\angle A = \angle F$

2. In ΔLMN, $\angle L = 60°$, $\angle M = 50°$. If $\Delta LMN \sim \Delta PQR$ then the value of $\angle R$ is
   A) 40° B) 70°
   C) 30° D) 110°

3. If ΔABC is an isosceles triangle with $\angle C = 90°$ and AC = 5 cm, then AB is
   A) 2.5 cm B) 5 cm
   C) 10 cm D) $5\sqrt{2}$ cm

4. In a given figure ST || QR, PS = 2 cm and SQ = 3 cm. Then the ratio of the area of ΔPQR to the area of ΔPST is
   A) 25 : 4
   B) 25 : 7
   C) 25 : 11
   D) 25 : 13

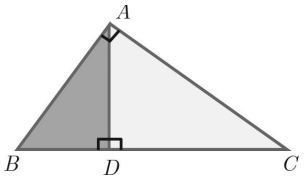


5. The perimeters of two similar triangles ΔABC and ΔPQR are 36 cm and 24 cm respectively. If PQ = 10 cm, then the length of AB is
   A) $6\frac{2}{3}$ cm B) $\frac{10\sqrt{6}}{3}$ cm
   C) $66\frac{2}{3}$ cm D) 15 cm

6. If in ΔABC , DE || BC. AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm then the length of AE is
   A) 1.4 cm B) 1.8 cm
   C) 1.2 cm D) 1.05 cm

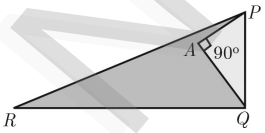
7. In a ΔABC, AD is the bisector of $\angle BAC$. If AB = 8 cm, BD = 6 cm and DC = 3 cm. The length of the side AC is
   A) 6 cm B) 4 cm
   C) 3 cm D) 8 cm

8. In the adjacent figure $\angle BAC = 90°$ and $AD \perp BC$ then
   A) $BD \cdot CD = BC^2$
   B) $AB \cdot AC = BC^2$
   C) $BD \cdot CD = AD^2$
   D) $AB \cdot AC = AD^2$



9. Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m, what is the distance between their tops?
   A) 13 m B) 14 m
   C) 15 m D) 12.8 m

10. In the given figure, PR = 26 cm, QR = 24 cm, $\angle PAQ = 90°$, PA = 6 cm and QA = 8 cm. Find $\angle PQR$
    A) 80° B) 85°
    C) 75° D) 90°

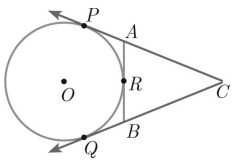


11. A tangent is perpendicular to the radius at the
    A) centre B) point of contact
    C) infinity D) chord

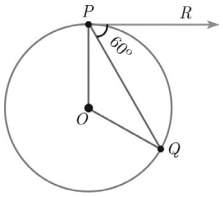
12. How many tangents can be drawn to the circle from an exterior point?
    A) one B) two
    C) infinite D) zero

13. The two tangents from an external points P to a circle with centre at O are PA and PB. If $\angle APB = 70°$ then the value of $\angle AOB$ is
    A) 100° B) 110°
    C) 120° D) 130°

14. In figure CP and CQ are tangents to a circle with centre at O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7 cm, then the length of BR is
    A) 6 cm
    B) 5 cm
    C) 8 cm
    D) 4 cm



15. In figure if PR is tangent to the circle at P and O is the centre of the circle, then $\angle POQ$ is
    A) 120°
    B) 100°
    C) 110°
    D) 90°



5. Coordinate Geometry

1. The area of triangle formed by the points (−5, 0), (0, −5) and (5, 0) is

A) 0 sq.units B) 25 sq.units
C) 5 sq.units D) none of these

2. A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall to be the Y axis. The path travelled by the man is

A) $x = 10$ B) $y = 10$
C) $x = 0$ D) $y = 0$

3. The straight line given by the equation $x = 11$ is

A) parallel to X axis
B) parallel to Y axis
C) passing through the origin
D) passing through the point (0, 11)

4. If (5, 7), (3, p) and (6, 6) are collinear, then the value of p is

A) 3 B) 6
C) 9 D) 12

5. The point of intersection of $3x - y = 4$ and $x + y = 8$ is

A) (5, 3) B) (2, 4)
C) (3, 5) D) (4, 4)

6. The slope of the line joining (12, 3), (4, a) is $\frac{1}{8}$. The value of 'a' is

A) 1 B) 4
C) –5 D) 2

7. The slope of the line which is perpendicular to a line joining the points (0, 0) and (–8, 8) is

A) –1 B) 1
C) $\frac{1}{3}$ D) –8

8. If slope of the line PQ is $\frac{1}{\sqrt{3}}$ then slope of the perpendicular bisector of PQ is

A) $\sqrt{3}$ B) $-\sqrt{3}$
C) $\frac{1}{\sqrt{3}}$ D) 0

9. If A is a point on the Y axis whose ordinate is 8 and B is a point on the X axis whose abscissae is 5 then the equation of the line AB is

A) $8x + 5y = 40$ B) $8x - 5y = 40$
C) $x = 8$ D) $y = 5$

10. The equation of a line passing through the origin and perpendicular to the line $7x - 3y + 4 = 0$ is

A) $7x - 3y + 4 = 0$ B) $3x - 7y + 4 = 0$
C) $3x + 7y = 0$ D) $7x - 3y = 0$

11. Consider four straight lines

(i) $l_1 : 3y = 4x + 5$; (ii) $l_2 : 4y = 3x - 1$
(iii) $l_3 : 4y + 3x = 7$ (iv) $l_4 : 4x + 3y = 2$

Which of the following statement is true?

A) l_1 and l_2 are perpendicular
B) l_1 and l_4 are parallel
C) l_2 and l_4 are perpendicular
D) l_2 and l_3 are parallel

12. A straight line has equation $8y = 4x + 21$. Which of the following is true?

A) The slope is 0.5 and the y intercept is 2.6
B) The slope is 5 and the y intercept is 1.6
C) The slope is 0.5 and the y intercept is 1.6
D) The slope is 5 and the y intercept is 2.6

13. When proving that a quadrilateral is a trapezium, it is necessary to show

A) Two sides are parallel
B) Two parallel and two non-parallel sides
C) Opposite sides are parallel
D) All sides are of equal length

14. When proving that a quadrilateral is a parallelogram by using slopes you must find

A) The slopes of two sides
B) The slopes of two pair of opposite sides
C) The lengths of all sides
D) Both the lengths and slopes of two sides

15. (2, 1) is the point of intersection of two lines.

A) $x - y - 3 = 0$; $3x - y - 7 = 0$
B) $x + y = 3$; $3x + y = 7$
C) $3x + y = 3$; $x + y = 7$
D) $x + 3y - 3 = 0$; $x - y - 7 = 0$

6. Trigonometry

1. The value of $\sin^2\theta + \frac{1}{1+\tan^2\theta}$ is equal to
   A) $\tan^2\theta$ B) 1
   C) $\cot^2\theta$ D) 0

2. $\tan\theta\operatorname{cosec}^2\theta - \tan\theta$ is equal to
   A) $\sec\theta$ B) $\cot 2\theta$
   C) $\sin\theta$ D) $\cot\theta$

3. If $(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha$, then the value of k is equal to
   A) 9 B) 7
   C) 5 D) 3

4. If $\sin\theta + \cos\theta = a$ and $\sec\theta + \operatorname{cosec}\theta = b$, then the value of $b(a^2 - 1)$ is equal to
   A) 2a B) 3a
   C) 0 D) 2ab

5. If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$, then $x^2 - \frac{1}{x^2}$ is equal to
   A) 25 B) $\frac{1}{25}$
   C) 5 D) 1

6. If $\sin\theta = \cos\theta$, then $2\tan^2\theta + \sin^2\theta - 1$ is equal to
   A) $\frac{-3}{2}$ B) $\frac{3}{2}$
   C) $\frac{2}{3}$ D) $\frac{-2}{3}$

7. If $x = a\tan\theta$ and $y = b\sec\theta$ then
   A) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
   C) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ D) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

8. $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$ is equal to
   A) 0 B) 1
   C) 2 D) –1

9. $a\cot\theta + b\operatorname{cosec}\theta = p$ and $b\cot\theta + a\operatorname{cosec}\theta = q$ then $p^2 - q^2$ is equal to
   A) $a^2 - b^2$ B) $b^2 - a^2$
   C) $a^2 + b^2$ D) $b - a$

10. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3} : 1$, then the angle of elevation of the sun has measure
   A) 45° B) 30°
   C) 90° D) 60°

11. The electric pole subtends an angle of 30° at a point on the same level as its foot. At a second point 'b' metres above the first, the depression of the foot of the pole is 60°. The height of the pole (in metres) is equal to
   A) $\sqrt{3}\,b$ B) –
   C) $\frac{b}{2}$ D) $\frac{b}{\sqrt{3}}$

12. A tower is 60 m heigh. Its shadow is x metres shorter when the sun's altitude is 45° than when it has been 30°, then x is equal to
   A) 41.92 m B) 43.92 m
   C) 43 m D) 45.6 m

13. The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried building are 30° and 60° respectively. The height of the multistoried building and the distance between two buildings (in metres) is
   A) 20, $10\sqrt{3}$ B) 30, $5\sqrt{3}$
   C) 20, 10 D) 30, $10\sqrt{3}$

14. Two persons are standing 'x' metres apart from each other and the height of the first person is double that of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the shorter person (in metres) is
   A) $\sqrt{2}\,x$ B) $\frac{x}{2\sqrt{2}}$
   C) $\frac{x}{\sqrt{2}}$ D) $2x$

15. The angle of elevation of a cloud from a point h metres above a lake is b. The angle of depression of its reflection in the lake is 45°. The height of location of the cloud from the lake is
   A) $\frac{h(1+\tan\beta)}{1-\tan\beta}$ B) $\frac{h(1-\tan\beta)}{1+\tan\beta}$
   C) $h\tan(45° - \beta)$ D) none of these

7. Mensuration

1. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is

A) 60π cm^2 B) 68π cm^2

C) 120π cm^2 D) 136π cm^2

2. If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is

A) $4\pi r^2$ sq. units B) $6\pi r^2$ sq. units

C) $3\pi r^2$ sq. units D) $8\pi r^2$ sq. units

3. The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be

A) 12 cm B) 10 cm

C) 13 cm D) 5 cm

4. If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is

A) 1:2 B) 1:4

C) 1:6 D) 1:8

5. The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is

A) $\frac{9\pi h^2}{8}$ sq.units B) $24\pi h^2$ sq.units

C) $\frac{8\pi h^2}{9}$ sq.units D) $\frac{56\pi h^2}{9}$ sq.units

6. In a hollow cylinder, the sum of the external and internal radii is 14 cm and the width is 4 cm. If its height is 20 cm, the volume of the material in it is

A) 5600π cm^3 B) 11200π cm^3

C) 56π cm^3 D) 3600π cm^3

7. If the radius of the base of a cone is tripled and the height is doubled then the volume is

A) made 6 times B) made 18 times

C) made 12 times D) unchanged

8. The total surface area of a hemi-sphere is how much times the square of its radius.

A) π B) 4π

C) 3π D) 2π

9. A solid sphere of radius x cm is melted and cast into a shape of a solid cone of same radius. The height of the cone is

A) $3x$ cm B) x cm

C) $4x$ cm D) $2x$ cm

10. A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm. Then, the volume of the frustum is

A) 3328π cm^3 B) 3228π cm^3

C) 3240π cm^3 D) 3340π cm^3

11. A shuttle cock used for playing badminton has the shape of the combination of

A) a cylinder and a sphere

B) a hemisphere and a cone

C) a sphere and a cone

D) frustum of a cone and a hemisphere

12. A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r^2 units. Then $r_1 : r_2$ is

A) 2:1 B) 1:2

C) 4:1 D) 1:4

13. The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is

A) $\frac{4}{3}\pi$ B) $\frac{10}{3}\pi$

C) 5π D) $\frac{20}{3}\pi$

14. The height and radius of the cone of which the frustum is a part are h_1 units and r_1 units respectively. Height of the frustum is h_2 units and radius of the smaller base is r_2 units. If $h_2 : h_1 : = 1 : 2$ then $r_2 : r_1$ is

A) 1 : 3 B) 1 : 2

C) 2 : 1 D) 3 : 1

15. The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is

A) 1 : 2 : 3 B) 2 : 1 : 3

C) 1 : 3 : 2 D) 3 : 1 : 2

8. Statistics and Probability

1. Which of the following is not a measure of dispersion?

A) Range B) Standard deviation

C) Arithmetic mean D) Variance

2. The range of the data 8, 8, 8, 8, 8, ..., 8 is

A) 0 B) 1

C) 8 D) 3

3. The sum of all deviations of the data from its mean is

A) Always positive B) always negative

C) zero D) non-zero integer

4. The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all deviations is

A) 40000 B) 160900

C) 160000 D) 30000

5. Variance of first 20 natural numbers is

A) 32.25 B) 44.25

C) 33.25 D) 30

6. The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is

A) 3 B) 15

C) 5 D) 225

7. If the standard deviation of x, y, z is p then the standard deviation of $3x$ + 5, 3y + 5, 3z + 5 is

A) 3p + 5 B) 3p

C) p + 5 D) 9p +15

8. If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is

A) 3.5 B) 3

C) 4.5 D) 2.5

9. Which of the following is incorrect?

A) $P(A) > 1$ B) $0 \le P(A) \le 1$

C) $P(\varphi) = 0$ D) $P(A) + P(\overline{A}) = 1$

10. The probability a red marble selected at random from a jar containing p red, q blue and r green marbles is

A) $\frac{q}{p+q+r}$ B) $\frac{p}{p+q+r}$

C) $\frac{p+q}{p+q+r}$ D) $\frac{p+r}{p+q+r}$

11. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is

A) $\frac{3}{10}$ B) $\frac{7}{10}$

C) $\frac{3}{9}$ D) $\frac{7}{9}$

12. The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$ then the value of x is

A) 2 B) 1

C) 3 D) 1.5

13. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is $\frac{1}{9}$, then the number of tickets bought by Kamalam is

A) 5 B) 10

C) 15 D) 20

14. If a letter is chosen at random from the English alphabets {a, b, ..., z}, then the probability that the letter chosen precedes x

A) $\frac{12}{13}$ B) $\frac{1}{13}$

C) $\frac{23}{26}$ D) $\frac{3}{26}$

15. A purse contains 10 notes of ₹ 2000, 15 notes of ₹ 500, and 25 notes of ₹ 200. One note is drawn at random. What is the probability that the note is either a ₹ 500 note or ₹ 200 note?

A) $\frac{1}{5}$ B) $\frac{3}{10}$

C) $\frac{2}{3}$ D) $\frac{4}{5}$

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ANSWERS

1. Relations and Functions							
1.C	2.C	3.A	4.B	5.C	6.D	7.C	8.A
9.C	10.C	11.A	12.D	13.C	14.B	15.D	

2. Numbers and Sequences							
1.C	2.A	3.B	4.C	5.D	6.A	7.D	8.C
9.A	10.C	11.C	12.D	13.B	14.B	15.C	

3. Algebra							
1.D	2.A	3.B	4.A	5.B	6.C	7.D	8.B
9.C	10.C	11.B	12.A	13.B	14.D	15.B	16.B
17.D	18.B	19.C	20.A				

4. Geometry							
1.C	2.B	3.D	4.A	5.D	6.A	7.B	8.C
9.A	10.D	11.B	12.B	13.B	14.D	15.A	

5. Coordinate Geometry							
1.B	2.A	3.B	4.C	5.C	6.D	7.B	8.B
9.A	10.C	11.C	12.A	13.B	14.B	15.B	

6. Trigonometry							
1.B	2.D	3.B	4.A	5.B	6.B	7.A	8.C
9.B	10.D	11.B	12.B	13.D	14.B	15.A	

7. Mensuration							
1.D	2.A	3.A	4.B	5.C	6.B	7.B	8.C
9.C	10.A	11.D	12.A	13.A	14.B	15.D	

8. Statistics and Probability							
1.C	2.A	3.C	4.B	5.C	6.D	7.B	8.A
9.A	10.B	11.B	12.B	13.C	14.C	15.D	

Problems for Practice

GRAPH

1. Discuss the nature of solutions of the following quadratic equations.
   (i) $x^2+x-12=0$ (ii) $x^2-8x+16=0$
   (iii) $x^2+2x+5=0$
2. Draw the graph of $y=2x^2$ and hence solve $2x^2-x-6=0$
3. Draw the graph of $y=x^2+4x+3$ and hence find the roots of $x^2+x+1=0$
4. Draw the graph of $y=x^2+x-2$ and hence solve $x^2+x-2=0$.
5. Draw the graph of $y=x^2+x-2$ and hence solve $x^2+x-2=0$.
6. Graph the following quadratic equations and state their nature of solutions.
   (i) $x^2-9x+20=0$ (ii) $x^2-4x+4=0$
   (iii) $x^2+x+7=0$ (iv) $x^2-9=0$
   (v) $x^2-6x+9=0$ (vi) $(2x-3)(x+2)=0$
7. Draw the graph of $y=x^2-4$ and hence solve $x^2-x-12=0$
8. Draw the graph of $y=x^2+x$ and hence solve $x^2+1=0$
9. Draw the graph of $y=x^2+3x+2$ and use it to solve $x^2+2x+1=0$
10. Draw the graph of $y=x^2+3x-4$ and hence use it to solve $x^2+3x-4=0$
11. Draw the graph of $y=x^2-5x-6$ and hence solve $x^2-5x-14=0$
12. Draw the graph of $y=2x^2-3x-5$ and hence solve $2x^2-4x-6=0$
13. Draw the graph of $y=(x-1)(x+3)$ and hence solve $x^2-x-6=0$

GEOMETRY

1. Construct a ΔPQR in which PQ = 8 cm, R = 60° and the median RG from R to PQ is 5.8 cm. Find the length of the altitude from R to PQ.
2. Construct a triangle ΔPQR such that QR = 5 cm, $\angle P = 30°$ and the altitude from P to QR is of length 4.2 cm.
3. Draw a triangle ABC of base BC = 8 cm, A = 60° and the bisector of $\angle A$ meets BC at D such that BD = 6 cm.
4. Construct a ΔPQR which the base PQ = 4.5 cm, $\angle R = 35°$ and the median from R to RG is 6 cm.
5. Construct a ΔPQR in which QR = 5cm, P = 40° and the median PG from P to QR is 4.4 cm. Find the length of the altitude from P to QR.
6. Construct a ΔPQR such that QR = 6.5 cm, P = 60° and the altitude from P to QR is of length 4.5 cm.
7. Construct a ΔABC such that AB = 5.5 cm, C = 25° and the altitude from C to AB is 4 cm.
8. Draw a triangle ABC of base BC = 5.6 cm, A = 40° and the bisector of $\angle A$ meets BC at D such that CD = 4 cm.
9. Draw ΔPQR such that PQ = 6.8 cm, vertical angle is 50° and the bisector of the vertical angle meets the base at D where PD = 5.2 cm.

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