CHENNAI DISTRICT

Class : 11 Register Number

COMMON HALF YEARLY EXAMINATION - 2024 - 25

BUSINESS MATHEMATICS AND STATISTICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

PART - I $20 \times 1 = 20$

1. Answer all the questions by choosing the correct answer from the given 4 alternatives
2. Write question number, correct option and corresponding answer
3. Each question carries 1 mark

1. The cofactor of -7 in the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

(a) -18 (b) 18 (c) -7 (d) 7

2. If A and B non-singular matrix then, which of the following is incorrect?

(a) $A^2 = I$ implies $A^{-1} = A$ (b) $I^{-1} = I$

(c) If $AX = B$ then $X = B^{-1}A$ (d) If A is square matrix of order 3 then $|\text{adj } A| = |A|^2$

3. The greatest positive integer which divide $n(n+1)(n+2)(n+3)$ for all $n \in N$ is

(a) 2 (b) 6 (c) 20 (d) 24

4. There are 10 true or false questions in an examination. Then these questions can be answered in

(a) 240 ways (b) 120 ways (c) 1024 ways (d) 100 ways

5. The x-intercept of the straight line $3x + 2y - 1 = 0$ is

(a) 3 (b) 2 (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

6. The equation of the circle with centre on the x axis and passing through the origin is

(a) $x^2 - 2ax + y^2 = 0$ (b) $y^2 - 2ay + x^2 = 0$ (c) $x^2 + y^2 = a^2$ (d) $x^2 - 2ay + y^2 = 0$

7. The value of $\cos(-480°)$ is

(a) $\sqrt{3}$ (b) $-\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

8. The value of $\frac{3\tan 10° - \tan^3 10°}{1 - 3\tan^2 10°}$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$

9. The minimum value of the function $f(x) = |x|$ is

(a) 0 (b) -1 (c) $+1$ (d) $-\infty$

10. If $y = \log x$, then $y_2 =$

(a) $\frac{1}{x}$ (b) $-\frac{1}{x^2}$ (c) $-\frac{2}{x^2}$ (d) e^2

11. Relationship among MR, AR and η_d is

(a) $\eta_d = \frac{AR}{AR - MR}$ (b) $\eta_d = AR - MR$ (c) $MR = AR = \eta_d$ (d) $AR = \frac{MR}{\eta_d}$

12. If $q = 1000 + 8p_1 - p_2$, then $\frac{\partial q}{\partial p_1}$ is

(a) -1 (b) 8 (c) 1000 (d) $1000 - p_2$

13. If ' a ' is the annual payment, ' n ' is the number of periods and ' i ' is compound interest for ₹ 1 then future amount of the ordinary annuity is

(a) $A = \frac{a}{i}(1+i)[(1+i)^n - 1]$ (b) $A = \frac{a}{i}[(1+i)^n - 1]$

(c) $P = \frac{a}{i}$ (d) $P = \frac{a}{i}(1+i)[1-(1+i)^{-n}]$

14. Example of contingent annuity is

(a) Installments of payment for a plot of land

(b) An endowment fund to give scholarships to a student

(c) Personal loan from a bank

(d) All the above

15. The correct relationship among A.M.,G.M.and H.M.is

(a) A. M. $< G.M. < H.M.$ (b) G. M. ≥ A. M. ≥ H. M. (c) H.M. ≥ G.M. ≥ A.M. (d) A · M. ≥ G · M. ≥ H · M.

16. The events A and B are independent if

(a) $P(A \cap B) = 0$ (b) $P(A \cap B) = P(A) \times P(B)$ (c) $P(A \cap B) = P(A) + P(B)$ (d) $P(A \cup B) = P(A) \times P(B)$

17. Correlation co-efficient lies between

(a) 0 to ∞ (b) -1 to $+1$ (c) -1 to 0 (d) -1 to ∞

18. The correlation coefficient

(a) $r = \pm\sqrt{b_{xy} \times b_{yx}}$ (b) $r = \frac{1}{b_{xy} \times b_{yx}}$ (c) $r = b_{xy} \times b_{yx}$ (d) $r = \pm\sqrt{\frac{1}{b_{xy} \times b_{yx}}}$

19. A solution which maximizes or minimizes the given LPP is called

(a) a solution (b) a feasible solution (c) an optimal solution (d) none of these

20. Network problems have advantage in terms of project

(a) Scheduling (b) Planning (c) Controlling (d) All the above

PART – II

1. **Answer any 7 questions** **7 x 2 = 14**
2. **Each question carries 2 marks**
3. **Question number 30 is compulsory**

21. The technology matrix of an economic system of two industries is $\begin{bmatrix} 0.50 & 0.30 \\ 0.41 & 0.33 \end{bmatrix}$. Test whether the system is viable as per Hawkins Simon conditions.

22. Find the equation of the circle having the centre (3,5) and radius 5 units

23. For the function $y = x^3 + 19$, find the values of x when its marginal value is equal to 27 .

24. A man travelled by car for 3 days. He covered 480 km each day. On the first day he drove for 10 hours at 48 km. an hour. On the second day, he drove for 12 hours at 40 km an hour and for the last day he drove for 15 hours at 32 km. What is his average speed?

25. From a class of 32 students, 4 students are to be chosen for a competition. In how many ways can this be done?

26. Find y_2 for the function : $y = e^{3x+2}$

27. The chairman of a society wishes to award a gold medal to a student getting highest marks in Business Mathematics and Statistics. If this medal costs to ₹ 9,000 every year and the rate of compound interest is 15%, then what amount is to be deposited now.

28. From the following data calculate the correlation coefficient $\Sigma xy = 120, \Sigma x^2 = 90, \Sigma y^2 = 640$.

29. Prove that $\frac{\sin(-\theta)\tan(90^\circ-\theta)\sec(180^\circ-\theta)}{\sin(180+\theta)\cot(360-\theta)\operatorname{cosec}(90^\circ-\theta)} = 1$

30. Find the elasticity of supply for the supply function $x = 2p^2 + 5$ when $p = 3$.

PART - III

1. **Answer any 7 questions**
2. **Each question carries 3 marks** $\underline{7 \times 3 = 21}$
3. **Question number 40 is compulsory**

31. If $\lim_{x\to 2} \frac{x^n - 2^n}{x-2} = 448$, then find the least positive integer n.

32. If $X = \begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & p & q \end{bmatrix}$ then, find p, q if $Y = X^{-1}$

33. If the production of a firm is given by $P = 4LK - L^2 + K^2, L > 0, K > 0$, Prove that $L\frac{\partial P}{\partial L} + K\frac{\partial P}{\partial K} = 2P$.

34. Evaluate : $\frac{(3!)! \times 2!}{5!}$

35. If the dividend received from 9% of ₹20 shares is ₹1,620, then find the number of shares.

36. The parabola $y^2 = kx$ passes through the point $(4, -2)$. Find its latus rectum and focus.

37. An unbiased die is thrown twice. Let the event A be odd number on the first throw and B the event odd number on the second throw. Check whether A and B events are independent.

38. If $\tan(x + y) = 42$ and $x = \tan^{-1}(2)$, then find y.

39. Draw the event oriented network for the following data:

Events	1	2	3	4	5	6	7
Immediate Predecessors	–	1	1	2,3	3	4,5	5,6

40. If $\sin y = x\sin(a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

PART – IV

1. **Answer all the questions** $\underline{7 \times 5 = 35}$
2. **Each question carries 5 marks**

41. a) Find the equation of the circle passing through the points (0,0), (1,2) and (2,0).

(OR)

b) Solve by matrix inversion method: $3x - y + 2z = 13; 2x + y - z = 3; x + 3y - 5z = -8$

42. a) Resolve into partial fractions : $\frac{x-2}{(x+2)(x-1)^2}$

(OR)

b) If $A + B = 45^\circ$, prove that $(1 + \tan A)(1 + \tan B) = 2$ and hence deduce the value of $\tan 22\frac{1}{2}^\circ$

43. a) Calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity of the project given below and determine the Critical path of the project and duration to complete the project.

Activity	1 – 2	1 – 3	1 – 5	2 – 3	2 – 4	3 – 4	3 – 5	3 – 6	4 – 6	5 – 6
Duration (In week)	8	7	12	4	10	3	5	10	7	4

(OR)

(OR)

b) Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5...(2n-1)2^n x^n}{n!}$

44. a) Calculate Karl Pearson's coefficient of correlation from the following data:

X:	6	8	12	15	18	20	24	28	31
Y:	10	12	15	15	18	25	22	26	28

(OR)

b) If $y = a\cos mx + b\sin mx$, then show that $y_2 + m^2 y = 0$.

45. a) Find the stationary values and stationary points for the function: $f(x) = 2x^3 + 9x^2 + 12x + 1$.

(OR)

b) Suppose the inter-industry flow of the product of two industries are given as under.

Production Sector	Consumption sector X	Consumption sector Y	Domestic demand	Total output
X	30	40	50	120
Y	20	10	30	60

Determine the technology matrix and test Hawkin's -Simon conditions for the viability of the system. If the domestic demand changes to 80 and 40 units respectively, what should be the gross output of each sector in order to meet the new demands.

46. a) Which is better investment: 12% ₹ 20 shares at ₹ 16 (or) 15% ₹ 20 shares at ₹ 24 .

(OR)

b) Solve the linear programming problem by graphical method.

Maximize $Z = 6x_1 + 8x_2$ subject to constraints $30x_1 + 20x_2 \le 300$; $5x_1 + 10x_2 \le 110$; and $x_1, x_2 \ge 0$.

47. a) The heights (in cm.) of a group of fathers and sons are given below:

Heights of the fathers	158	166	163	165	167	170	167	172	177	181
Heights of the sons	163	158	167	170	160	180	170	175	172	175

Find the lines of regression and estimate the height of son when the height of the father is 164 cm.

(OR)

b) In a shooting test the probability of hitting the target are $\frac{3}{4}$ for A, $\frac{1}{2}$ for B and $\frac{2}{3}$ for C. If all of them fire at the same target, calculate the probabilities that

(i) All the three hit the target

(ii) Only one of them hits the target

(iii) At least one of them hits the target.

Half yearly Exam

11th std Business Maths

Part - I.

S.No	OPTION	ANSWER.
1.	b	18
2.	c	If $Ax = B$, then $x = B^{-1}A$
3.	d	24
4	c	1024 Ways
5.	c	$1/3$
6	a	$x^2 - 2ax + y^2 = 0$
7.	d	$-1/2$
8	a	$1/\sqrt{3}$
9.	a	0
10	b	$-1/x^2$
11	a	$\eta_d = \frac{AR}{AR - MR}$
12	b	8.
13	b	$A = \frac{a}{i}\left[(1+i)^{n} - 1\right]$
14	b	An endowment fund to give Scholarships to a student
15	d	$AM \geq GM \geq HM$
16	b	$p(A \cap B) = p(A) - p(B)$
17	b	-1 to $+1$
18	a	$\gamma = \pm\sqrt{b_{xy} \times b_{yx}}$
19	c	an optimal Solution
20	d	All of the Above.

Part - II.

21.

$$I-B = \begin{bmatrix} 0.50 & -0.30 \\ -0.41 & 0.67 \end{bmatrix}$$

$$|I-B| = \begin{vmatrix} 0.50 & -0.30 \\ -0.41 & 0.67 \end{vmatrix}$$

$$= 0.218 > 0$$

∴ Hawkins Simon Condition Satisfied

22.

Center (3,5)

$r = 5$

$(x-3)^2 + (y-5)^2 = 5^2$

$x^2 + y - 6x - 10y + 9 = 0$

23.

$\frac{dy}{dx} = 3x^2 + 0$

$\frac{dy}{dx} = 27$

$3x^2 = 27$

$x^2 = 9$ $\boxed{x = 3}$

24.

x	f	f/x
48	10	0.2083
40	12	0.3
32	15	0.47
	37	0.9771

$HM = \frac{N}{\Sigma(f/x)}$

$= \frac{37}{0.9771}$

$= 37.86$ km/hr.

25

$32C_4 = \frac{32!}{4!(28!)}$

26.

$y_1 = e^{3x+2}$

$\frac{d}{dx}(3x+2) = e^{3x+2}(3)$

$y_2 = \frac{d}{dx}(3e^{3x+2})$

$= 3e^{3x+2}(3)$

$= 9(e^{3x+2})$

27. $a = 9000$

$i = 0.15$

$P = \frac{a}{i}$

$= \frac{9000}{0.15}$

$=$ ₹ 60,000

$\boxed{P = ₹ 60,000}$

28.

$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2}\sqrt{\Sigma y^2}} = \frac{120}{\sqrt{57600}}$

$= r = \frac{120}{240}$

$= 0.5$

$\boxed{r = 0.5}$

29.

$$\frac{\sin(-\theta)\tan(90^\circ-\theta)\sec(180-\theta)}{\sin(180+\theta)\cot(360^\circ-\theta)\csc(90^\circ-\theta)}$$

$$= \frac{(-\sin\theta)\cot(-\sec\theta)}{(-\sin\theta)(-\cot\theta)\sec\theta} = 1.$$

Hence proved.

30.

$x = 2p^2+5$ $p = 3.$

$$\frac{dx}{dp} = 2(2p)$$

$$= 4p$$

$$\eta_s = \frac{p}{x}\frac{dx}{dp}$$

$$= \frac{4p^2}{2p^2+5}$$

$$= \frac{4(9)}{18+5}$$

$$= 36/23$$

Part - III.

31. $\lim_{x\to 2} \frac{x^n - 2^n}{x-2} = 448$

$n(2)^{n-1} = 7(2)^6$

$n(2)^{n-1} = 7(2^{7-1})$

$= \boxed{n=7}$

32.

$$X^{-1} = \frac{1}{|X|}\operatorname{adj} X$$

$$|X| = 8\begin{vmatrix}1 & 2\\ -1 & -4\end{vmatrix} + 1\begin{vmatrix}-5 & 2\\ 10 & -4\end{vmatrix} - 3\begin{vmatrix}5 & 1\\ 10 & -1\end{vmatrix}$$

$= -16+15$

$= -1$

$|X| = -1$

$$\operatorname{adj} X = \begin{pmatrix}-2 & -1 & 1\\ 0 & -2 & -1\\ -5 & -2 & 3\end{pmatrix}$$

$$X^{-1} = \frac{1}{-1}\begin{pmatrix}-2 & -1 & 1\\ 0 & -2 & -1\\ -5 & -2 & 3.\end{pmatrix}$$

$y = x^{-1}$

$$\begin{pmatrix}2 & 1 & -1\\ 0 & 2 & 1\\ 5 & p & q\end{pmatrix} = \begin{pmatrix}2 & 1 & -1\\ 0 & 2 & 1\\ 5 & 2 & -3\end{pmatrix}$$

$$\boxed{\begin{array}{l} p = 2\\ q = -3\end{array}}$$

33.

$P = 4LK - L^2 + K^2$

$P(tL, tK) = 4t^2LK - t^2L^2 + t^2K^2$

$= t^2(4LK - L^2 + K^2)$

$= t^2 P$

$\therefore$ It is a homogeneous function of degree 2.

$\boxed{n=2}$

$$L\frac{\partial P}{\partial L}+k\frac{\partial P}{\partial k}=2p$$

Hence proved.

34 $\frac{3!\times 2!}{5!}$

$$=\frac{3\times2\times1\times2\times1}{5\times4\times3\times2\times1}=\frac{1}{10}$$

35. Dividend = No of Shares × FV × Rate %

$$x=\frac{1620\times100}{120\times9}$$

$\boxed{x=900 \text{ Shares}}$

36. $k=1$

$y^2=x$

Latus rectum

$4a=1$

$a=\frac{1}{4}$

focus $(a,0)$

$=(\frac{1}{4},0)$

37. $n(S)=36$

Let,

A = {(1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) (5,1) (5,2) (5,3) (5,4) (5,5) (5,6)}

$n(A)=18$

B = {(1,1) (1,3) (1,5) (2,1) (2,3) (2,5) (3,1) (3,3) (3,5) (4,1) (4,3) (4,5) (5,1) (5,3) (5,5) (6,1) (6,3) (6,5)}

$n(B)=18$

$p(A)=\frac{1}{2}$ $p(B)=\frac{1}{2}$

$p(A\cap B)=\frac{1}{4}$

$p(A\cap B)=p(A)\cdot p(B)$

$=\frac{1}{4}$

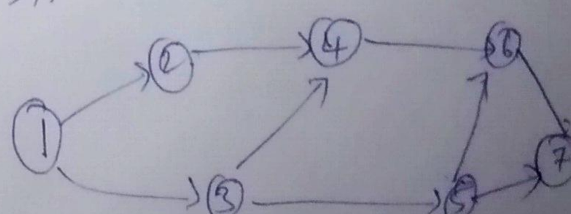
∴ A and B are Independent.

38. $y=\tan^{-1}(42)-\tan^{-1}(2)$

$$y=\tan^{-1}\left[\frac{42-2}{1+42(2)}\right]$$

$$=\tan^{-1}\left(\frac{8}{17}\right)$$

39.



40 $y=x\sin(a+y)$

$$\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin A}$$ H.P

Part - IV.

41.

a)

(0,0)

$0+0+0+0+c=0$

$c=0$ — (1)

(1,2)

$1^2+2^2+2g(1)+2f(2)+c=0$

$2g+4f+c=-5$ — (2)

(2,0)

$2^2+0+2g(2)+0+c=0$

$4g+c=-4$ — (3)

Solving (1), (2) & (3)

$g=-1 \quad f=-3/4 \quad c=0$

$x^2+y^2-2x-3/2\, y+0=0$

$2x^2+2y^2-4x-3y=0$

41.

b)

$$\begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & -1 \\ 1 & 3 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 13 \\ 3 \\ -8 \end{pmatrix}$$

$A \quad X = B$

$X=A^{-1}B$

$A^{-1}=\frac{1}{|A|}\operatorname{adj} A$

$|A|=3(-5+3)+1(-10+1)+2(6-1)$

$=-5$

$$\operatorname{adj} A=\begin{pmatrix} 1 & -1 & 2 & 1 \\ 3 & -5 & 1 & 3 \\ -1 & 2 & 3 & -1 \\ 1 & -1 & 2 & 1 \end{pmatrix}^T$$

$$=\begin{pmatrix} -2 & 1 & -1 \\ 9 & -17 & 7 \\ 5 & -10 & 5 \end{pmatrix}$$

$$A^{-1}=\frac{1}{-5}\begin{pmatrix} -2 & 1 & -1 \\ 9 & -17 & 7 \\ 5 & -10 & 5 \end{pmatrix}$$

$$=\frac{1}{5}\begin{pmatrix} 2 & -1 & 1 \\ -9 & 17 & -7 \\ -5 & 10 & -5 \end{pmatrix}$$

$X=A^{-1}B$

$$=\frac{1}{5}\begin{pmatrix} 2 & -1 & 1 \\ -9 & 17 & -7 \\ -5 & 10 & 5 \end{pmatrix}\begin{pmatrix} 13 \\ 3 \\ 8 \end{pmatrix}$$

$$=\frac{1}{5}\begin{pmatrix} 15 \\ -10 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \quad \boxed{\therefore x=3,\ y=-2,\ z=1}$$

42.

a)

$$\frac{x-2}{(x+2)(x-1)^2}$$

$$\frac{x-2}{(x+2)(x-1)^2}=\frac{A}{x+2}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2} \quad (1)$$

$$\frac{x-2}{(x+2)(x-1)^2} = \frac{A(x-1)^2 + B(x-1)(x+2) + C(x+2)}{(x+2)(x-1)^2}$$

$$x-2 = A(x-1)^2 + B(x-1)(x+2) + C(x+2)$$

$x = 1$

$-1 = 0 + 0 + C(3)$

$\boxed{C = -1/3}$

$x = -2$

$-4 = A(9)$

$\boxed{A = 4/9}$

$A + B = 0$

$\frac{4}{9} + B = 0$

$\boxed{B = -4/9}$

Apply A, B, & C in (1)

$$\frac{x-2}{(x+2)(x-1)^2} = \frac{4}{9(x+2)} - \frac{4}{9(x-1)} - \frac{1}{3(x-1)^2}$$

42

b) $A + B = 45^\circ$

$\tan(A+B) = \tan 45^\circ$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$\tan 45^\circ = 1$

$\tan A + \tan B = 1 - \tan A \tan B$

$\tan A + \tan B + \tan A \tan B = 1$

Add (1) on both sides

$1 + \tan A + \tan B + \tan A \tan B = 1 + 1$

$(1 + \tan A) + \tan B(1 + \tan A) = 2$

$(1 + \tan A)(1 + \tan B) = 2$

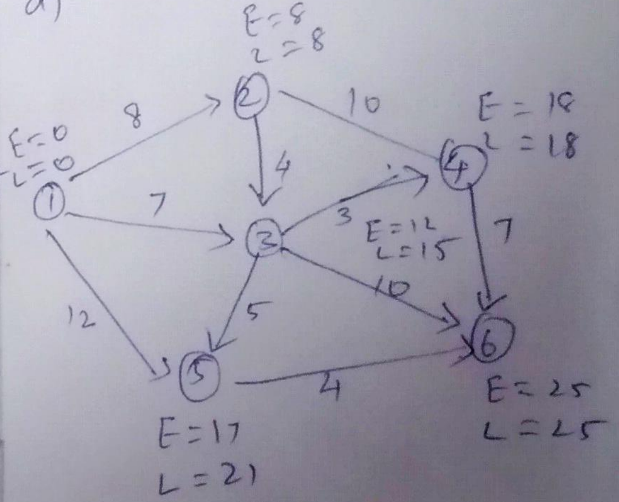
Applying $A = B = 22\frac{1}{2}^\circ$

$\left(1 + \tan 22\frac{1}{2}^\circ\right)\left(1 + \tan 22\frac{1}{2}^\circ\right) = 2$

$\tan 22\frac{1}{2}^\circ = \pm\sqrt{2} - 1$

43.

a)



Activity	Duration	EST	EFT	LST	LFT
1-2	8	0	8	0	8
1-3	7	0	7	8	15
1-5	12	0	12	9	21
2-3	4	8	12	11	15
2-4	10	8	18	8	18
3-4	3	12	15	15	18
3-5	5	12	17	16	21
3-6	10	12	22	15	25
4-6	7	18	25	18	25
5-6	4	17	21	21	25

Critical path is 1-2-4-6

project Completion = 25weeks.

43.(b)

$(1+x)^{2n}$

$n = 2n \quad x = 1 \quad a = x$

General term

$t_{r+1} = 2nC_r (1)^{2n-r} x^r = 2nC_r x^r$ → ①

Since $n = 2n$ is even the Middle term is

$t_{\frac{n}{2}+1}$

$t_{\frac{2n}{2}+1} = t_{n+1}$

Apply $r = n$ in ①

$t_{n+1} = 2nC_n x^n = \dfrac{2n!}{n!\,n!} x^n$

$= \dfrac{(2n)(2n-1)(2n-2)\cdots 4\cdot 3\cdot 2\cdot 1\cdot x^n}{n!\,n!}$

Separate odd and even terms in Numerator

$t_{n+1} = \dfrac{[(2n)(2n-2)(2n-4)\cdots 4\cdot 2]\,[(2n-1)(2n-3)\cdots 3\cdot 1]\,x^n}{n!\,n!}$

$= \dfrac{2^n[(n)(n-1)(n-2)\cdots 2\cdot 1]\,[1\cdot 3\cdot 5\cdots(2n-3)(2n-1)\,x^n]}{n!\,n!}$

$= \dfrac{2^n n!\,[1\cdot 3\cdot 5\cdots(2n-3)(2n-1)]\,x^n}{n!\,n!}$

$= \dfrac{1\cdot 3\cdot 5\cdot(2n-1)\,2^n x^n}{n!}$

Hence proved//.

44.a)

$N = 9$

$\bar{x} = \dfrac{\Sigma x}{N} = \dfrac{162}{9}$

$= 18$

$\bar{y} = \dfrac{\Sigma y}{N} = 19.$

x	y	X = x − 18	Y = y − 19	X²	Y²	XY
6	10	−12	−9	144	81	108
8	12	−10	−7	100	49	70
12	15	−6	−4	36	16	24
15	15	−3	−4	9	16	12
18	18	0	−1	0	1	0
20	25	2	6	4	36	12
24	22	6	3	36	9	18
28	26	10	7	100	49	70
31	28	13	9	169	81	117
Σx = 162	Σy = 171	ΣX = 0	ΣY = 0	ΣX² = 598	ΣY² = 338	ΣXY = 431

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}}$$

$$= \frac{431}{\sqrt{598 \times 338}}$$

$$= \frac{431}{449.562}$$

$$\boxed{r = +0.959}$$

44. b)

$y = a\cos mx + b\sin mx$ —— (1)

Diff w.r.t. x

$y_1 = a(-\sin mx)(m) + b(\cos mx)(m)$

Diff w.r.t. x

$y_2 = a(-\cos x)(m)(m) + b(-\sin x)(m)(m)$

$y_2 = -m^2(a\cos mx + b\sin mx)$

$= -m^2 y$

Using (1)

$y_2 + m^2 y = 0$

Hence proved.

45. (a)

$f(x) = 2x^3 + 9x^2 + 12x + 1$ —— (1)

$f'(x) = 6x^2 + 18x + 12$

$= 6(x^2 + 3x + 2)$

$= 6(x+2)(x+1)$

$f'(x) = 0$

$6(x+2)(x+1) = 0$

$x = -2, x = -1$

Apply $x = -2, x = -1$ in (1)

$f(-2) = 2(-8) + 9(4) + 12(-2) + 1$

$= -3$

$f(-1) = 2(-1) + 9(1) + 12(-1) + 1$

$= -4$

$(-2, -3)$ $(-1, -4)$

45. (b)

$b_{11} = 1/4$ $b_{21} = 1/6$

$b_{12} = 2/3$ $b_{22} = 1/6$

$$B = \begin{bmatrix} 1/4 & 2/3 \\ 1/6 & 1/6 \end{bmatrix}$$

$$I-B = \begin{bmatrix} 3/4 & -2/3 \\ -1/6 & 5/6 \end{bmatrix}$$

$$|I-B| = \begin{vmatrix} 3/4 & -2/3 \\ -1/6 & 5/6 \end{vmatrix}$$

$$= \frac{37}{72} \neq 0.$$

$$(I-B)^{-1} = \frac{1}{|I-B|} \text{adj } I-B.$$

$$= \frac{72}{37} \begin{bmatrix} 5/6 & 2/3 \\ 1/6 & 3/4 \end{bmatrix}$$

$$X = (I-B)^{-1} D$$

$$= \frac{72}{37} \begin{pmatrix} 5/6 & 2/3 \\ 1/6 & 3/4 \end{pmatrix} \begin{pmatrix} 80 \\ 40 \end{pmatrix}$$

$$= \frac{72}{37} \begin{bmatrix} \frac{400}{6} + \frac{80}{3} \\ \frac{80}{6} + \frac{120}{4} \end{bmatrix}$$

$$= \frac{72}{37} \begin{bmatrix} 560/6 \\ 1040/24 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{40320}{222} \\ \frac{74880}{888} \end{bmatrix}$$

$$= \begin{bmatrix} 181.62 \\ 84.32 \end{bmatrix}$$

46. (a)

Income from 12% of 20 shares

$$\frac{12}{16} (16 \times 24)$$

$$= ₹\,288$$

Income from 15% of 20 shares

$$\frac{15}{24} (16 \times 24)$$

$$= ₹\,240$$

∴ Income from 12% of 20 shares at ₹16 is the better investment.

46. (b)

$$30x_1 + 20x_2 = 300$$

$$5x_1 + 10x_2 = 110$$

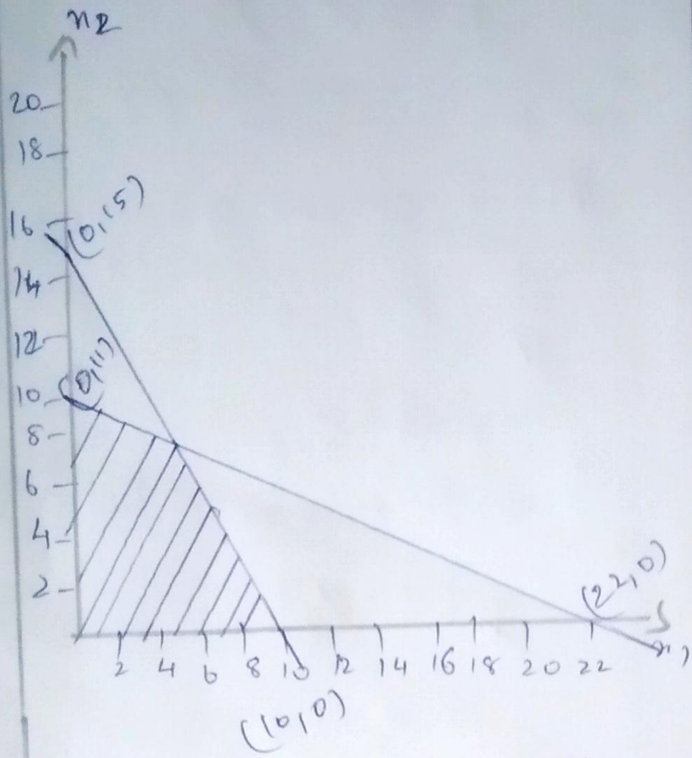
$$x_1, x_2 = 0$$

x_1	0	10
x_2	15	0

x_1	0	22
x_2	11	0

$30x_1 + 20x_2 = 300$ passes through $(0,15)(10,0)$

$5x_1 + 10x_2 = 110$ passes through the $(0,11)$ $(22,0)$



feasible region satisfying all the conditions is on BC

$30x_1 + 20x_2 = 300$ — ①

$5x_1 + 10x_2 = 110$ — ②

② × 2

$10x_1 + 20x_2 = 220$

① − ②

$30x_1 + 20x_2 = 300$

$10x_1 + 20x_2 = 220$

(−) (−) (−)

$20x_1 = 80$

$x_1 = 4$.

Sub $x_1 = 4$ in ①

$x_2 = 9$.

B is $(4,9)$

Corner points	$z = 6x_1 + 8x_2$
$O(0,0)$	0
$A(10,0)$	60
$B(4,9)$	$24 + 72 = 96$
$C(0,11)$	88

$Z_{max} = 96$.

47. (a)

$N = 10$

$\bar{x} = \frac{1686}{10} = 168.6$

$\bar{y} = \frac{1690}{10} = 169$

x	y	$dx = x - 168$	$dy = x - 169$	dx^2	dy^2	$dxdy$
158	163	−10	−6	100	26	60
166	158	−2	−11	4	121	22
163	167	−5	−2	25	4	10
165	170	−3	1	9	1	−3
167	166	−1	−9	1	81	9
170	180	2	11	4	121	22
167	170	−1	1	1	1	−1
172	175	4	6	−16	36	24
177	172	9	3	81	9	27
181	175	13	6	169	36	78
Σx	Σy	Σdx	Σdy	Σdx^2	Σdy^2	$\Sigma dx dy$
1086	1690	6	0	410	446	248

Regression on x on y

$x - \bar{x} = b_{xy}(y - \bar{y})$

$b_{xy} = \dfrac{N\Sigma dxdy - (\Sigma dx)(\Sigma dy)}{N\Sigma dy^2 - (\Sigma dy)^2}$

$\boxed{b_{xy} = 0.5560}$

$x - 168.6 = 0.556(y - 169)$

$x = 0.556y + 74.64$

Regression y on x

$y - \bar{y} = b_{yx}(x - \bar{x})$

$b_{yx} = \dfrac{N\Sigma dxdy - (\Sigma dx)(\Sigma dy)}{N\Sigma dx^2 - (\Sigma dx)^2}$

$\boxed{b_{yx} = 0.61}$

$y - 169 = (0.61)(x - 168.6)$

$y = 0.61x + 16.154$

$x = 164 \quad y = ?$

$y = 0.61(164) + 66.154$

$\boxed{y = 166.194.}$

47(b) $p(A) = 3/4 \quad p(B) = 1/2$

$p(C) = 2/3$

$p(\bar{A}) = 1/4 \quad p(\bar{B}) = 1/2$

$p(\bar{C}) = 1/3$

(i) $p(\bar{A}) - p(A \cap B \cap C) = p(A)p(B)p(C)$

$= \frac{3}{4} \cdot (1/2)(2/3)$

$= 1/4$

(ii) p(only one of them hits the target)

$= p\{(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)\}$

$= \left(\frac{3}{4} \cdot \frac{1}{2} \cdot \frac{1}{3}\right) + \left(\frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{3}\right) + \left(\frac{1}{4} \cdot \frac{1}{2} \cdot \frac{2}{3}\right) = 1/4$

(iii) p(at least one of them hit the target)

$= 1 - p(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 - p(\bar{A})p(\bar{B})p(\bar{C})$

$= 1 - \frac{1}{24}$

$= 23/24.$