SIDDHIKSHA EDUCATION CARE 2024-25

MATHEMATICS

Chapter 1

XI STD

1. By taking suitable sets A, B, C Verify the following results : $(B-A) \cup C = (B \cup C) - (A - C)$

Soln:-

Take $U = \{1, 2, 5, 7, 8, 9, 10\}$

$A = \{1, 2, 5, 7\}$

$B = \{2, 7, 8, 9\}$

$C = \{1, 5, 8, 7\}$

$B - A = \{\cancel{2}, \cancel{7}, 8, 9\} - \{1, \cancel{2}, 5, \cancel{7}\}$

$= \{8, 9\}$

$(B-A) \cup C = \{8, 9\} \cup \{1, 5, 8, 7\}$

$= \{1, 5, 7, 8, 9\}$ ——— (1)

$B \cup C = \{2, 7, 8, 9\} \cup \{1, 5, 8, 7\}$

$= \{1, 2, 5, 7, 8, 9\}$

$A - C = \{\cancel{1}, 2, \cancel{5}, \cancel{7}\} - \{\cancel{1}, \cancel{5}, 8, \cancel{7}\}$

$= \{2\}$

$(B \cup C) - (A - c) = \{1, \cancel{2}, 5, 7, 8, 9\} - \{\cancel{2}\}$

$= \{1, 5, 7, 8, 9\}$ ——— (2)

From (1) & (2) we get

$(B-A) \cup C = (B \cup C) - (A - c)$

1. By taking suitable sets A, B, C Verify the following results : $A \times (B \cup C) = (A \times B) \cup (A \times C)$

2. In the set of Z of integers, define mRn if $m-n$ is a multiple of 12. Prove that R is an equivalence relation.

soln:-

As $m-m=0$ and $0 \times 12 = 0$, hence mRm proving that R is reflexive.

Let mRn. Then $m-n = 12k$

$\Rightarrow n-m = 12(-k)$ is also integer

hence nRm.

R is symmetric

Let mRn and nRp then $m-n=12k$ and $n-p = 12l$ for some integers k and l

So $m-p = m-n+n-p$

$= 12k+12l$

$= 12(k+l) \Rightarrow mRp$

R is transitive.

Thus R is an equivalence relation.

2. Let P be the set of all triangles in a plane and R be the relation defined on P as aRb if a is similar to b. Prove that R is an equivalence relation.

3. On the set of natural numbers let R be the relation defined by aRb if $a+b \leq 6$. Write down the relation by listing all the pairs. Check whether it is

(i) reflexive (ii) symmetric (iii) transitive

(iv) equivalence.

soln:-

set of all natural numbers aRb if $a+b \le 6$

$R = \{(1,1)(1,2)(1,3)(1,4)(1,5)(2,1)(2,2)(2,3)(2,4)$
$(3,1)(3,2)(3,3)(4,1)(4,2)(5,1)\}$

(i) $(4,4), (5,5) \notin R$.
It is not reflexive.

(ii) $aRb \Rightarrow bRa$
It is symmetric

(iii) $(4,2), (2,3) \in R \Rightarrow (4,3) \notin R$
It is not transitive
$\therefore$ It is not an equivalence relation.

3. On the set of natural numbers let R be the relation defined by aRb if $2a+3b = 30$. Write down the relation by listing all the pairs. check whether it is (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence.

4. If $R \rightarrow R$ is defined by $f(x) = 2x-3$ prove that f is a bijection and find its inverse.

Soln:-

Given $f(x) = 2x-3$

$\Rightarrow y = 2x-3$

$y+3 = 2x \Rightarrow x = \frac{y+3}{2}$

$g(y) = \frac{y+3}{2}$

$(g \circ f)(x) = g(f(x)) = g(2x-3)$

$= \frac{2x-3+3}{2} = x.$

$(f \circ g)(y) = f(g(y)) = f\left(\frac{y+3}{2}\right)$

$= \cancel{2}\left(\frac{y+3}{\cancel{2}}\right) - 3 = y$

Thus, $g \circ f = I_x$ and $f \circ g = I_y$

$\Rightarrow$ f and g are bijections and inverses to each other. Hence f is a bijection and $f^{-1}(y) = \frac{y+3}{2}$.

Replace y by x, we get

$f^{-1}(x) = \frac{x+3}{2}$.

4. If $f: R \rightarrow R$ is defined by $f(x) = 3x-5$, prove that f is a bijection and find its inverse.

5. Write the values of f at $-4, 1, -2, 7, 0$ if

$$f(x) = \begin{cases} -x+4 & \text{if } -\infty < x \leq -3 \\ x+4 & \text{if } -3 < x < -2 \\ x^2-x & \text{if } -2 \leq x < 1 \\ x-x^2 & \text{if } 1 \leq x < 7 \\ 0 & \text{otherwise} \end{cases}$$

soln:-

If $x = -4$, $f(x) = -x+4$

$f(-4) = -(-4)+4$

$= 4+4 = 8.$

If $x = 1$, $f(x) = x - x^2$

$f(1) = 1 - 1^2$

$= 1 - 1 = 0.$

If $x = -2$, $f(x) = x^2 - x$

$f(-2) = (-2)^2 - (-2)$

$= 4 + 2 = 6$

If $x = 7$, $f(x) = 0$

$f(7) = 0$

If $x = 0$, $f(x) = x^2 - x$

$f(0) = 0^2 - 0$

$= 0.$

5. Write the values of f at $-3, 5, 2, -1, 0$ if

$$f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0, 2) \\ x^2 - 3 & \text{otherwise} \end{cases}$$

6. Let $f, g : R \rightarrow R$ be defined as $f(x) = 2x - |x|$ and $g(x) = 2x + |x|$. Find $f \circ g$.

Soln:

$$|x| = \begin{cases} -x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$

So,

$$f(x) = \begin{cases} 2x - (-x) & \text{if } x \leq 0 \\ 2x - x & \text{if } x > 0 \end{cases}$$

$$= \begin{cases} 3x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$

Also,

$$g(x) = \begin{cases} 2x + (-x) & \text{if } x \le 0 \\ 2x + x & \text{if } x > 0 \end{cases}$$

$$= \begin{cases} x & \text{if } x \le 0 \\ 3x & \text{if } x > 0 \end{cases}$$

Let $x \le 0$. Then

$$(f \circ g)(x) = f(g(x))$$
$$= f(x)$$
$$= 3x.$$

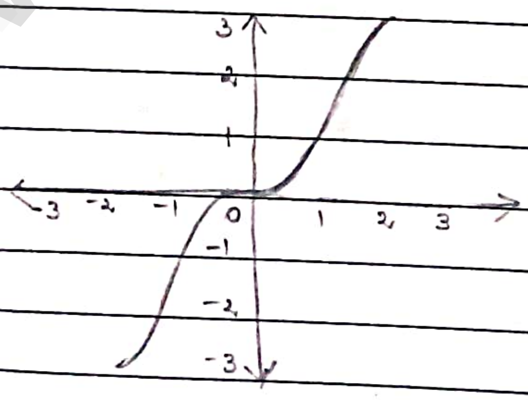
Let $x > 0$ Then

$$(f \circ g)(x) = f(g(x))$$
$$= f(3x)$$
$$= 3x$$

Thus, $(f \circ g)(x) = 3x$ for all x.

6. If $f, g : R \to R$ are defined by $f(x) = |x| + x$ and $g(x) = |x| - x$, find $g \circ f$ and $f \circ g$.

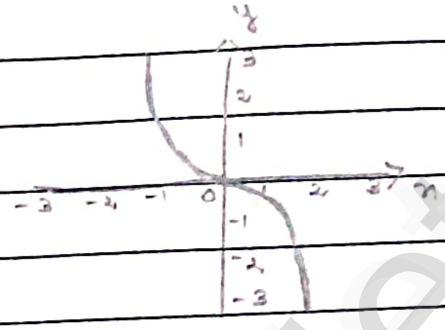
7. For the curve $y = x^3$ given in the figure, draw (i) $y = -x^3$ (ii) $y = x^3 + 1$ (iii) $y = x^3 - 1$ (iv) $y = (x+1)^3$ with the same scale.



Soln: -

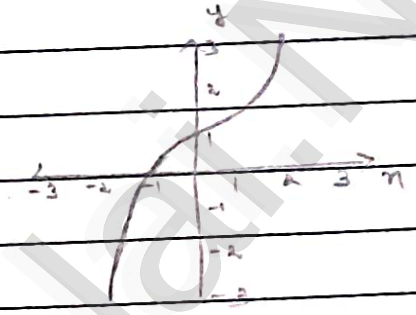
(i) $y = -x^3$

The graph $y = -x^3$ is the reflection of the graph $y = x^3$ about x-axis.



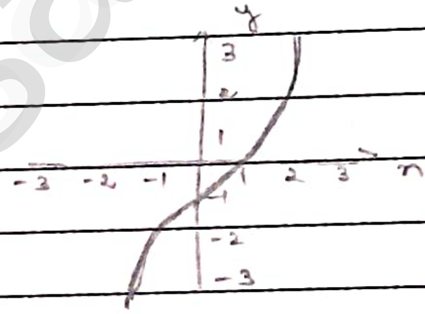
(ii) $y = x^3 + 1$

The graph $y = x^3+1$ causes the graph $y = x^3$ a shift to the upward by 1 unit.



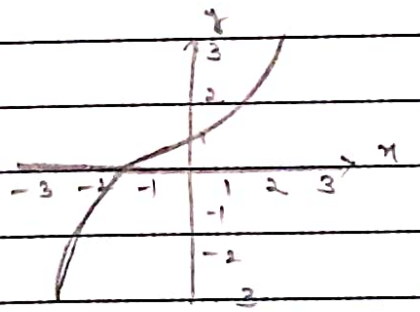
(iii) $y = x^3 - 1$

The graph $y = x^3 - 1$ causes the graph $y = x^3$ a shift to the downward by 1 unit.



(iv) $y = (x+1)^3$

The graph $y = (x+1)^3$ causes the graph $y = x^3$ a shift to the left by 1 unit.



7. From the curve $y = \sin x$, graph the functions

(i) $y = \sin(-x)$ (ii) $y = -\sin(-x)$

(iii) $y = \sin\left(\frac{\pi}{2} + x\right)$ (iv) $y = \sin\left(\frac{\pi}{2} - x\right)$