XI STD

# SIDDHIKSHA EDUCATION CARE 2024-25

## MATHEMATICS

### Chapter 2

1. Use the method of undetermined coefficients to find the sum of $1+2+3+\dots+(n-1)+n$, $n \in N$.

Soln:-

$$s(n) = n+(n-1)+(n-2)+\dots+2+1$$

$$= n+(n-1)+(n-2)+\dots+[n-(n-2)]+[n-(n-1)]$$

$$= n\left[1+\frac{n-1}{n}+\frac{n-2}{n}+\dots+\frac{n-(n-1)}{n}\right]$$

$$\leq n[1+1+\dots+1] \quad \text{since } \frac{n-1}{n}<1,\ \frac{n-2}{n}<1$$

$$s(n) \leq n^2$$

Let $s(n) = a+bn+cn^2$ where $a, b, c \in R$

$$s(n+1)-s(n) = n+1$$

$$a+b(n+1)+c(n+1)^2-a-bn-cn^2 = n+1$$

$$b+2cn+c = n+1$$

$$2cn+(b+c) = n+1$$

Equating like terms, we get

$$2c = 1 \Rightarrow c = \frac{1}{2}$$

$$b+c = 1 \Rightarrow b+\frac{1}{2} = 1 \Rightarrow b = 1-\frac{1}{2}$$

$$b = \frac{1}{2}.$$

$$s(1) = 1 \Rightarrow a+b+c = 1$$

$$a+\frac{1}{2}+\frac{1}{2} = 1 \Rightarrow a = 1-1$$

$$a = 0$$

$$s(n) = \frac{1}{2}n+\frac{1}{2}n^2 = \frac{n(n+1)}{2},\ n \in N.$$

1. The equations $x^2 - bx + a = 0$ and $x^2 - bx + b = 0$ have one root in Common. The other root of the first and the second equations are integers in the ratio 4 : 3. Find the common root.

2. Resolve into partial fractions: $\frac{2x}{(x^2+1)(x-1)}$

soln:

$$\frac{2x}{(x^2+1)(x-1)} = \frac{A}{x-1} + \frac{(Bx+C)}{x^2+1}$$

$$\frac{2x}{\cancel{(x^2+1)(x-1)}} = \frac{A(x^2+1) + (Bx+C)(x-1)}{\cancel{(x^2+1)(x-1)}}$$

$$2x = A(x^2+1) + (Bx+C)(x-1)$$

When $x = 1$,

$$2(1) = A(1^2+1) + (Bx+C)(1-1)$$
$$2 = A(2)$$
$$A = 1$$

When $x = 0$,

$$2(0) = A(0^2+1) + (B(0)+C)(0-1)$$
$$0 = A - C$$
$$0 = 1 - C \implies C = 1.$$

When $x = -1$,

$$2(-1) = A((-1)^2+1) + (B(-1)+C)(-1-1)$$
$$-2 = 2A + 2B - 2C$$
$$-2 = 2(1) + 2B - 2(1)$$
$$-2 - \cancel{2} + \cancel{2} = 2B$$
$$-2 = 2B \implies B = -1.$$

Thus, $$\frac{2x}{(x^2+1)(x-1)} = \frac{1}{x-1} + \frac{1-x}{x^2+1}.$$

3. Resolve into partial fractions: $\dfrac{x+1}{x^2(x-1)}$.

soln:

$$\frac{x+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$\frac{x+1}{\cancel{x^2(x-1)}} = \frac{Ax(x-1)+B(x-1)+C(x^2)}{\cancel{x^2(x-1)}}$$

Then, $x+1 = Ax(x-1)+B(x-1)+C(x^2)$

When $x=1$,

$$1+1 = A(1)(1-1)+B(1-1)+C(1^2)$$

$$2 = C$$

When $x=0$

$$0+1 = A(0)(0-1)+B(0-1)+C(0^2)$$

$$1 = -B$$

$$B = -1$$

When $x=-1$,

$$-1+1 = A(-1)(-1-1)+B(-1-1)+C(-1)^2$$

$$0 = 2A-2B+C$$

$$0 = 2A-2(-1)+2$$

$$0 = 2A+2+2$$

$$-4 = 2A$$

$$A = -2.$$

Thus,

$$\frac{x+1}{x^2(x-1)} = \frac{-2}{x} - \frac{1}{x^2} + \frac{2}{x-1}.$$

2. Resolve into partial fractions: $\frac{x+12}{(x+1)^2(x-2)}$

3. Resolve into partial fractions: $\frac{7+x}{(1+x)(1+x^2)}$

4. Resolve into partial fractions: $\frac{x^3+2x+1}{x^2+5x+6}$

Soln:-

$$\begin{array}{r|l} & x-5 \\ \hline x^2+5x+6 & \cancel{x^3}+0x^2+2x+1 \\ & \underline{\overset{(-)}{\cancel{x^3}}\overset{(-)}{+5x^2}\overset{(-)}{+6x}} \\ & -5x^2-4x+1 \\ & \underline{\overset{(+)}{-\cancel{5x^2}}\overset{(+)}{-25x}\overset{(+)}{-30}} \\ & 21x+31 \end{array}$$

$$\frac{x^3+2x+1}{x^2+5x+6} = (x-5) + \frac{21x+31}{x^2+5x+6} \quad \text{--- (1)}$$

From (1),

$$\frac{21x+31}{x^2+5x+6} = \frac{21x+31}{(x+2)(x+3)}$$

$$\frac{21x+31}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$\frac{21x+31}{\cancel{(x+2)(x+3)}} = \frac{A(x+3)+B(x+2)}{\cancel{(x+2)(x+3)}}$$

Thus,

$$21x+31 = A(x+3)+B(x+2)$$

When $x=-3$,

$$21(-3)+31 = A(-3+3)+B(-3+2)$$

$$-63+31 = B(-1)$$

$$-32 = -B$$

$$B = 32$$

When $x = -2$.

$$21(-2)+31 = A(-2+3) + B(-2+2)$$

$$-42+31 = A(1)$$

$$-11 = A$$

$$\therefore \frac{21x+31}{(x+2)(x+3)} = \frac{-11}{x+2} + \frac{32}{x+3}$$

From (1),

$$\frac{x^3+2x+1}{x^2+5x+6} = (x-5) - \frac{11}{x+2} + \frac{32}{x+3}$$

4. Resolve into partial fractions: $\dfrac{x^2+x+1}{x^2-5x+6}$.

5. Simplify: $\dfrac{1}{3-\sqrt{8}} - \dfrac{1}{\sqrt{8}-\sqrt{7}} + \dfrac{1}{\sqrt{7}-\sqrt{6}} - \dfrac{1}{\sqrt{6}-\sqrt{5}} + \dfrac{1}{\sqrt{5}-2}$

6. If $\log_2 x + \log_4 x + \log_{16} x = \dfrac{7}{2}$, find the value of $x$.

soln:

Note that $x > 0$.

$$\log_2 x + \log_4 x + \log_{16} x = \frac{7}{2}$$

$$\frac{1}{\log_x 2} + \frac{1}{\log_x 4} + \frac{1}{\log_x 16} = \frac{7}{2}$$

(change of base rule)

$$\frac{1}{a} + \frac{1}{2a} + \frac{1}{4a} = \frac{7}{2}$$ where $a = \log_x 2$

$$\Rightarrow \frac{4+2+1}{4a} = \frac{7}{2}$$

$$\frac{7}{4a} = \frac{7}{2}$$

$$\Rightarrow 4a = 2$$

$$a = \frac{2}{4} \Rightarrow a = \frac{1}{2}$$

So,

$$\log_x 2 = \frac{1}{2}$$

$$\Rightarrow x^{1/2} = 2$$

Squaring on both sides, we get

$$x = 2^2$$

$$x = 4.$$

6. Prove $\log \frac{75}{16} - 2\log \frac{5}{9} + \log \frac{32}{243} = \log 2.$

7. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$ then prove that $xyz = 1.$

soln:-

$$\frac{\log x}{y-z} = k \Rightarrow \log x = k(y-z)$$

$$x = e^{k(y-z)}$$

$$x = e^{ky-kz} \quad \text{---(1)}$$

$$\frac{\log y}{z-x} = k \Rightarrow \log y = k(z-x)$$

$$y = e^{k(z-x)}$$

$$y = e^{kz-kx} \quad \text{---(2)}$$

$$\frac{\log z}{x-y} = k \Rightarrow \log z = k(x-y)$$

$$z = e^{k(x-y)}$$

$$z = e^{kx-ky} \quad \text{— (3)}$$

from (1), (2) and (3) sub in $xyz$

$$= e^{ky-kz} \times e^{kz-kx} \times e^{kx-ky}$$

$$= e^{ky-kz+kz-kx+kx-ky}$$

$$= e^{0}$$

$$= 1$$

Hence proved

7. Prove that $\log_{10} 2 + 16 \log_{10} \frac{16}{15} + 12 \log_{10} \frac{25}{24} + 7 \log_{10} \frac{81}{80} = 1$.

8. Prove $\log a + \log a^2 + \log a^3 + \dots + \log a^n = \frac{n(n+1)}{2} \log a$.