SIDDHIKSHA EDUCATION CARE
A place to learn!

XI STD

SIDDHIKSHA EDUCATION CARE 2024-25

MATHEMATICS

Chapter 4

1. By the principle of mathematical Induction, prove that, for all integers $n \geq 1$,

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

soln:-

$$P(n) := 1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

Put $n=1$,

$$P(1) = \frac{1(1+1)(2(1)+1)}{6}$$

$$= \frac{1 \times 2 \times 3}{6}$$

$$= \frac{6}{6}$$

$$= 1.$$

$P(1)$ is true.

Assume $n=k$.

$$P(k) = 1^2+2^2+3^2+\cdots+k^2 = \frac{k(k+1)(2k+1)}{6}$$

To prove $P(k+1)$ is true.

$$P(k+1) = 1^2+2^2+3^2+\cdots+(k+1)^2$$

$$= 1^2+2^2+3^2+\cdots+k^2+(k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$

$$= \frac{(k+1)\left[k(2k+1)+6(k+1)\right]}{6}$$

$$= \frac{(k+1)\left[2k^2+k+6k+6\right]}{6}$$

$$= \frac{(k+1)\left[(k+2)(2k+3)\right]}{6}$$

$$= \frac{(k+1)\left[(k+1+1)(2(k+1)+1)\right]}{6}$$

That is,

$$P(k+1) = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

$P(k+1)$ is true.

By Mathematical Induction, $P(n)$ is true for all $n \in N$.

$12 < \frac{4/2}{3/2}$

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2. Prove that for any natural number n, $a^n - b^n$ is divisible by $a-b$ where $a > b$.

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$P(n) = a^n - b^n$ is divisible by $a-b$.

Put $n=1$,

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$P(k) = a^k - b^k = \lambda(a-b)$, $\lambda \in N$.

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Assume $n=k$,

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$P(k) = a^k - b^k = \lambda(a-b)$, $\lambda \in N$.

To prove that

$P(k+1) = a^{k+1} - b^{k+1}$ is divisible by a-b.

$P(k+1) = a^{k+1} - b^{k+1}$

$= a^{k+1} - ab^k + ab^k - b^{k+1}$

$= a(a^k - b^k) + b^k(a-b)$

$= a(\lambda(a-b)) + b^k(a-b)$

$= (a-b)(a\lambda + b^k)$

$= (a-b)\lambda_1$ where $\lambda_1 = a\lambda + b^k$

$\lambda_1 \in N$.

which is divisible by a-b.

P(k+1) is true.

By Mathematical Induction, $a^n - b^n$ is divisible by a-b where a > b for all natural numbers n.

2. Using the mathematical induction, show that for any natural number n; $x^{2n} - y^{2n}$ is divisible by $x+y$.

3. Prove that $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \geq 1$.

Soln:-

$P(n) : = 3^{2n+2} - 8n - 9$ is divisible by 8

Put n = 1,

$P(1) = 3^{2(1)+2} - 8(1) - 9$

$= 3^4 - 8 - 9$

$= 81 - 8 - 9$

$= 64$ is divisible by 8

P(1) is true.

Assume $P(k) = 3^{2k+2} - 8k - 9$ is divisible by 8

$P(k) = 3^{2k+2} - 8k - 9 = 8k_1, \; k_1 \geq 1$

$\Rightarrow 3^{2k+2} = 8k_1 + 8k + 9$

To show that

$P(k+1) = 3^{2(k+1)+2} - 8(k+1) - 9$ is divisible by 8.

$P(k+1) = 3^{2(k+1)+2} - 8(k+1) - 9$

$= 3^2 \cdot 3^{2k+2} - 8k - 8 - 9$

$= 3^2(3^{2k+2}) - 8k - 17$

$= 3^2(8k_1 + 8k + 9) - 8k - 17$

$= 72k_1 + 72k + 81 - 8k - 17$

$= 72k_1 + 64k + 64$

$= 8(9k_1 + 8k + 8)$

$= 8k_2$ where $k_2 = 9k_1 + 8k + 8$

is divisible by 8.

$P(k+1)$ is true.

By principle of mathematical induction, $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \geq 1$.

3. Use induction to prove that $n^3 - 7n + 3$ is divisible by 3, for all natural numbers n.

4. If $^{(n+2)}C_7 : {}^{(n-1)}P_4 = 13 : 24$ find n.

Soln:

Given $^{(n+2)}C_7 : {}^{(n-1)}P_4 = 13 : 24$

$$\frac{^{(n+2)}C_7}{^{(n-1)}P_4} = \frac{13}{24}$$

$$\frac{(n+2)!}{(n-5)! \times 7!} \times \frac{(n-5)!}{(n-1)!} = \frac{13}{24}$$

$$\frac{(n+2)(n+1) \cdot n(n-1)!}{7!\,(n-1)!} = \frac{13}{24}$$

$$(n+2)(n+1)n = \frac{13}{24} \times 7!$$

$$= \frac{13}{\cancel{24}} \times 7 \times 6 \times 5 \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1$$

$$= 13 \times 7 \times 2 \times 3 \times 5$$

$$n(n+1)(n+2) = 13 \times 14 \times 15$$

Equating like terms, we get

$$n = 13.$$

4. If $^nP_r = 720$ and $^nC_r = 120$ find n, r.

5. Find the number of strings of 5 letters that can be formed with the letters of the word PROPOSITION.

Soln:-

There are 11 letters in the word.

4 distinct letters → (R, S, T, N)

2 sets of two alike letters → (PP, II)

1 set of three alike letters → (OOO)

option 1:-

5 letters from (R, S, T, N, P, I, O)

$$^7C_5 \times 5! = \frac{7 \times 6 \times 5 \times 4 \times 3}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}$$

$$= 21 \times 30 \times 4$$

$$= 21 \times 120$$

$$= 2520.$$

Option 2:-

1 set of 3 alike (OOO), 1 set of 2 alike (PP, II)

$$^1C_1 \times {}^2C_1 \times \frac{5!}{3!\,2!}$$

$$= 1 \times 2 \times \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3!} \times \cancel{2} \times 1}$$

$$= 1 \times 2 \times 5 \times 2$$

$$= 20$$

option 3:

1 set of 3 alike (ooo), 2 distinct (R, S, T, N, P, I)

$$1C_1 \times 6C_2 \times \frac{5!}{3!}$$

$$= 1 \times \frac{\cancel{6}^{3} \times 5}{\cancel{2} \times 1} \times \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}}$$

$$= 1 \times 15 \times 20$$

$$= 300$$

option 4:

2 sets of 2 alike (PP, II, OO), 1 distinct (R, S, T, N and remaining one in 2 alike)

$$3C_2 \times 5C_1 \times \frac{5!}{2! \times 2!}$$

$$= \frac{3 \times \cancel{2}}{\cancel{2} \times 1} \times 5 \times \frac{5 \times 4 \times 3 \times \cancel{2!}}{2! \times \cancel{2!}}$$

$$= \frac{3 \times 5 \times 5 \times \cancel{4}^{2} \times 3}{\cancel{2} \times 1}$$

$$= 15 \times 10 \times 3$$

$$= 15 \times 30$$

$$= 450$$

option 5:

1 set of 2 alike (PP, II, OO), 3 distinct R, S, T, N and remaining two in 2 alike

$$3C_1 \times 6C_3 \times \frac{5!}{2!}$$

$$= 3 \times \frac{\cancel{6} \times 5 \times 4}{\cancel{3} \times \cancel{2} \times 1} \times \frac{5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}}$$

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$$= (a-b)\lambda_1 \quad \text{where } \lambda_1 = a\lambda + b^k$$
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$$\frac{{}^{(n+2)}C_7}{{}^{(n-1)}P_4} = \frac{13}{24}$$

$$\frac{(n+2)!}{\cancel{(n-5)!} \times 7!} \times \frac{\cancel{(n-5)!}}{(n-1)!} = \frac{13}{24}$$

$$\frac{(n+2)(n+1).n\cancel{(n-1)!}}{7!\cancel{(n-1)!}} = \frac{13}{24}$$

$$(n+2)(n+1)n = \frac{13}{24} \times 7!$$

$$= \frac{13}{\cancel{24}} \times 7 \times 6 \times 5 \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1$$

$$= 13 \times 7 \times 2 \times 3 \times 5$$

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$$= 21 \times 30 \times 4$$

$$= 21 \times 120$$

$$= 2520.$$

Option 2:-

1 set of 3 alike (OOO), 1 set of 2 alike (PP, II)

$$1C_1 \times 2C_1 \times \frac{5!}{3!\,2!}$$

$$= 1 \times 2 \times \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3!} \times \cancel{2} \times 1}$$

$$= 1 \times 2 \times 5 \times 2$$

$$= 20$$

option 3:

1 set of 3 alike (000), 2 distinct (R, S, T, N, P, I)

$${}^{1}C_1 \times {}^{6}C_2 \times \frac{5!}{3!}$$

$$= 1 \times \frac{\cancel{6}^{3} \times 5}{\cancel{2} \times 1} \times \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}}$$

$$= 1 \times 15 \times 20$$

$$= 300$$

option 4:

2 sets of 2 alike (PP, II, OO), 1 distinct (R, S, T, N and remaining one in 2 alike)

$${}^{3}C_2 \times {}^{5}C_1 \times \frac{5!}{2! \times 2!}$$

$$= \frac{3 \times \cancel{2}}{\cancel{2} \times 1} \times 5 \times \frac{5 \times 4 \times 3 \times \cancel{2!}}{2! \times \cancel{2!}}$$

$$= \frac{3 \times 5 \times 5 \times \cancel{4}^{2} \times 3}{\cancel{2} \times 1}$$

$$= 15 \times 10 \times 3$$

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$$= 450$$

option 5:

1 set of 2 alike (PP, II, OO), 3 distinct R, S, T, N and remaining two in 2 alike

$${}^{3}C_1 \times {}^{6}C_3 \times \frac{5!}{2!}$$

$$= 3 \times \frac{\cancel{6} \times 5 \times 4}{\cancel{3} \times \cancel{2} \times 1} \times \frac{5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}}$$

$= 3 \times 20 \times 60$

$= 3600.$

Hence, the total number of strings are

$2520 + 20 + 300 + 450 + 3600 = 6890.$

5. Find the number of strings of 4 letters that can be formed with the letters of the word EXAMINATION ?

6. A committee of 7 peoples has to be formed from 8 men and 4 women. In how many ways can this be done when the committee consists of
(i) exactly 3 women?
(ii) atleast 3 women?
(iii) atmost 3 women?

Soln:-

Number of men = 8 ; Number of women = 4

Number of peoples in the committee = 7

(i) Exactly 3 women

In a 7 member committee, women must be 3, the remaining 4 must be men.

$\Rightarrow 4C_3 \times 8C_4$

$= \frac{4!}{3!\,1!} \times \frac{8!}{4!\,4!}$

$= \frac{4 \times \cancel{3!}}{\cancel{3!}\,1!} \times \frac{8 \times 7 \times 6 \times 5 \times \cancel{4!}}{4! \times \cancel{4!}}$

$= 4 \times \frac{\cancel{8} \times 7 \times \cancel{6}^{2} \times 5}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$

$= 4 \times 70$

$= 280$

(ii) Atleast 3 women

a) 4 women + 3 men b) 3 women + 4 men

a) 4 women + 3 men

$4C_4 * 8C_3$

$= 1 * \frac{8!}{5!\,3!}$

$= 1 * \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!} \times 3!}$

$= 1 * \frac{8 \times 7 \times \cancel{6}}{\cancel{3} \times \cancel{2} \times 1}$

$= 1 \times 56 = 56$ ways

b) 3 women + 4 men

$4C_3 \times 8C_4$

$= 280$ ways

The required number of ways of forming the Committee $= 56 + 280 = 336$ ways

(iii) Atmost 3 women

a) 0 women + 7 men b) 1 women + 6 men

c) 2 women + 5 men d) 3 women + 4 men

$= 4C_0 \times 8C_7 + 4C_1 \times 8C_6 + 4C_2 \times 8C_5 + 4C_3 \times 8C_4$

$= 1 \times 8 + 4 \times 28 + 6 \times 56 + 4 \times 70$

$= 8 + 112 + 336 + 280$

$= 736$ ways.

6. How many different selections of 5 books can be made from 12 different books if,

(i) Two particular books are always selected?

(ii) Two particular books are never selected?

7. Find the sum of all 4 digit numbers that can be formed using the digits 1, 2, 4, 6, 8.

soln:-

The number of 4 digit numbers that can be formed using the given 5 digits is

$$5P_4 = 5 \times 4 \times 3 \times 2$$

$$= 120.$$

First, find the sum of the digits in the unit place of all these 120 numbers. Remaining $4P_3$.

Therefore,

$$(4P_3 \times 1) + (4P_3 \times 2) + (4P_3 \times 4) + (4P_3 \times 6) + (4P_3 \times 8) = 4P_3 (1+2+4+6+8)$$

$$= 4P_3 \times 21.$$

III'ly 10^{th} place $= 4P_3 \times 21 \times 10$

100^{th} place $= 4P_3 \times 21 \times 100$

1000^{th} place $= 4P_3 \times 21 \times 1000$

Hence, the sum of all the 4 digit numbers formed by using the digits 1, 2, 4, 6, 8 is

$$= (4P_3 \times 21) + (4P_3 \times 21 \times 10) + (4P_3 \times 21 \times 100) + (4P_3 \times 21 \times 1000)$$

$$= 4P_3 \times 21 \; (1 + 10 + 100 + 1000)$$

$$= 4 \times 3 \times 2 \times 21 \; (1111)$$

$$= 24 \times 21 \times 1111$$

$$= 559944.$$

7. Find the sum of all 4-digit numbers that can be formed using digits 1, 2, 3, 4 and 5 repetitions not allowed?

7. Find the sum of all 4-digit numbers that can be formed using digits 0, 2, 5, 7, 8 without repetition?

8. Prove that $\frac{(2n)!}{n!} = 2^n (1.3.5 \ldots (2n-1))$.

soln:.

$$\frac{(2n)!}{n!} = \frac{1.2.3.4 \ldots (2n-2)(2n-1)2n}{n!}$$

$$= \frac{(1.3.5 \ldots (2n-1))(2.4.6 \ldots (2n-2)2n)}{n!}$$

$$= \frac{(1.3.5 \ldots (2n-1))\, 2^n (1.2.3 \ldots (n-1).n)}{n!}$$

$$= \frac{(1.3.5 \ldots (2n-1)).\, 2^n.\, \cancel{n!}}{\cancel{n!}}$$

$$= 2^n (1.3.5 \ldots (2n-1)).$$

Hence proved.

8. Find the value of n if

(i) $(n+1)! = 20\,(n-1)!$ (ii) $\frac{1}{8!} + \frac{1}{9!} = \frac{n}{10!}$

9. How many strings of length 6 can be formed using letters of the word FLOWER if

(i) either starts with F or ends with R?

(ii) neither starts with F nor ends with R?

soln:.

(i) starts with F

F	_	_	_	_	_
↓	↓	↓	↓	↓	↓
1	5	4	3	2	1

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$ ways

ends with R

$$\underset{5}{\_}\ \underset{4}{\_}\ \underset{3}{\_}\ \underset{2}{\_}\ \underset{1}{\_}\ \underset{1}{R}$$

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$ ways.

starts with F and also ends with R

$$\underset{1}{F}\ \underset{4}{\_}\ \underset{3}{\_}\ \underset{2}{\_}\ \underset{1}{\_}\ \underset{1}{R}$$

$= 4 \times 3 \times 2 \times 1$

$= 24.$

By the principle of inclusion - exclusion, the number of strings of length 6, either starting with F or ending with R is $120 + 120 - 24$

$= 216$ ways.

(ii) Neither starts with F nor ends with R

= (Total number of letter strings) − (starts with F or end with R)

First,

Total number of letter strings (6 letters)

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$ ways

$\therefore$ Neither starts with F nor ends with R

$= 720 - 216$

$= 504$ ways.

9. How many strings can be formed using the letters of the word LOTUS if the word

(i) either starts with L or ends with S?

(ii) neither starts with L nor ends with S?