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HYM

11 - Std

HALF YEARLY EXAMINATION - 2024

MATHEMATICS

Time : 3.00 hrs. Marks : 90

Part - I

Note : i) All questions are compulsory $20 \times 1 = 20$

ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1. If $n((A \times B) \cap (A \times C)) = 8$ and $n(B \cap C) = 2$, then $n(A)$ is

(1) 6 (2) 4 (3) 8 (4) 16

2. If $\frac{|x-2|}{x-2} \geq 0$, then x belongs to

(1) $[2, \infty)$ (2) $(2, \infty)$ (3) $(-\infty, 2)$ (4) $(-2, \infty)$

3. If 8 and 2 are the roots of $x^2 + ax + c = 0$ and 3,3 are the roots of $x^2 + dx + b = 0$, then the roots of the equation $x^2 + ax + b = 0$ are

(1) 1,2 (2) -1,1 (3) 9,1 (4) -1,2

4. Which of the following is not true?

(1) $\sin\theta = -\frac{3}{4}$ (2) $\cos\theta = -1$ (3) $\tan\theta = 25$ (4) $\sec\theta = \frac{1}{4}$

5. If $f(\theta) = |\sin\ \theta| + |\cos\ \theta|, \theta \in R$, then $f(\theta)$ is in the interval

(1) $[0,2]$ (2) $[1, \sqrt{2}]$ (3) $[1,2]$ (4) $[0,1]$

6. If P_r stands for ${}^rP_r$ then the sum of the series $1 + P_1 + 2P_2 + 3P_3 + \cdots + nP_n$ is

(1) P_{n+1} (2) $P_{n+1} - 1$ (3) $P_{n-1} + 1$ (4) ${}^{(n+1)}P_{(n-1)}$

7. Value of ${}^{10}C_8$

(1) 10 (2) 8 (3) 45 (4) 108

8. The sum up to n terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \cdots$ is

(1) $\frac{n(n+1)}{2}$ (2) $2n(n+1)$ (3) $\frac{n(n+1)}{\sqrt{2}}$ (4) 1 .

9. The angle between the two straight lines $2x + y = 4$ and $x + 3y = 5$ is

(1) 0 (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) π

10. A number is selected from the set $\{1,2,3, \ldots, 20\}$. The probability that the selected number is divisible by 3 or 4 is

(1) $\frac{2}{5}$ (2) $\frac{1}{8}$ (3) $\frac{1}{2}$ (4) $\frac{2}{3}$

11 . The value of the determinant of $A = \begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix}$ is

(1) $-2abc$ (2) abc (3) 0 (4) $a^2 + b^2 + c^2$

12 . The matrix A satisfying the equation $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ is

(1) $\begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & -4 \\ 1 & 0 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$

13 . If $|\vec{a}| = 13, |\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 60°$ then $|\vec{a} \times \vec{b}|$ is

(1) 15 (2) 35 (3) 45 (4) 25

14 . If $|\vec{a} + \vec{b}| = 60, |\vec{a} - \vec{b}| = 40$and $|\vec{b}| = 46$, then $|\vec{a}|$ is

(1) 42 (2) 12 (3) 22 (4) 32

15 . The value of $\lim_{x \to 0} \frac{\sin 5x}{5}$is

(1) 5 (2) $\frac{1}{5}$ (3) 0 (4) 1

16 . The differential coefficient of $\log_{10} x$ with respect to $\log_x 10$ is

(1) 1 (2) $-(\log_{10} x)^2$ (3) $(\log_x 10)^2$(4) $\frac{x^2}{100}$

17 . If $y = f(x^2 + 2)$and $f'(3) = 5$, then $\frac{dy}{dx}$ at $x = 1$ is

(1) 5 (2) 25 (3) 15 (4) 10

18 . The gradient (slope) of a curve at any point $(x, y)$is $\frac{x^2 - 4}{x^2}$. If the curve passes through the point (2,7), then the equation of the curve is

(1) $y = x + \frac{4}{x} + 3$ (2) $y = x + \frac{4}{x} + 4$ (3) $y = x^2 + 3x + 4$ (4) $y = x^2 - 3x + 6$

19 . $\int \frac{1}{e^x} \, dx =$ (1) $\log e^x + c$ (2) $-\frac{1}{e^x} + c$ (3) $\frac{1}{e^x} + c$ (4) $x + c$

20 . If two events A and B are such that $P(\bar{A}) = \frac{3}{10}$ and $P(A \cap \bar{B}) = \frac{1}{2}$, then $P(A \cap B)$ is

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) $\frac{1}{5}$

Part – II

Note : i) Answer any Seven questions.

ii) Question number 30 is compulsory. 7 ×2 = 14

21 . Justify the trueness of the statement :

"An element of a set can never be a subset of itself.

22. Construct a quadratic equation with roots 7 and −3.

23. Find the values of : $sin(480^c)$

24. If $\frac{1}{7!} + \frac{1}{8!} = \frac{A}{9!}$ then find the value of A.

25. Find the middle term in the expansion of $(x + y)^6$.

26. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, show that A^2 is a unit matrix.

27. Find a unit vector parallel along the vector $5\hat{i} - 3\hat{j} + 4\hat{k}$.

28. If two coins are tossed simultaneously, then find the probability of getting
(i) one head and one tail (ii) at most two tails

29. Integrate the with respect to x : $\frac{1}{x^7}$

30. If $y = \sin^2 x$, find $\frac{dy}{dx}$.

Part – III

Note : i) Answer any Seven questions. 7 ×3 = 21
ii) Question number 40 is compulsory.

31. In the set Z of integers , define mRn if m-n is divisible by 7 .Prove that R is an equivalence relation .

32. Solve : $2|x + 1| - 6 \leq 7$ and graph the solution set in a number line.

33. If the letters of the word FUNNY are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, find the rank of the word FUNNY.

34. The length of the perpendicular drawn from the origin to a line is 12 and makes an angle 150°with positive direction of the x-axis. Find the equation of the line.

35. Show that $\begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ca - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$.

36. Find a direction ratio and direction cosines of the following vectors.
(i) $3\hat{i} + 4\hat{j} - 6\hat{k}$, (ii) $3\hat{i} - 4\hat{k}$.

37. Evaluate the limits : $\lim_{\sqrt{x} \to 3} \frac{x^2 - 81}{\sqrt{x} - 3}$.

38. Find the derivatives of the $y = x^{\cos x}$.

39. If $P(A) = 0.5, P(B) = 0.8$ and $P(B/A) = 0.8$, find $P(A/B)$ and $P(A \cup B)$.

40. Prove that $2 \tan^{-1}\frac{2}{3} = \tan^{-1}\frac{12}{5}$

Part – IV

Note : i) Answer all the questions 7× 5 = 35

41 . a) Let f and g be the two functions from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = 3x - 4$ and $g(x) = x^2 + 3$. Find $g \circ f$ and $f \circ g$. **(OR)**

b) Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines. Find the distance between them.

42 . a) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1 - x^2)y_2 - 3xy_1 - y = 0$. **(OR)**

b) Resolve into partial fractions : $\frac{2x}{(x^2+1)(x-1)}$.

43 . a)Prove that the line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram. **(OR)**

b)If θ is an acute angle, then find : $\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$, when $\sin\theta = \frac{1}{25}$

44 . a) By the principle of mathematical induction, prove that, for $n \geq 1$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$ **(OR)**

b) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$.

45 . a)If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3x - 5$, prove that f is a bijection and find its inverse. **(OR)**

b) Show that $\lim_{x\to 0^+} x\left[\left\lfloor\frac{1}{x}\right\rfloor + \left\lfloor\frac{2}{x}\right\rfloor + \cdots + \left\lfloor\frac{15}{x}\right\rfloor\right] = 120$.

46 . a) Evaluate the integrals : $\int \frac{1}{x^2-2x+5}\,dx$. **(OR)**

b) Prove that the points whose position vectors $2\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$, $4\hat{\imath} + \hat{\jmath} + 9\hat{k}$ and $10\hat{\imath} - \hat{\jmath} + 6\hat{k}$ form a right angled triangle.

47 . a) Prove that $\sqrt[3]{x^3 + 7} - \sqrt[3]{x^3 + 4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

(OR) b) A factory has two machines I and II. Machine - I produces 40% of items of the output and Machine-II produces 60% of the items. Further 4% of items produced by Machine - I are defective and 5% produced by Machine - II are defective. If an item is drawn at random, find the probability that it is a defective item.