(Maths) 11^{th} Half Yearly Examination -(2024-2025)

TIRUPATTUR Dt

PART-I

1. $\mathbb{N}$
2. 4
3. 0
4. 40
5. $10C_6\, 2^{10}$
6. $(-3,-2)$
7. 2
8. $[1,\sqrt{2}]$
9. $\frac{n(n+1)}{\sqrt{2}}$
10. 3
11. 2/3
12. $-1/\log 3$
13. 1
14. $2(\log 2)^2$
15. 22
16. $\begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$
17. $a+b=0$
18. $\theta=0$
19. $1/t$
20. 0.

PART-II

21. $f(-2)=-16,\ f(2)=16$ (1)

$B=[-16,16]$ (1)

22. $(102)^4=(100+2)^4$ (1)

$=4C_0(100)^4 2^0+4C_1(100)^3 2^1+4C_2(100)^2 2^2+4C_3(100)^1 2^3+4C_4(100)^0 2^4$

$=100000000+8000000+240000+3200+16$

$=108243216.$ (1)

23. $\log_9 27=\log_9 3^3=3\log_9 3=3/\log_3 3^2$

$=\frac{3}{2\log_3 3}=\frac{3}{2}$ (½)

$\log_{27}9=\log_{27}3^2=2\log_{27}3$

$=2\frac{1}{\log_3 27}=2\frac{1}{\log_3 3^3}=\frac{2}{3\log_3 3}$

$=\frac{2}{3}.$ (½)

$\log_9 27-\log_{27}9=\frac{3}{2}-\frac{2}{3}=\frac{5}{6}$ (1)

24. D.R's $=3,0,-4.$ (1)

D.C's $=\frac{3}{5},0,\frac{-4}{5}.$ (1)

25. $RHS=4\sin A\cos A(\cos^2 A-\sin^2 A)$

$=2(2\sin A\cos A)\cos 2A$

$=2\sin 2A\cos 2A$ (1)

$=\sin 4A=LHS.$ (1)

26. $\frac{dy}{dx}=100(x^3-1)^{99}(3x^2)$

$=300x^2(x^3-1)^{99}$ (2)

27. $a_1b_2=a_2b_1$ (1)

$(3)(8)=(2)(12)=24.$

∴ Lines are parallel. (1)

28. (i) $P(A)=\frac{5}{10}=\frac{1}{2}$ (1)

(ii) $P(B)=\frac{3}{10}.$ (1)

29. $\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}$ (1)

$\lim_{x\to 1} \frac{x^3-1^3}{x-1} = 3(1)^{3-1} = 3(1)^2 = 3$ (1)

30. LHS $= s\begin{vmatrix} 1 & a^2 & a^2+b^2+c^2 \\ 1 & b^2 & a^2+b^2+c^2 \\ 1 & c^2 & a^2+b^2+c^2 \end{vmatrix}$ $R_3 \to R_3 + R_2$

$= s(a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & 1 \\ 1 & b^2 & 1 \\ 1 & c^2 & 1 \end{vmatrix}$ (1)

$= 0 =$ RHS. (1)

PART – III

31. LHS $= \begin{vmatrix} 1 & x & x^2 \\ x & 1 & x \\ x & x & 1 \end{vmatrix} \times \begin{vmatrix} 1 & x & x \\ -x & -1 & -x \\ -x & -x & -1 \end{vmatrix}$ (2)

$=$ RHS (1)

32. $\vec{AB} = -\hat{i} + \hat{j}$; $\vec{AC} = -\hat{i} + \hat{k}$ (1)

$\vec{AB} \times \vec{AC} = \hat{i} + \hat{j} + \hat{k}$ (1)

$|\vec{AB} \times \vec{AC}| = \sqrt{3}$

Area $= \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{\sqrt{3}}{2}$.

33. $\sin\theta = -\frac{\sqrt{3}}{2} = -\sin\frac{\pi}{3} = \sin\left(-\frac{\pi}{3}\right)$ (2)

$\Rightarrow \theta = -\frac{\pi}{3}$ is the prin. sln (1)

34. (i) $P(A\cup B) = P(A) + P(B) = 0.8$ (1)

(ii) $P(A \cap \bar{B}) = P(A) - P(A\cap B) = 0.5$ (1)

(iii) $P(\bar{A} \cap B) = P(B) - P(A \cap B) = 0.3$ (1)

35. $\frac{(n+1)C_8}{(n-3)P_4} = \frac{57}{16}$

$\frac{(n+1)!}{(n-3)!} = 8!\,\frac{57}{16}$ (1)

$(n+1)n(n-1)(n-2) = \frac{21\times20\times19\times18}{\cancel{18\times}}$ (1)

$\Rightarrow n = 20$ (1)

36. $a + 4d = 19$; $a + 8d = 35$

$\Rightarrow a = 3, d = 4$ (1)

12^{th} term in A.P. $= a + 11d = 47$. (1)

12^{th} term in H.P $= \frac{1}{47}$. (1)

37. $y = \sqrt{x + \sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x+\sqrt{x}}}\left(1 + \frac{1}{2\sqrt{x}}\right)$ (2)

$\frac{dy}{dx} = \frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$ (1)

38. $a = 4$; $2h = 4 \Rightarrow h = 2$; $b = 1$ (1)

$h^2 - ab = 4 - 4(1) = 0$. (1)

∴ It represents pair of 11^{el} lines (1)

39. $\int f'(x)dx = \int[3x^2 - 4x + 5]dx$

$f(x) = x^3 - 2x^2 + 5x + c$ (1)

$f(1) = 3 \Rightarrow c = -1$ (1)

∴ $f(x) = x^3 - 2x^2 + 5x - 1$ (1)

40. $\alpha = 1 + \sqrt{5}$; $\beta = 1 - \sqrt{5}$

$\alpha + \beta = 2$; $\alpha\beta = -4$. (1)

$p(x) = k(x^2 - 2x - 4)$. (1)

$k = -\frac{2}{5}$; $p(x) = \frac{-2}{5}(x^2 - 2x - 4)$ (1)

PART – IV

41(a)

$f(-3) = 1$ (1)

$f(5) = 38$ (1)

$f(2) = 1$ (1)

$f(-1) = -5$ (1)

$f(0) = -3$ (1)

41(b) For $a = 0$, $|A| = 0$.

$\Rightarrow a$ is a factor (1)

$|||^{ly}$ b & c are also factors (1)

$\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = k\,abc$ (1)

$k = 8$ (1)

$\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$ (1)

42(A) $\frac{x^2+x+1}{x^2-5x+6} = 1 + \frac{6x-5}{(x-2)(x-3)}$ (2)

$\frac{6x-5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$ (1)

$\frac{x^2+x+1}{x^2-5x+6} = 1 + \frac{-7}{x-2} + \frac{13}{x-3}$ (2)

42(B) $LHS = \lim_{x\to 0^+} x\frac{1}{x}[1+2+\dots+15]$ (1)

$= 1+2+\dots+15$ (1)

$= \frac{15(16)}{2}$ (2)

$= 120 = RHS$ (1)

(or) Alternate Method.

43(A) $\Rightarrow LHS = \frac{1}{2}(2\cos^2A+2\cos^2B+2\cos^2C)$ (1)

$= \frac{3}{2} + \frac{1}{2}(\cos 2A+\cos 2B+\cos 2C)$ (1)

$= \frac{3}{2} + \frac{1}{2}[-2\cos C\cos(A-B) + 2\cos^2C - 1]$ (2)

$= 1 - 2\cos A\cos B\cos C = RHS$ (1)

43(B) $\frac{x+1}{x+3} - 3 < 0$ (1)

$\frac{x+4}{x+3} > 0$ (1)

x	$x+3$	$x+4$	$\frac{x+4}{x+3}$
$x<-4$	$-$	$-$	$+$
$-4<x<-3$	$-$	$+$	$-$
$x>-3$	$+$	$+$	$+$
$x=-4$	$-$	0	0

(2)

Solution $= (-\infty,-4)\cup(-3,\infty)$ (1)

44(A) P(1) is true (1)

Assume P(K) is true (1)

To prove: P(K+1) is true (2)

Conclusion (1)

44(B) $2x-3 = (2x+4) - 7$ (1)

$I_1 = \int \frac{2x+4}{x^2+4x-12}\,dx$

$= \log|x^2+4x-12|$ (1)

$I_2 = -7\int \frac{1}{x^2+4x-12}\,dx$

$= \frac{-7}{8}\log\left(\frac{x-2}{x+6}\right)$ (2)

$I = I_1 + I_2$ (1)

45(A) $h = \frac{a}{2}(\operatorname{cosec}\theta+\sin\theta)$; $k = \frac{b}{2}(\operatorname{cosec}\theta-\sin\theta)$ —(1)

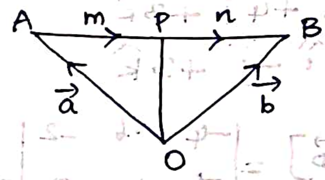
$\frac{2h}{a} = \operatorname{cosec}\theta+\sin\theta$; $\frac{2k}{b} = \operatorname{cosec}\theta-\sin\theta$ —(1)

$\left(\frac{2h}{a}\right)^2 - \left(\frac{2k}{b}\right)^2 = (\operatorname{cosec}\theta+\sin\theta)^2 - (\operatorname{cosec}\theta-\sin\theta)^2$ (1)

$\frac{h^2}{a^2} - \frac{k^2}{b^2} = 1$ (1)

$b^2x^2 - a^2y^2 = a^2b^2$ —— (1)

45(B) Statement (1)



—(1)

$n\,\vec{AP} = m\,\vec{PB}$ —(1)

$n(\vec{r}-\vec{a}) = m(\vec{b}-\vec{r})$ —(1)

$\vec{r} = \vec{OP} = \frac{n\vec{a}+m\vec{b}}{n+m}$ —(1)

46(A) $\sqrt{\dfrac{1-x}{1+x}} = (1-x)^{1/2}(1+x)^{-1/2}$ — (1)

$= \left[1-\frac{1}{2}x + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}x^2 + \dots\right] \times \left[1-\frac{1}{2}x + \frac{\frac{1}{2}\left(\frac{3}{2}\right)}{2!}x^2 + \dots\right]$ — (1)

$= \left[1-\frac{x}{2}-\frac{x^2}{8}+\dots\right] \times \left[1-\frac{x}{2}+\frac{3x^2}{8}+\dots\right]$ — (1)

$\simeq 1 - x + \frac{x^2}{2}$ — (2)

46(B) $y\sqrt{1-x^2} = \sin^{-1}x$ — (1)

$y'(1-x^2) - xy = 1$ — (2)

$(1-x^2)y_2 - 3xy_1 - y = 0$ — (2)

47(A) $P(A_1) = \frac{60}{100}$; $P(A_2) = \frac{40}{100}$ — (1)

$P(B|A_1) = \frac{2}{100}$; $P(B|A_2) = \frac{4}{100}$ — (1)

$P(B) = P(A_1)\,P(B|A_1) + P(A_2)\,P(B|A_2)$ — (1)

$P(B) = 0.028$ — (2)

47(B)
$$\left.\begin{aligned} \vec{AB} &= -4\hat{i} - 6\hat{j} - 2\hat{k} \\ \vec{AC} &= -\hat{i} + 4\hat{j} + 3\hat{k} \\ \vec{AD} &= -8\hat{i} - \hat{j} + 3\hat{k} \end{aligned}\right\} \text{— (2)}$$

$$[\vec{AB}\ \vec{AC}\ \vec{AD}] = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0 \text{ — (2)}$$

Vectors are coplanar — (1)

— x —