HALF YEARLY EXAMINATION - 2024

CLASS:XI **MATHEMATICS** Reg.No

Time : 3.00 Hours MARKS : 90

PART – I

Answer all the questions . **20 x 1 = 20**

1. The value of x, for which the matrix $A=\begin{bmatrix} e^{x-2} & e^{7+x} \\ e^{2+x} & e^{2x+3} \end{bmatrix}$ is singular is
   (1) 9 (2) 8 (3) 7 (4) 6
2. If A is skew – symmetric of order n and C is a column matrix of order n x 1, then $C^T AC$ is
   (1) an identify matrix of order n (2) an identify matrix of order 1
   (3) a zero matrix of order 1 (4) an identify matrix of order 2
3. If $\vec{r}=\frac{9\vec{a}+7\vec{b}}{16}$, then the point P whose position vector $\vec{r}$ divides the line joining the points with position vectors $\vec{a}$ and $\vec{b}$ in the ratio
   (1) 7 : 9 internally (2) 9 : 7 internally (3) 9 : 7 externally (4) 7 : 9 externally
4. If the points whose position vectors $10\hat{i}+3\hat{j}, 12\hat{i}-5\hat{j}$ and $a\hat{i}+11\hat{j}$ are collinear then a is equal to
   (1) 6 (2) 3 (3) 5 (4) 8
5. The value of $\lim\limits_{x\to k^-} x-[x]$, where k is an integer is
   (1) – 1 (2) 1 (3) 0 (4) 2
6. The derivative of $f(x)=x|x|$ at $x=-3$ is
   (1) 6 (2) – 6 (3) does not exist (4) 0
7. $\int \frac{\sqrt{\tan x}}{\sin 2x}dx$ is
   (1) $\sqrt{\tan x}+c$ (2) $2\sqrt{\tan x}+c$ (3) $\frac{1}{2}\sqrt{\tan x}+c$ (4) $\frac{1}{4}\sqrt{\tan x}+c$
8. $\int e^{-4x}\cos x\,dx$ is
   (1) $\frac{e^{-4x}}{17}[4\cos x-\sin x]+c$ (2) $\frac{e^{-4x}}{17}[-4\cos x+\sin x]+c$
   (3) $\frac{e^{-4x}}{17}[4\cos x+\sin x]+c$ (4) $\frac{e^{-4x}}{17}[-4\cos x-\sin x]+c$
9. A letter is taken at random from the letters of the word 'ASSISTANT' and another letter is taken at random from the letters of the word 'STATISTICS'. The probability that the selected letters are the same is
   (1) $\frac{7}{45}$ (2) $\frac{17}{90}$ (3) $\frac{29}{90}$ (4) $\frac{19}{90}$
10. The probability of two events A and B are 0.3 and 0.6 respectively. The probability that both A and B occur simultaneously is 0.18. The probability that neither A nor B occurs is
    (1) 0.1 (2) 0.72 (3) 0.42 (4) 0.28
11. If $A=\{(x,y): y=e^x, x\in R\}$ and $B=\{(x,y): y=e^{-x}, x\in R\}$ then $n(A\cap B)$ is
    (1) Infinity (2) 0 (3) 1 (4) 2
12. The function $f: R\to R$ is defined by $f(x)=\sin x+\cos x$ is
    (1) an odd function (2) neither an odd function nor an even function
    (3) an even function (4) both odd function and even function

13. The number of solutions of $x^2+|x-1|=1$ is

(1) 1 (2) 0 (3) 2 (4) 3

14. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)=$

(1) $\frac{1}{8}$ (2) $\frac{1}{2}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{\sqrt{2}}$

15. If $\sin\alpha+\cos\alpha=b$, then $\sin 2\alpha$ is equal to

(1) b^2-1, if $b\le\sqrt{2}$ (2) b^2-1, if $b>\sqrt{2}$ (3) b^2-1, if $b\ge 1$ (4) b^2-1, if $b\ge\sqrt{2}$

16. Number of sides of a polygon having 44 diagonals is ......

(1) 4 (2) 4 ! (3) 11 (4) 22

17. 1 + 3 + 5 + 7 +........ + 17 is equal to

(1) 101 (2) 81 (3) 71 (4) 61

18. The value of $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+$........ is

(1) $\frac{e^2+1}{2e}$ (2) $\frac{(e+1)^2}{2e}$ (3) $\frac{(e-1)^2}{2e}$ (4) $\frac{e^2-1}{2e}$

19. The slope of the line which makes an angle 45° with the line $3x-y=-5$ are

(1) $1,-1$ (2) $\frac{1}{2},-2$ (3) $1,\frac{1}{2}$ (4) $2,-\frac{1}{2}$

20. If the lines represented by the equation $6x^2+41xy-7y^2=0$ makes angles α and β with $x-axis$, then $\tan\alpha\tan\beta=$

(1) $-\frac{6}{7}$ (2) $\frac{6}{7}$ (3) $-\frac{7}{6}$ (4) $\frac{7}{6}$

PART – II

Answer any seven questions. Question No.30 is compulsory. **7 x 2 = 14**

21. If $n(A\cap B)=3$ and $n(A\cup B)=10$, then find $n(P(A\Delta B))$.

22. Solve : $3x^2+5x-2\le 0$.

23. Find the general solution of $\sin\theta=-1$

24. If $^nC_4=495$, What is n?

25. If P (r, c) is mid point of a line segment between the axes, then show that $\frac{x}{r}+\frac{y}{c}=2$.

26. If Consider the matrix $A_\alpha=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ Show that $A_\alpha A_\beta=A_{(\alpha+\beta)}$.

27. Find the direction cosines of $\overrightarrow{AB}$ where A is (2, 3, 1) and B is (3, - 1, 2).

28. Compute $\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}$

29. Determine whether the function $f(x)=x\,|x|$ is differentiable at $x=0$.

30. Integrate : $\frac{(\sin^{-1}x)^2}{\sqrt{1-x^2}}$

PART – III

Answer any Seven questions. Question No.40 is compulsory. **7 x 3 = 21**

31. In the set Z of integers, define mRn if $m-n$ is a multiple of 12. Prove that R is an equivalence relation.

32. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, then prove that $xyz = 1$

33. Show that $\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9} = 2$.

34. If the letters of the word FUNNY are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, find the rank of the word FUNNY?

35. Prove that one of the straight lines given by $ax^2 + 2hxy + by^2 = 0$ will bisect the angle between the co – ordinate axes if $(a + b)^2 = 4h^2$.

36. Show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$.

37. Find the area of the parallelogram whose two adjacent sides are determined by the vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$

38. Integrate : $\frac{1}{\sqrt{x+3} - \sqrt{x-4}}$.

39. A year is selected at random. What is the probability that (i) it contains 53 Sundays (ii) it is a leap year which contains 53 Sundays.

40. Find $\frac{dy}{dx}$ if $x = a(t + \sin t), y = a(1 + \cos t)$.

PART – IV

Answer all the questions. **7 x 5 = 35**

41. **(a)** Let $f, g : R \to R$ be defined as $f(x) = 2x - |x|$ and $g(x) = 2x + |x|$. Find $f \circ g$ and $g \circ f$

(OR)

(b) Resolve into partial fractions: $\frac{2x}{(x^2+1)(x-1)}$.

42. **(a)** By the principle of mathematical induction, prove that, for all integers $n \geq 1$,

$$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(OR)

(b) If p is length of perpendicular from origin to the line whose intercepts on the axes are a and b, then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

43. **(a)** A consulting firm rents car from three agencies such that 50% from agency L, 30% from agency M and 20% from agency N. If 90% of the cars from L, 70% of cars from M and 60% of the cars from N are in good conditions

(i) what is the probability that the firm will get a car in good conditions?

(ii) if a car is in good condition, what is the probability that it has come from agency N?

(OR)

(b) Integrate: $\frac{3x+5}{x^2+4x+7}$

44.**(a)** If a, b, c are all positive, and are p^{th}, q^{th} and r^{th} terms of a G.P, show that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$

(OR)

(b) Prove that $\sqrt[3]{x^3+6} - \sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.

45.**(a)** Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines. Find the distance between them.

(OR)

(b) Show that the medians of a triangle are concurrent.

46.**(a)** Find the value of k if $\lim_{x \to 1} \frac{x^4-1}{x-1} = \lim_{x \to k} \frac{x^3-k^3}{x^2-k^2}$

(OR)

(b) Find the derivative of $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ with respect to $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.

47. **(a)** State and prove any one of the Napier's formulae.

(OR)

(b) Show that the following function is not differentiable at the indicated value of x.

$f(x) = \begin{cases} -x+2, & x \le 2 \\ 2x-4, & x > 2 \end{cases}; \ x = 2$