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Time : 3.00 Hrs.

**Half Yearly Examination - 2024**  
**MATHEMATICS**

Register No.

40

Marks : 90

## SECTION - A

10 x 1 = 10

Choose the correct answer:

- The function  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = \sin x + \cos x$  is  
a) an odd function b) neither an odd function nor an even function c) an even function d) both odd function and even function
- Given that  $x, y$  and  $b$  are real numbers  $x < y, b > 0$ , then a)  $xb < yb$  b)  $xb > yb$  c)  $xb \leq yb$  d)  $\frac{x}{b} \geq \frac{y}{b}$
- The maximum value of  $4\sin^2 x + 3\cos^2 x + \sin \frac{x}{2} + \cos \frac{x}{2}$  is a)  $4 + \sqrt{2}$  b)  $3 + \sqrt{2}$  c) 9 d) 4
- If  $\sin \alpha + \cos \alpha = b$ , then  $\sin 2\alpha$  is equal to a)  $b^2 - 1$ , if  $b \leq \sqrt{2}$  b)  $b^2 - 1$ , if  $b > \sqrt{2}$  c)  $b^2 - 1$ , if  $b \geq 1$  d)  $b^2 - 1$ , if  $b \geq \sqrt{2}$
- The number of 5 digit numbers all digits of which are odd is  
a) 25 b)  $5^5$  c)  $5^6$  d) 625
- Everybody in a room shake hands with everybody else. The total number of shake hands is 66. The number of persons in the room is a) 11 b) 12 c) 10 d) 6
- If  $a, 8, b$  are in AP,  $a, 4, b$  are in GP, and if  $a, x, b$  are in HP then  $x$  is a) 2 b) 1 c) 4 d) 16
- Which of the following point lie on the locus of  $3x^2 + 3y^2 - 8x - 12y + 17 = 0$  a) (0, 0) b) (-2, 3) c) (1, 2) d) (0, -1)
- If  $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$  and  $(A + B)^2 = A^2 + B^2$ , then the values of  $a$  and  $b$  are  
a)  $a = 4, b = 1$  b)  $a = 1, b = 4$  c)  $a = 0, b = 4$  d)  $a = 2, b = 4$
- The value of  $\vec{AB} + \vec{BC} + \vec{DA} + \vec{CD}$  is a)  $\vec{AD}$  b)  $\vec{CA}$  c)  $\vec{0}$  d)  $-\vec{AD}$
- A vector makes equal angle with the positive direction of the coordinate axes. Then each angle is equal to  
a)  $\cos^{-1} \left( \frac{1}{3} \right)$  b)  $\cos^{-1} \left( \frac{2}{3} \right)$  c)  $\cos^{-1} \left( \frac{1}{\sqrt{3}} \right)$  d)  $\cos^{-1} \left( \frac{2}{\sqrt{3}} \right)$
- $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{x}$  a) 0 b) 1 c)  $\sqrt{2}$  d) does not exist
- $\lim_{n \rightarrow \infty} \left( \frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \dots + \frac{n}{n^2} \right)$  is a)  $\frac{1}{2}$  b) 0 c) 1 d)  $\infty$
- If the derivative of  $(ax - 5)e^{3x}$  at  $x = 0$  is  $-13$ , then the value of  $a$  is a) 8 b)  $-2$  c) 5 d) 2
- $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$  is a)  $\cot(xe^x) + c$  b)  $\sec(xe^x) + c$  c)  $\tan(xe^x) + c$  d)  $\cos(xe^x) + c$
- A number is elected from the set  $\{1, 2, 3, \dots, 20\}$ . The probability that the selected number is divisible by 3 or 4 is  
a)  $\frac{2}{5}$  b)  $\frac{1}{8}$  c)  $\frac{1}{2}$  d)  $\frac{2}{3}$
- A man has 3 fifty rupee notes, 4 hundred rupees notes, and 6 five hundred rupees notes in his pocket. If 2 notes are taken at random, what are the odds in favour of both notes being of hundred rupee denomination? a) 1 : 12 b) 12 : 1 c) 13 : 1 d) 1 : 13
- $2 \sin 5x \cos x \dots$  a)  $\sin 6x + \cos 4x$  b)  $\sin 6x + \sin 4x$  c)  $\cos 6x + \sin 4x$  d)  $\cos 6x + \cos 4x$
- If  $\alpha$  and  $\beta$  are the roots of  $2x^2 - 3x - 4 = 0$  find the value of  $\alpha^2 + \beta^2$  a)  $\frac{41}{4}$  b)  $\frac{\sqrt{14}}{2}$  c) 0 d) none of these
- If  $mC_1 = nC_2$ , then..... a)  $2m = n$  b)  $2m = n(n+1)$  c)  $2m = n(n-1)$  d)  $2n = m(m-1)$

## SECTION - B

Answer any seven questions. Question No.30 is compulsory

7 x 2 = 14

- If  $A = \{1, 2, 3, 4\}$  and  $B = \{3, 4, 5, 6\}$ , find  $n(A \cup B) \times (A \cap B) \times (A \Delta B)$
- If  $a$  and  $b$  are the roots of the equation  $x^2 - px + q = 0$ , find the value of  $\frac{1}{a} + \frac{1}{b}$
- Find the middle term in the expansion of  $(x + y)^6$ .
- Find the path traced out by the point  $\left( ct, \frac{c}{t} \right)$ , here  $t \neq 0$  is the parameter and  $c$  is a constant.
- If  $(k, 2), (2, 4)$  and  $(3, 2)$  are vertices of the triangle of area 4 square units then determine the value of  $k$ .

26. Find  $|\vec{a} \times \vec{b}|$ , where  $\vec{a} = 3\hat{i} + 4\hat{j}$  and  $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

27. Evaluate the following limits :  $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$

28. Differentiate :  $y = e^{\tan x}$ .

29. Integrate the following with respect to  $x$  :  $\frac{1}{x^{10}}$

30. If A and B are two independent events such that,  $P(A) = 0.4$  and  $P(A \cup B) = 0.9$ . Find  $P(B)$ .

**SECTION - C**

7 x 3 = 21

Answer any seven questions. Question No.40 is compulsory.

31. Find the range of the function  $f(x) = \frac{1}{1 - 3 \cos x}$

32. A girl A is reading a book having 446 pages and she has already finished reading 271 pages. She wants to finish reading this book within a week. What is the minimum number of pages she should read per day to complete reading the book within a week?

33. If in two Circles, arcs of the same length subtend angles  $60^\circ$  and  $75^\circ$  at the center, find the ratio of their radii.

34. Prove that  ${}^{10}C_2 + 2 \cdot {}^{10}C_3 + {}^{10}C_4 = {}^{12}C_4$ .

35. If the 5<sup>th</sup> and 9<sup>th</sup> terms of a harmonic progression are  $\frac{1}{19}$  and  $\frac{1}{35}$ , find the 12<sup>th</sup> term of the sequence.

36. If  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$  and  $A^3 - 6A^2 + 7A + KI = 0$ , find the value of  $k$ .

37. Differentiate the following :  $y = \sqrt{1 + 2 \tan x}$

38. Evaluate the following integrals :  $\int e^x (\sin x + \cos x) dx$

39. What is the chance that (i) non-leap year (ii) leap year should have fifty three Sundays?

40. If  $\frac{\log x}{y - z} = \frac{\log y}{z - x} = \frac{\log z}{x - y}$  then prove that  $xyz = 1$ .

**SECTION - D**

7 x 5 = 35

Answer all questions.

41. a) If  $f : \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = 3x - 5$ , prove that  $f$  is a bijection and find its inverse. (OR)

b) If one root of  $k(x - 1)^2 = 5x - 7$  is double the other root, show that  $k = 2$  or  $-25$ .

42. a) Express the matrix  $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$  as the sum of a symmetric and a skew-symmetric matrices. (OR)

b) State and prove the law of sines.

43. a) What will Rs 500 amount to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually? (OR)

b) Use induction to prove that  $n^3 - 7n + 3$  is divisible by 3, for all natural numbers  $n$ .

44. a) Show that the points  $(1, 3)$ ,  $(2, 1)$  and  $(\frac{1}{2}, 4)$  are collinear, by using (i) concept of slope (ii) a straight line (iii) any other method. (OR)

b) The slope of one of the straight lines  $ax^2 + 2hxy + by^2 = 0$  is twice that of the other, show that  $8h^2 = 9ab$ .

45. a) Prove that  $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$  (OR)

b) Find the derivative with  $\left(\frac{\sin x}{1 + \cos x}\right)$  with respect to  $\left(\frac{\cos x}{1 + \sin x}\right)$

46. a) Show that the vectors  $5\hat{i} + 6\hat{j} + 7\hat{k}$ ,  $7\hat{i} - 8\hat{j} + 9\hat{k}$ ,  $3\hat{i} + 20\hat{j} + 5\hat{k}$  are coplanar. (OR)

b) Evaluate the following integrals :  $\int \frac{3x+5}{x^2+4x+7} dx$

47. a) The chances of X, Y and Z becoming managers of a certain company are 4 : 2 : 3. The probabilities that bonus scheme will be introduced if X, Y and Z become managers are 0.3, 0.5 and 0.4 respectively. If the bonus scheme has been introduced, What is the probability that Z was appointed as the manager? (OR)

b) Sum upto  $n$  terms the series.  $7 + 77 + 777 + \dots$