Class : 11

Register Number

COMMON HALFYEARLY EXAMINATION - 2024 - 25

MATHEMATICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

PART - I

Answer all the questions. 20x1=20

1. Let A and B be subsets of the universal set $\mathbb{N}$, the set of natural numbers. Then $A' \cup [(A \cap B) \cup B']$ is

(1) A (2) A' (3) B (4) $\mathbb{N}$

2. The value of $\log_3 11 \cdot \log_{11} 13 \cdot \log_{13} 15 \cdot \log_{15} 27 \cdot \log_{27} 81$ is

(1) 1 (2) 2 (3) 3 (4) 4

3. $\cos 1° + \cos 2° + \cos 3° + \cdots + \cos 179° =$

(1) 0 (2) 1 (3) -1 (4) 89

4. There are 10 points in a plane and 4 of them are collinear. The number of straight lines joining any two points is

(1) 45 (2) 40 (3) 39 (4) 38 .

5. The coefficient of x^6 in $(2+2x)^{10}$ is

(1) ${}^{10}C_6$ (2) 2^6 (3) ${}^{10}C_6 2^6$ (4) ${}^{10}C_6 2^{10}$.

6. The image of the point (2,3) in the line $y = -x$ is

(1) $(-3,-2)$ (2) $(-3,2)$ (3) $(-2,-3)$ (4) $(3,2)$

7. Find a so that the sum and product of the roots of the equation $2x^2 + (a-3)x + 3a - 5 = 0$ are equal is

(1) 1 (2) 2 (3) 0 (4) 4

8. If $f(\theta) = |\sin\theta| + |\cos\theta|, \theta \in R$, then $f(\theta)$ is in the interval

(1) $[0,2]$ (2) $[1,\sqrt{2}]$ (3) $[1,2]$ (4) $[0,1]$

9. The sum up to n terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \cdots$ is

(1) $\frac{n(n+1)}{2}$ (2) $2n(n+1)$ (3) $\frac{n(n+1)}{\sqrt{2}}$ (4) 1 .

10. If the points $(x,-2), (5,2), (8,8)$ are collinear, then x is equal to

(1) -3 (2) $\frac{1}{3}$ (3) 1 (4) 3

11. If X and Y be two events such that $P(X/Y) = \frac{1}{2}, P(Y/X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$, then $P(X \cup Y)$ is

(1) $\frac{1}{3}$ (2) $\frac{2}{5}$ (3) $\frac{1}{6}$ (4) $\frac{2}{3}$

12. If $\int \frac{3^{\frac{1}{x}}}{x^2} dx = k\left(3^{\frac{1}{x}}\right) + c$, then the value of k is

(1) $\log 3$ (2) $-\log 3$ (3) $-\frac{1}{\log 3}$ (4) $\frac{1}{\log 3}$

13. If $f(x) = x + 2$, then $f'(f(x))$ at $x = 4$ is

(1) 8 (2) 1 (3) 4 (4) 5

14. $\lim_{x\to 0} \frac{8^x - 4^x - 2^x + 1^x}{x^2} =$

(1) 2 log2 (2) $2(\log 2)^2$ (3) log2 (4) 3log2

15. If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ and $|\vec{b}| = 46$, then $|\vec{a}|$ is

(1) 42 (2) 12 (3) 22 (4) 32

16. If $A + I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$, then $(A + I)(A - I)$ is equal to

(1) $\begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$ (2) $\begin{bmatrix} -5 & 4 \\ -8 & 9 \end{bmatrix}$ (3) $\begin{bmatrix} 5 & 4 \\ 8 & 9 \end{bmatrix}$ (4) $\begin{bmatrix} -5 & -4 \\ -8 & -9 \end{bmatrix}$

17. Condition for perpendicular to the pair of straight lines $ax^2 + 2hxy + by^2 = 0$

(1) $a + b = 0$ (2) $a - b = 0$ (3) $h^2 - ab = 0$ (4) $h^2 + ab = 0$

18. Two vectors are parallel in the same direction then

(1) $\theta = 0$ (2) $\theta = \frac{\pi}{2}$ (3) $\theta = \frac{\pi}{3}$ (4) $\theta = \frac{\pi}{4}$

19. If $x = at^2, y = 2at$ then $\frac{dy}{dx} =$

(1) t (2) $1/t$ (3) $-t$ (4) $2t$

20. The probability of impossibe event is ______

(1) 1 (2) 0 (3) 1/2 (4) 1/4

PART - II

Answer any seven questions. Question No:30 is compulsory **7x2=14**

21. If $f: [-2,2] \to B$ is given by $f(x) = 2x^3$, then find B so that f is onto.

22. Compute : 102^4

23. Compute : $log_9 27 - log_{27} 9$.

24. Find a direction ratio and direction cosines of $3\hat{\imath} - 4\hat{k}$.

25. Prove that $\sin 4A = 4 \sin A \cos^3 A - 4 \cos A \sin^3 A$.

26. Differentiate : $y = (x^3 - 1)^{100}$.

27. Show that the lines are $3x + 2y + 9 = 0$ and $12x + 8y - 15 = 0$ are parallel lines.

28. An integer is chosen at random from the first ten positive integers. Find the probability that it is (i) an even number (ii) multiple of three.

29. Compute $\lim_{x\to 1} \frac{x^3-1}{x-1}$.

30. Without expanding the determinant, prove that $\begin{vmatrix} s & a^2 & b^2 + c^2 \\ s & b^2 & c^2 + a^2 \\ s & c^2 & a^2 + b^2 \end{vmatrix} = 0$.

PART - III

Answer any seven questions. Question No:40 is compulsory **7x3=21**

31. Prove that $\begin{vmatrix} 1 & x & x^2 \\ x & 1 & x \\ x & x & 1 \end{vmatrix} = \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix}$.

32. Find the area of a triangle having points $A(1,0,0), B(0,1,0)$ and $C(0,0,1)$ as its vertices.

33. Find the principal solution of $\sin\theta = -\frac{\sqrt{3}}{2}$.

34. The probability of an event A occurring is 0.5 and B occurring is 0.3. If A and B are mutually exclusive events, then find the probability of

    (i) $P(A \cup B)$ (ii) $P(A \cap \bar{B})$ (iii) $P(\bar{A} \cap B)$

35. If $(n+1)C_8 : {}^{(n-3)}P_4 = 57:16$, find the value of n.

36. If the 5th and 9th terms of a harmonic progression are $\frac{1}{19}$ and $\frac{1}{35}$, find the 12th term of the sequence.

37. Differentiate: $y = \sqrt{x + \sqrt{x}}$

38. Show that $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines.

39. If $f'(x) = 3x^2 - 4x + 5$ and $f(1) = 3$, then find $f(x)$.

40. A quadratic polynomial has one of its zeros $1 + \sqrt{5}$ and it satisfies $p(1) = 2$. Find the quadratic polynomial.

PART - IV

Answer all the questions. **7x5=35**

41. A) Write the values of f at $-3,5,2,-1,0$ if A)

$$f(x) = \begin{cases} x^2 + x - 5, & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2, & \text{if } x \in (3, \infty) \\ x^2, & \text{if } x \in (0,2) \\ x^2 - 3, & \text{otherwise} \end{cases}$$

(OR)

B) Show that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$.

42. A) Resolve the following rational expressions into partial fractions $\frac{x^2+x+1}{x^2-5x+6}$

(OR)

B) Show that $\lim_{x\to\infty^+} x\left[\left\lfloor\frac{1}{x}\right\rfloor + \left\lfloor\frac{2}{x}\right\rfloor + \cdots + \left\lfloor\frac{15}{x}\right\rfloor\right] = 120$.

43. A) If $A + B + C = \pi$, prove that $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A\cos B\cos C$.

(OR)

B) Solve : $\frac{x+1}{x+3} < 3$.

44. A) By the principle of mathematical induction, prove that, for $n \geq 1$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

(OR)

B) Integrate with respect to x : $\frac{2x-3}{x^2+4x-12}$

45. A) The coordinates of a moving point P are $\left(\frac{a}{2}(\text{cosec}\theta + \sin\theta), \frac{b}{2}(\text{cosec}\theta - \sin\theta)\right)$, where θ is a variable parameter. Show that the equation of the locus P is $b^2x^2 - a^2y^2 = a^2b^2$.

(OR)

B) Prove that Section Formula – Internal Division.

46. A) Prove that $\sqrt{\frac{1-x}{1+x}}$ is approximately equal to $1 - x + \frac{x^2}{2}$ when x is very small.

(OR)

B) If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2 - 3xy_1 - y = 0$.

47. A) A factory has two Machines-I and II. Machine-I produces 60% of items and Machine-II produces 40% of the items of the total output. Further 2% of the items produced by Machine-I are defective whereas 4% produced by Machine-II are defective. If an item is drawn at random what is the probability that it is defective?

(OR)

B) Show that the points whose position vectors $4\hat{i} + 5\hat{j} + \hat{k}, -\hat{j} - \hat{k}, 3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ are coplanar.