11 - STD

Reg. No.

HALF YEARLY EXAMINATION - 2024

TIME : 3.00 Hrs MATHEMATICS MARKS : 90

PART – A

i. *Answer all the questions*

ii. *Choose the most suitable answer from the given four alternatives.* *20X 1 = 20*

1. For non-empty sets A and B, if $A \subset B$ then $(A \times B) \cap (B \times A)$ is equal to
   (1) $A \cap B$ (2) $A \times A$ (3) $B \times B$ (4) none of these

2. If the function $f: [-3,3] \to S$ defined by $f(x) = x^2$ is onto, then S is
   (1) $[-9,9]$ (2) $\mathbb{R}$ (3) $[-3,3]$ (4) $[0,9]$

3. The equation whose roots are numerically equal but opposite in sign to the roots of $3x^2 - 5x - 7 = 0$ is
   (1) $3x^2 - 5x - 7 = 0$ (2) $3x^2 + 5x - 7 = 0$
   (3) $3x^2 - 5x + 7 = 0$ (4) $3x^2 + x - 7 = 0$

4. $\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$ is equal to
   (1) $\sin 2(\theta + \phi)$ (2) $\cos 2(\theta + \phi)$
   (3) $\sin 2(\theta - \phi)$ (4) $\cos 2(\theta - \phi)$

5. A wheel is spinning at 2 radians/second. How many seconds will it take to make 10 complete rotations?
   (1) 10π seconds (2) 20π seconds
   (3) 5π seconds (4) 15π seconds

6. The number of ways of choosing 5 cards out of a deck of 52 cards which include at least one king is
   (1) ${}^{52}C_5$ (2) ${}^{48}C_5$ (3) ${}^{52}C_5 + {}^{48}C_5$ (4) ${}^{52}C_5 - {}^{48}C_5$

7. If ${}^{n}C_4, {}^{n}C_5, {}^{n}C_6$ are in AP the value of n can be
   (1) 14 (2) 11 (3) 9 (4) 5

8. The sequence $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}+\sqrt{2}}, \frac{1}{\sqrt{3}+2\sqrt{2}}, \cdots$ form an
   (1) AP (2) GP (3) HP (4) AGP.

9. The value of the series $\frac{1}{2} + \frac{7}{4} + \frac{13}{8} + \frac{19}{16} + \cdots$ is
   (1) 14 (2) 7 (3) 4 (4) 6.

10. If the point $(8, -5)$ lies on the locus $\frac{x^2}{16} - \frac{y^2}{25} = k$, then the value of k is
    (1) 0 (2) 1 (3) 2 (4) 3

11. If $|\vec{a} + \vec{b}| = 60, |\vec{a} - \vec{b}| = 40$ and $|\vec{b}| = 46$, then $|\vec{a}|$ is
    (1) 42 (2) 12 (3) 22 (4) 32

12. The matrix A satisfying the equation $\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix} A = \begin{bmatrix}1 & 1\\0 & -1\end{bmatrix}$ is

(1) $\begin{bmatrix}1 & 4\\-1 & 0\end{bmatrix}$ (2) $\begin{bmatrix}1 & -4\\1 & 0\end{bmatrix}$ (3) $\begin{bmatrix}1 & 4\\0 & -1\end{bmatrix}$ (4) $\begin{bmatrix}1 & -4\\1 & 1\end{bmatrix}$

13. If $\overrightarrow{BA} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and the position vector of B is $\hat{\imath} + 3\hat{\jmath} - \hat{k}$, then the position vector A is

(1) $4\hat{\imath} + 2\hat{\jmath} + \hat{k}$ (2) $4\hat{\imath} + 5\hat{\jmath}$ (3) $4\hat{\imath}$ (4) $-4\hat{\imath}$

$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$
$\overrightarrow{OA} = \overrightarrow{BA} + \overrightarrow{OB}$
$= 3\hat{i} + 2\hat{j} + \hat{k} + \hat{i} + 3\hat{j} - \hat{k}$
$\overrightarrow{OA} = 4\hat{i} + 5\hat{j}$

14. If $\Delta = \begin{vmatrix}a & b & c\\x & y & z\\p & q & r\end{vmatrix}$, then $\begin{vmatrix}ka & kb & kc\\kx & ky & kz\\kp & kq & kr\end{vmatrix}$ is

(1) Δ (2) $k\Delta$ (3) $3k\Delta$ (4) $k^3\Delta$

15. $\lim\limits_{x\to\infty} \frac{\sqrt{x^2-1}}{2x+1} =$

(1) 1 (2) 0 (3) -1 (4) $\frac{1}{2}$

$\frac{\sqrt{\infty^2-1}}{2(\infty+1)}$
$= \frac{\sqrt{\infty^2}}{2(\infty)}$
$= \frac{\infty}{\infty}$
$= 0$

16. At $x = \frac{3}{2}$ the function $f(x) = \frac{|2x-3|}{2x-3}$ is

(1) continuous (2) discontinuous
(3) differentiable (4) non-zero

17. If $f(x) = \begin{cases} x+2, & -1 < x < 3\\ 5, & x = 3\\ 8-x, & x > 3\end{cases}$, then at $x = 3$, $f'(x)$ is

(1) 1 (2) -1 (3) 0 (4) does not exist

18. If $f(x) = x + 2$, then $f'(f(x))$ at $x = 4$ is

(1) 8 (2) 1 (3) 4 (4) 5

19. $\int e^{\sqrt{x}} dx$ is

(1) $2\sqrt{x}\left(1 - e^{\sqrt{x}}\right) + c$ (2) $2\sqrt{x}\left(e^{\sqrt{x}} - 1\right) + c$
(3) $2e^{\sqrt{x}}(1 - \sqrt{x}) + c$ (4) $2e^{\sqrt{x}}(\sqrt{x} - 1) + c$

$x^{\frac{1}{2}+1}$
$\frac{x^{3/2}}{3/2}$
$e^{\sqrt{x}}$
$y = x + 2$
$x = y - 2$
$f(x) = y - 2 + 2$
$f(x) = y$
$\frac{2e^{\sqrt{x}} x\sqrt{x}}{3} + c$

20. In a certain college 4% of the boys and 1% of the girls are taller than 1.8 meter. Further 60% of the students are girls. If a student is selected at random and is taller than 1.8 meters, then the probability that the student is a girl is

(1) $\frac{2}{11}$ (2) $\frac{3}{11}$ (3) $\frac{5}{11}$ (4) $\frac{7}{11}$

PART – B

i. *Answer any seven questions :-*
ii. *Question No. 30 is compulsory :-.* 7X 2= 14

21. If $\wp(A)$ denotes the power set of A, then find $n\left(\wp(\wp(\wp(\phi)))\right)$.

22. What is the chance that
(i) non-leap year (ii) leap year should have fifty three Sundays?

23. Find the general solution of $\sin\theta = -\frac{\sqrt{3}}{2}$.
24. If $nC_{12} = {}^{n}C_9$ find $21C_n$.
25. Find the middle term in the expansion of $(x+y)^6$.
26. Find the nearest point on the line $2x + y = 5$ from the origin.
27. If $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 0 & -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 \\ -1 & 1 \\ 1 & 2 \end{bmatrix}$ find AB and BA if they exists.
28. If D is the midpoint of the side BC of a triangle ABC, prove that $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$.
29. Differentiate : $y = e^{\sin x}$.
30. If the logarithm of 324 to base a is 4, then find a.

PART – C

i. Answer any seven questions:–
ii. Question No. 40 is compulsory:– 7X 3= 21

31. Find the range of the function $\frac{1}{2\cos x - 1}$.
32. Resolve into partial fractions : $\frac{x}{(x+3)(x-4)}$.
33. Find $\cos(x-y)$, given that $\cos x = -\frac{4}{5}$ with $\pi < x < \frac{3\pi}{2}$ and $\sin y = -\frac{24}{25}$ with $\pi < y < \frac{3\pi}{2}$.
34. If $(n+1)C_8 : {}^{(n-3)}P_4 = 57:16$, find the value of n.
35. Prove that if a, b, c are in HP, if and only if $\frac{a}{c} = \frac{a-b}{b-c}$.
36. Find the points on the line $x + y = 5$, that lie at a distance 2 units from the line $4x + 3y - 12 = 0$.
37. Prove that $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}^2 = \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix}$.
38. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{3}, \frac{1}{4}$, and $\frac{1}{5}$ (i) What is the probability that the problem is solved? (ii) What is the probability that exactly one of them will solve it?
39. Find $\frac{dy}{dx}$ if $x = a(t - \sin t), y = a(1 - \cos t)$.
40. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + 2\vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{c}$.

PART – D

i. Answer all the questions:– 7X 5= 35

41. (a) Write the values of f at $-3,5,2,-1,0$ if

$$f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0,2) \\ x^2 - 3 & \text{otherwise} \end{cases}$$

(or)

(b) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

42. (a) Find the condition that one of the roots of $ax^2 + bx + c$ may be

(i) negative of the other, (ii) thrice the other, (iii) reciprocal of the other **(or)**

(b) Using the Mathematical induction, show that for any natural number $n \geq 2$,

$$\frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \cdots + \frac{1}{1+2+3+\cdots+n} = \frac{n-1}{n+1}.$$

43. (a) A straight tunnel is to be made through a mountain. A surveyor observes the two extremities A and B of the tunnel to be built from a point P in front of the mountain. If $AP = 3$km, BP = 5 km and $\angle APB = 120°$, then find the length of the tunnel to be built.

(or)

(b) If $A + B + C = \pi$, prove that $cos^2 A + cos^2 B + cos^2 C = 1 - 2\cos A \cos B \cos C$.

44. (a) Evaluate : $\int \frac{1}{x^2-2x+5} dx$ **(or)**

(b) Given that $P(A) = 0.52$, $P(B) = 0.43$ and $P(A \cap B) = 0.24$, find

(i) $P(A \cap \bar{B})$ (ii) $P(A \cup B)$ (iii) $P(\bar{A} \cap \bar{B})$ (iv) $P(\bar{A} \cup \bar{B})$

45. (a) Show that the straight lines $x^2 - 4xy + y^2 = 0$ and $x + y = 3$ form an equilateral triangle.

(or)

(b) Prove that $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$.

46. (a) If $y = \frac{sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2 - 3xy_1 - y = 0$.

(or)

(b) The chances of X, Y and Z becoming managers of a certain company are 4 : 2 : 3. The probabilities that bonus scheme will be introduced if X, Y and Z become managers are 0.3, 0.5 and 0.4 respectively. If the bonus scheme has been introduced, what is the probability that Z was appointed as the manager?

47. (a) The position vectors of the vertices of a triangle are $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$; $3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $-2\hat{\imath} + 3\hat{\jmath} - 7\hat{k}$. Find the perimeter of the triangle. **(or)**

(b) Show that $\lim_{x \to \infty^+} x \left[\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \cdots + \left[\frac{15}{x}\right] \right] = 120$.

$|\vec{a}+\vec{b}| = 60$

$|\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2 = 3600$

$|\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + (46)^2 = 3600$

$|\vec{a}|^2 + 2\vec{a}\cdot\vec{b} = 3600 - 2116 \to (1)$

$|\vec{a}-\vec{b}| = 40$

$|\vec{a}-\vec{b}|^2 = 1600$

$|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b} = 1600$

$|\vec{a}|^2 - 2\vec{a}\cdot\vec{b} = -2116 + 1600 \to (2)$

$2|\vec{a}|^2 = 1484 + (-516)$

$|\vec{a}|^2 = 484$

$46 \times 46 = 276 + 184 = 2116$

$3600 - 2116 = 1484$

$2116 - 1600 = 516$

$1484 - 516 = 968$

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} a+3c & b+3d \\ 0+c & d \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$\boxed{c=0}$, $\boxed{d=-1}$, $a = -3(0)+1$

$\boxed{a=1}$

$b = 1 - 3d$

$b = 1 - 3(-1)$

$\boxed{b=4}$

$A = \begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$