

HALF YEARLY EXAMINATION -2024

CLASS: XI

MATHEMATICS

TIME: 3.00 HRS.

MAX. MARKS: 90

PART-I

Choose the correct answer:

20*1=20

1. Let A and B be subsets of the universal set \mathbb{N} , the set of natural numbers then $A' \cup [(A \cap B) \cup B']$ is
 (1) A (2) A' (3) B (4) \mathbb{N}
2. The number of constant functions from a set containing m elements to a set containing n elements is
 (1) mn (2) m (3) n (4) $m+n$
3. Find a so that the sum and product of the roots of the equation $2x^2 + (a-3)x + 3a - 5 = 0$ are equal is
 (1) 1 (2) 2 (3) 0 (4) 4
4. The value of $\cos^3\left(\frac{\pi}{8}\right) \cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right) \sin\left(\frac{3\pi}{8}\right)$ is
 (1) $\frac{1}{4}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{2}$ (4) $\frac{1}{\sqrt{2}}$
5. If $f(\theta) = |\sin\theta| + |\cos\theta|$, $\theta \in R$, then $f(\theta)$ is in the interval
 (1) $[0,2]$ (2) $[1, \sqrt{2}]$ (3) $[1,2]$ (4) $[0,1]$
6. The product of r consecutive positive integers is divisible by
 (1) $r!$ (2) $(r-1)!$ (3) $(r+1)!$ (4) r
7. The number of 10 digit number that can be written by using the digits 2 and 3 is
 (1) ${}^{10}C_2 + {}^9C_2$ (2) 2^{10} (3) $2^{10} - 2$ (4) $10!$
8. The value of $2 + 4 + 6 + \dots + 2n$ is
 (1) $\frac{n(n-1)}{2}$ (2) $\frac{n(n+1)}{2}$ (3) $\frac{2n(2n+1)}{2}$ (4) $n(n+1)$
9. The coefficient of x^3 in the series e^{-2x} is
 (1) $\frac{4}{3}$ (2) $\frac{-4}{3}$ (3) $\frac{-4}{15}$ (4) $\frac{4}{15}$
10. Equation of the straight line that forms an isosceles triangle with coordinate axes in the I-quadrant with perimeter $4 + 2\sqrt{2}$ is
 (1) $x + y + 2 = 0$ (2) $x + y - 2 = 0$ (3) $x + y - \sqrt{2} = 0$ (4) $x + y + \sqrt{2} = 0$
11. If the two straight lines $x + (2k - 7)y + 3 = 0$ and $3kx + 9y - 5 = 0$ are perpendicular then the value of k is
 (1) $k = 3$ (2) $k = \frac{1}{3}$ (3) $k = \frac{2}{3}$ (4) $k = \frac{3}{2}$

12. The value of x , for which the matrix $A = \begin{bmatrix} e^{x-2} & e^{7+x} \\ e^{2+x} & e^{2x+3} \end{bmatrix}$ is singular is

- (1) 9 (2) 8 (3) 7 (4) 6

13. The value of the determinant of $A = \begin{bmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{bmatrix}$ is

- (1) $-2abc$ (2) abc (3) 0 (4) $a^2 + b^2 + c^2$

14. If $ABCD$ is a parallelogram, then $\overline{AB} + \overline{AD} + \overline{CB} + \overline{CD}$ is equal to

- (1) $2(\overline{AB} + \overline{AD})$ (2) $4\overline{AC}$ (3) $4\overline{BD}$ (4) 0

15. If \vec{a} , \vec{b} and \vec{c} are unit vectors then $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$ does not exceed

- (1) 4 (2) 9 (3) 8 (4) 6

16. The value of $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1}}{2x+1}$ is

- (1) 1 (2) 0 (3) -1 (4) $\frac{1}{2}$

17. Let f be a continuous function on $[2,5]$. If f takes only rational values for all x and $f(3) = 12$, then $f(4.5)$ is equal to

- (1) $\frac{f(3)+f(4.5)}{7.5}$ (2) 12 (3) 17.5 (4) $\frac{f(4.5)-f(3)}{1.5}$

18. If $f(x) = x \tan^{-1}x$, then $f'(1)$ is

- (1) $1 + \frac{\pi}{4}$ (2) $\frac{1}{2} + \frac{\pi}{4}$ (3) $\frac{1}{2} - \frac{\pi}{4}$ (4) 2

19. The value of $\log_a a^a \log_b b^b \log_c c^c$ is

- (1) 1 (2) $a^3 b^3 c^3$ (3) abc (4) $3a + 3b + 3c$

20. If $f(x) = x + 2$, then $f'(f(x))$ at $x = 4$ is

- (1) 8 (2) 1 (3) 4 (4) 5

PART-II

Answer any seven questions: (Q.No. 30 is compulsory)

7*2=14

21) If $P(A)$ denotes the power set of A , then find $n(P(P(P(\phi))))$.

22) Evaluate: $\left(\left((256)^{\frac{-1}{2}} \right)^{\frac{-1}{4}} \right)^3$.

23) Find the principal solution and general solutions for $\tan \theta = -\frac{1}{\sqrt{3}}$

24) How many ways a committee of six persons from 10 persons can be chosen along with a Chair person and a Secretary?

25) Find the distance between the parallel lines $12x + 5y = 7$ and $12x + 5y + 7 = 0$.

- 26) Determine the value of a so that the matrix $A = \begin{bmatrix} 7 & 3 \\ -2 & a \end{bmatrix}$ is singular.
- 27) Find a unit vector parallel along the vector $5\hat{i} - 3\hat{j} + 4\hat{k}$.
- 28) Find the positive integer n so that $\lim_{x \rightarrow 3} \frac{x^n - 3^n}{x - 3} = 27$.
- 29) Find $\frac{dy}{dx}$ if $x = a(t - \sin t)$, $y = a(1 - \cos t)$.
- 30) Find the middle terms in the expansion of $(x + y)^4$

PART-III

Answer any seven questions : (Q.No. 40 is compulsory)

7*3=21

- 31) Find the largest possible domain of the real valued function $f(x) = \frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$.
- 32) Solve : $\log_{5-x}(x^2 - 6x + 65) = 2$.
- 33) Prove that $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$.
- 34) If $nP_r = 11880$ and $nC_r = 495$, find n and r .
- 35) Write the first 6 terms of the sequences whose n^{th} term $a_n = \begin{cases} n & \text{if } n \text{ is } 1, 2 \text{ or } 3 \\ a_{n-1} + a_{n-2} + a_{n-3} & \text{if } n > 3 \end{cases}$
- 36) Find the equations of the straight lines, making the y -intercept of 7 and angle between the line and the y -axis is 30°
- 37) Show that $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix} = 0$
- 38) Find the angle between the vectors $2\hat{i} + 3\hat{j} - 6\hat{k}$ and $6\hat{i} - 3\hat{j} + 2\hat{k}$.
- 39) Show that $\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{3n^2+7n+2} = \frac{1}{6}$.
- 40) Find y' if $y = \frac{\cos 3x}{x^2}$.

PART-IV

Answer all:

7*5=35

- 41) a) On the set of natural numbers let R be the relation defined by aRb if $2a + 3b = 30$. Write down the relation by listing all the pairs. Check whether it is (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence. (OR)
- b) If a, b, c are respectively the $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms of a GP, show that $(q - r) \log a + (r - p) \log b + (p - q) \log c = 0$.

II-MAT EM - 3

42) a) Solve : $\frac{x+1}{x+3} < 3$ (OR)

b) If $A + B + C = 180^\circ$, prove that $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$

43) a) By the principle of mathematical induction, prove that, for $n \geq 1$,

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2} \right)^2 \quad (\text{OR})$$

b) Show that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$

44) a) Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines. Find the distance between them. (OR)

b) Show that the points whose position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ are coplanar.

45) a) Evaluate $\lim_{x \rightarrow \infty} x \left[3^{\frac{1}{x}} + 1 - \cos\left(\frac{1}{x}\right) - e^{\frac{1}{x}} \right]$. (OR)

b) Show that the medians of the triangle are concurrent.

46) a) If $y = (\cos^{-1} x)^2$, prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$. Hence find y_2 when $x = 0$. (OR)

b) Solve : $\sqrt{3} \sin \theta - \cos \theta = \sqrt{2}$

47) a) A committee of 5 peoples has to be formed from 4 men and 7 women. In how many ways can this be done when the committee consists of

(i) exactly 2 men? (ii) atleast 2 men? (iii) utmost 2 men?. (OR)

b) Find the derivative of (i) $y = e^{-2x} \sin x$ (ii) $y = x^{x^2}$

11-MAT EM -4