COMMON HALF YEARLY EXAMINATION - 2024

Standard XI

Reg.No. | | | | | | |

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MATHEMATICS

Time : 3.00 hrs **Part - A** *Marks : 90*

20 x 1 = 20

I. Choose the correct answer:

1. The number of constant functions from a set containing m elements to a set containing n element is
   a) mn b) m c) n d) m + n
2. The value of $\log_a b \log_b c \log_c a$ is _____.
   a) 2 b) 1 c) 3 d) 4
3. $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \ldots\ldots\ldots + \cos 179^\circ =$
   a) 0 b) 1 c) –1 d) 89
4. The function f : R → R is defined by f(x) = sin x + cos x is
   a) an odd function
   b) neither an odd function nor an even function
   c) an even function
   d) both odd function and even function
5. Which of the following is not true?
   a) $\sin\theta = \frac{-3}{4}$ b) $\cos\theta = -1$ c) $\tan\theta = 25$ d) $\sec\theta = \frac{1}{4}$
6. The product of r consecutive positive integers is divisible by
   a) r ! b) (r – 1) ! c) (r + 1) ! d) r^r
7. 1 + 3 + 5 + 7 + ... + 17 is equal to
   a) 101 b) 81 c) 71 d) 61
8. The coefficient of x^5 in the series e^{-2x} is
   a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $\frac{-4}{15}$ d) $\frac{4}{15}$
9. If the point (8, –5) lies on the locus $\frac{x^2}{16} - \frac{y^2}{25} = k$, then the value of k is
   a) 0 b) 1 c) 2 d) 3
10. If the two straight lines x + (2k – 7)y + 3 = 0 and 3kx + 9y – 5 = 0 are perpendicular, then the value of k is
    a) k = 3 b) $k = \frac{1}{3}$ c) $k = \frac{2}{3}$ d) $k = \frac{3}{2}$
11. The value of x, for which the matrix $A = \begin{bmatrix} e^{x-2} & e^{7+x} \\ e^{2+x} & e^{2x+3} \end{bmatrix}$ is singular
    a) 9 b) 8 c) 7 d) 6
12. If $A = \begin{vmatrix} -1 & 2 & 4 \\ 3 & 1 & 0 \\ -2 & 4 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} -2 & 4 & 2 \\ 6 & 2 & 0 \\ -2 & 4 & 8 \end{vmatrix}$, then B is given by
    a) B = 4A b) B = –4A c) B = –A d) B = 6A

13. If $\lambda\hat{i}+2\lambda\hat{j}+2\lambda\hat{k}$ is a unit vector then the value of λ is

a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{9}$ d) $\frac{1}{2}$

14. If $\bar{a},\bar{b},\bar{c}$ are the position vectors of three collinear points then which of the following is true?

a) $\bar{a}=\bar{b}+\bar{c}$ b) $2\bar{a}=\bar{b}+\bar{c}$ c) $\bar{b}=\bar{c}+\bar{a}$ d) $4\bar{a}+\bar{b}+\bar{c}=\bar{0}$

15. The value of $\lim_{x\to 0}\frac{\sin x}{\sqrt{x^2}}$ is

a) 1 b) –1 c) 0 d) limit does not exist

16. If $f(x) = x + 2$ then $f'(f(x))$ at $x = 4$ is

a) 8 b) 1 c) 4 d) 5

17. If $\int f(x)dx = g(x)+c$ then $\int f(x)\,g'(x)dx$

a) $\int (f(x))^2\,dx$ b) $\int f(x)g(x)dx$ c) $\int f'(x)g(x)dx$ d) $\int (g(x))^2\,dx$

18. $\int e^{-4x}\cos x\,dx$ is

a) $\frac{e^{-4x}}{17}(4\cos x-\sin x)+c$ b) $\frac{e^{-4x}}{17}(-4\cos x+\sin x)+c$

c) $\frac{e^{-4x}}{17}(4\cos x+\sin x)+c$ d) $\frac{e^{-4x}}{17}(-4\cos x-\sin x)+c$

19. A number is selected from the set {1,2,3,....., 20}. The probability that the selected number is divisible by 3 or 4 is

a) $\frac{2}{5}$ b) $\frac{1}{8}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

20. If two events A and B are independent such that P(A) = 0.35 and $P(A\cup B) = 0.6$ then P(B) is

a) $\frac{5}{13}$ b) $\frac{1}{13}$ c) $\frac{4}{13}$ d) $\frac{7}{13}$

Part - B

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. If a and b are the roots of the equation $x^2 - px + q = 0$, find the value of $\frac{1}{a}+\frac{1}{b}$

22. If A = {1,2,3,4} and B = {3,4,5,6}, find $n[(A\cup B)\times(A\cap B)\times(A\,\Delta\,B)]$

23. In a circle of diameter 40 cm, a chord is of length 20 cm. Find the length of the mirror arc of the chord.

24. Evaluate : $\frac{n!}{r!\,(n-r)!}$ when n = 6, r = 2

25. Find the equation of the line passing through the points (1,1) and (–2,3)

26. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\overrightarrow{O}$

27. Compute : $\lim_{x \to -2} (x^3 - 3x + 6)(-x^2 + 15)$

28. Evaluate : $\int (4x + 5)^6 \, dx$

29. If two coins are tossed simultaneously, then find the probability of getting one head and one tail.

30. Differentiate w.r. to 'x' if $y = x^3 + e^x + \sin x + \log x$

Part - C

III. **Answer any 7 questions. (Q.No.40 is compulsory)** 7 x 3 = 21

31. Find the range of the function $\frac{1}{2\cos x - 1}$

32. To secure A grade one must obtain an average of 90 marks or more in 5 subjects each of maximum 100 marks. If one scored 84, 87, 95, 91 in first four subjects, what is the minimum mark one scored in the fifth subject to get A grade in the course?

33. Prove that $\sin(45^\circ + \theta) - \sin(45^\circ - \theta) = \sqrt{2}\sin\theta$

34. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, then prove that $xyz = 1$

35. In an examination a student has to answer 5 questions. Out of 9 questions in which 2 are compulsory. In how many ways a student can answer the questions?

36. Find the equation of the line passing through the point of intersection lines $4x - y + 3 = 0$ and $5x + 2y + 7 = 0$, through the point $(-1, 2)$

37. Show that $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix}$

38. Evaluate : $\lim_{x \to 5} \frac{\sqrt{x+4} - 3}{x - 5}$

39. Find y', y'' and y''' if $y = x^3 - 6x^2 - 5x + 3$

40. If $f'(x) = 3x + 5$ and $f(2) = 20$, find $f(x)$

Part - D

IV. **Answer all the questions.** 7 x 5 = 35

41. a) If A and B are two sets so that $n(B - A) = 2n(A - B) = 4n(A \cap B)$ and if $n(A \cup B) = 14$ then find $n[P(A)]$.

(OR)

b) If ABCD is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that $\overline{AB} + \overline{AD} + \overline{CB} + \overline{CD} = 4\,\overline{EF}$

42. a) Resolve into partial fractions : $\frac{x+1}{x^2(x-1)}$

(OR)

b) Prove that $\sqrt[3]{x^3 + 7} - \sqrt[3]{x^3 + 4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

43. a) By the principle of mathematical induction, prove that for all integers $n \geq 1$,

$$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(OR)

b) If $y = e^{\tan^{-1}x}$, show that $(1 + x^2)y'' + (2x - 1)y' = 0$

44. a) Find the values of other five trigonometric functions for $\cos\theta = -\frac{1}{2}$, θ lies in the III quadrant.

(OR)

b) Show that $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x-a)^2\,(x+2a)$ by using factor theorem.

45. a) Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines. Find the distance between them.

(OR)

b) State and prove a Napier's formula.

46. a) A function f is defined as follows : $f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } 0 \leq x < 1 \\ -x^2 + 4x - 2 & \text{for } 1 \leq x < 3 \\ 4 - x & \text{for } x \geq 3 \end{cases}$

is the function continuous?

(OR)

b) Find all the values of x for which $\frac{x^3(x-1)}{x-2} > 0$

47. a) Evaluate : $\int \frac{3x+5}{x^2+4x+7}\,dx$

(OR)

b) A factory has two machines-I and II. Machine-I produces 60% of items and machine-II produces 40% of the items of the total output. Further 2% of the items produced by machine-I are defective where as 4% produced by a machine -II are defective. If an item is drawn at random, what is the probability that it is defective?

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