T

COMMON HALF YEARLY EXAMINATION - 2024

Standard - XI

Reg.No.

Time: 3.00 hrs. **MATHEMATICS** Marks:90

PART - I

i) All questions are compulsory.

ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer: 20×1=20

1. If A = {(x, y) : y = sinx, x ∈R} and B = {(x, y): y = cosx, x ∈R} then A∩B contains
   1) no element 2) infinitely many elements
   3) only one element 4) cannot be determined
2. Let A and B be subsets of the universal set N, the set of natural numbers. Then A'∪[(A∩B)∪B'] is
   1) A 2) A' 3) B 4) N
3. The number of solutions of $x^2 + |x - 1| = 1$ is
   1) 1 2) 0 3) 2 4) 3
4. The solution set of the following inequality $|x - 1| \geq |x - 3|$ is
   1) $[0, 2]$ 2) $[2, \infty]$ 3) $(0, 2)$ 4) $(-\infty, 2)$
5. One radian is equal to (interms of degree)
   1) $\frac{180°}{11}$ 2) $\frac{\pi}{180°}$ 3) $\frac{180°}{\pi}$ 4) $\frac{11}{180°}$
6. In a triangle ABC, $\sin^2A + \sin^2B + \sin^2C = 2$, then the triangle is
   1) equilateral triangle 2) isosceles triangle 3) right triangle 4) scalene triangle
7. In 3 fingers, the number of ways four rings can be worn is ______ ways.
   1) $4^3 - 1$ 2) 3^4 3) 68 4) 64
8. 1 + 3 + 5 + 7 + ... + 17 is equal to
   1) 101 2) 81 3) 71 4) 61
9. $e^{\log x}$ is equal to
   1) x 2) 1 3) e 4) $\log e^x$
10. Which of the following equation is the locus of $(at^2, 2at)$
    1) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 2) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 3) $x^2 + y^2 = a^2$ 4) $y^2 = 4ax$
11. The value of the determinant of $A = \begin{bmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{bmatrix}$ is
    1) -2abc 2) abc 3) 0 4) $a^2 + b^2 + c^2$
12. The value of $\begin{vmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \\ 3x & 3y & 3z \end{vmatrix}$ is
    1) 1 2) xyz 3) x + y + z 4) 0
13. The vectors $\vec{a} - \vec{b}$, $\vec{b} - \vec{c}$, $\vec{c} - \vec{a}$ are
    1) parallel to each other 2) unit vectors
    3) mutually perpendicular vectors 4) coplanar vectors

14. $\lim_{x \to 0} \frac{e^{\sin x} - 1}{x} =$

1) 1 2) e 3) 1/e 4) 0

15. The function $f(x) = \frac{x^2 - 1}{x^3 + 1}$ is not defined for x = -1. The value of f(-1) so that the function extended by this value is continuous is

1) $\frac{2}{3}$ 2) $-\frac{2}{3}$ 3) 1 4) 0

16. $x = \frac{1 - t^2}{1 + t^2}$, $y = \frac{2t}{1 + t^2}$ then $\frac{dy}{dx}$ is

1) $-\frac{y}{x}$ 2) $\frac{y}{x}$ 3) $-\frac{x}{y}$ 4) $\frac{x}{y}$

17. If pv = 81, then $\frac{dp}{dv}$ at v = 9 is

1) 1 2) -1 3) 2 4) -2

18. $\int \frac{1}{e^x} dx =$

1) $\log e^x + c$ 2) $-\frac{1}{e^x} + c$ 3) $\frac{1}{e^x} + c$ 4) $x + c$

19. If $\int f(x)\,dx = g(x) + c$, then $\int f(x) g'[x]\,dx$

1) $\int (f(x)^2)\,dx$ 2) $\int f(x)\,g(x)\,dx$ 3) $\int f'(x)\,g(x)\,dx$ 4) $\int (g(x))^2\,dx$

20. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

1) $P(A/B) = \frac{P(A)}{P(B)}$ 2) $P(A/B) < P(A)$ 3) $P(A/B) \geq P(A)$ 4) $P(A/B) > P(B)$

PART - II

Answer any seven questions. Question No.30 is compulsory: 7x2=14

21. Let f and g be the two functions from R to R defined by f(x) = x and g(x) = x^2. Find g o f and f o g.

22. Find the positive number smaller than $\frac{1}{2^{1000}}$ Justify.

23. An Engineer has to develop a triangular shaped park with a perimeter 120m in a village. The park to be developed must be of maximum area. Find out the dimensions of the park.

24. If n! + (n - 1)! = 30, then find the value of n.

25. Find the slope of the straight line passing through the points (5, 7) and (7, 5). Also find the angle of inclination of the line with the x axis.

26. Prove that $\begin{vmatrix} \sec^2\theta & \tan^2\theta & 1 \\ \tan^2\theta & \sec^2\theta & -1 \\ 38 & 36 & 2 \end{vmatrix} = 0$

27. Find a unit vector along the direction of the vector $5\hat{i} - 3\hat{j} + 4\hat{k}$

28. Compute $\lim_{\sqrt{x} \to 3} \frac{x^2 - 81}{\sqrt{x} - 3}$

29. Differentiate $y = \sqrt{x + \sqrt{x}}$

30. Find the probability of getting the number 7, when a usual die is rolled.

PART - III

Answer any seven questions. Question No.40 is compulsory. 7×3=21

31. Find the range of the function $f(x) = \frac{1}{1 - 3\cos x}$

32. Solve $(2x + 1)^2 - (3x + 2)^2 = 0$

33. Show that $\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9} = 2$

34. Prove that ${}^{2n}C_n = \frac{2^n \times 1 \times 3 \times \ldots (2n-1)}{n!}$

35. Find the coefficient of x^3 in the expansion of $(2 - 3x)^7$.

36. Find the value of x if $[2x \quad 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix} = 0$

37. Find the angle between the vectors $2\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ using vector product.

38. An integer is chosen at random from the first 100 positive integers. What is the probability that the integer chosen is a prime or multiple of 8?

39. If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$, find $\frac{dy}{dx}$.

40. Evaluate : $\int \frac{\sin x}{1 + \sin x} dx$

$(a+b)^2 = a^2$

PART - IV

Answer all the questions: 7×5=35

41. a) Prove that $\log\left(\frac{75}{16}\right) - 2\log\left(\frac{5}{9}\right) + \log\left(\frac{32}{243}\right) = \log 2$

(OR)

b) If $\theta + \phi = \alpha$ and $\tan\theta = k\tan\phi$, then prove that $\sin(\theta - \phi) = \frac{k-1}{k+1}\sin\alpha$

42. a) Find the number of strings of 4 letters that can be formed with the letters of the word EXAMINATION?

(OR)

b) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.

43. a) Show that the straight line joining the oring to the points of intersection of $3x - 2y + 2 = 0$ and $3x^2 + 5xy - 2y^2 + 4x + 5y = 0$ are at right angles.

(OR)

b) In the set Z of integers, define ${}_mR_n$ if m - n is a multiple of 12. Prove that R is an equivalence relation.

44. a) Prove that $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \geq 1$.

(OR)

b) Express the matrix $A = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$ as the sum of a symmetric and a skew - symmetric matrices.

45. a) Prove that the medians of a triangle are concurrent.

(OR)

b) Evaluate $\lim_{x \to 0} \frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\tan x}$

46. a) If $\sin y = x\sin(a+y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$, $a \neq n\pi$

(OR)

b) Evaluate : $\int \frac{3x+5}{x^2+4x+7}dx$

47. a) There are two identical urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn?

(OR)

b) Prove by factor method $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(ab + bc + ca)$
