

Answer Key - 2024

1 marks

- 1) a) 4
- 2) a) real and distinct
- 3) b) not a function
- 4) b) 18
- 5) c) 0
- 6) b) 6
- 7) d) 4
- 8) d) (-3, -2)
- 9) b) $\frac{e-1}{2e}$
- 10) d) 3
- 11) c) \vec{a} and \vec{b} are perpendicular
- 12) d) $(a^2-1)^2$
- 13) d) $\frac{2\hat{i}-\hat{j}}{\sqrt{5}}$
- 14) d) $\frac{1}{3}$
- 15) a) 4
- 16) a) $(a-z)^2$
- 17) a) 1
- 18) a) $\sqrt{1-x^2} + \sin^{-1}x + C$
- 19) a) $\log \sec x + C$
- 20) d) $\frac{7}{128}$

2 marks

- 21) $A = \{1, 2, 3, 4\}$
 $B = \{3, 4, 5, 6\}$
 $n(A \cup B) = 6, n(A \cap B) = 2$
 $n(A \Delta B) = 4$
 $= 6 \times 2 \times 4 = 48$
- 22) $f(x) = 4x^2 - 25$
 $4x^2 - 25 = 0$
 $x^2 = \frac{25}{4} \Rightarrow x = \pm \frac{5}{2}$
- 23) $\tan(45^\circ + A)$
 $= \frac{\tan 45^\circ + \tan A}{1 - \tan 45^\circ \tan A}$
 $= \frac{1 + \tan A}{1 - \tan A}$
- 24) BANANA
 $= \frac{6!}{3! \times 2!} = 60$
- 25) (1, 1) and (-2, 3)
 $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$$\frac{y-1}{3-1} = \frac{x-1}{-2-1}$$

$$2x + 3y = 5$$

$$26) \vec{a} = \hat{i} - \hat{j} + 5\hat{k}$$

$$\vec{b} = 3\hat{i} - 2\hat{k}$$

$$\vec{a} \cdot \vec{b} = 3 - 10 = -7$$

$$27) y = e^{\sin x}$$

$$y' = \cos x \cdot e^{\sin x}$$

$$28) P(A) = \frac{3}{8}, P(B) = \frac{1}{8}$$

$$P(\bar{A}) = 1 - P(A) = \frac{5}{8}$$

$$P(A \cup B) = P(A) + P(B) = \frac{3}{8} + \frac{1}{8} = \frac{1}{2}$$

$$29) \lim_{x \rightarrow 3} (x^3 - 2x + 6)$$

$$27 - 6 + 6 = 27$$

$$30) \int (6x^2 + 8) dx$$

$$6\left(\frac{x^3}{3}\right) + 8x + C$$

$$2x^3 + 8x + C$$

3 marks

$$31) f(x) = \frac{1}{1-3\cos x}$$

$$-1 \leq \cos x \leq 1$$

$$3 \geq -3\cos x \geq -3$$

$$-3 \leq -3\cos x \leq 3$$

$$1-3 \leq -3\cos x \leq 1+3$$

$$-2 \leq 1-3\cos x \leq 4$$

$$\frac{1}{1-3\cos x} \leq -\frac{1}{2}, \frac{1}{1-3\cos x} \geq \frac{1}{4}$$

$$\text{Range is } (-\infty, -\frac{1}{2}] \cup [\frac{1}{4}, \infty)$$

$$32) \frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$$

$$\frac{\log x}{y-z} = k \Rightarrow \log x = k(y-z)$$

$$x = e^{k(y-z)}$$

$$y = e^{k(z-x)}$$

$$z = e^{k(x-y)}$$

$$xyz = e^{k(y-z+z-x+x-y)} = e^{k(0)} = 1$$

$$33) \frac{1}{8!} + \frac{1}{a!} = \frac{n}{10!}$$

$$\frac{1}{8!} \left(1 + \frac{1}{a}\right) = \frac{n}{10 \times 9 \times 8!}$$

$$n = 100$$

$$34) a_1 = 1, a_2 = 2$$

$$a_3 = 3, a_4 = 5,$$

$$a_5 = 8, a_6 = 13$$

$$35) (-2, -3), (3, 2), (-1, -8)$$

$$\text{Area of } \Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$= \frac{1}{2} (-20 + 12 - 22) = -15$$

$$= 15 \text{ sq. units}$$

$$36) 3\hat{i} + 4\hat{j} - 6\hat{k}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r = \sqrt{61}$$

$$\text{Dir. cosines } \frac{3}{\sqrt{61}}, \frac{4}{\sqrt{61}}, \frac{-6}{\sqrt{61}}$$

$$\text{Dir. ratios } 3, 4, -6$$

$$37) \lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1}$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)} \Rightarrow \lim_{x \rightarrow 1} \frac{1}{\sqrt{x}+1}$$

$$\Rightarrow \frac{1}{2}$$

$$38) f''(x) = 12x - 6$$

$$f(1) = 30, f'(1) = 5$$

$$\int f''(x) = f'(x) = 12\frac{x^2}{2} - 6x + C$$

$$f'(x) = 6x^2 - 6x + C$$

$$5 = 6(1)^2 - 6(1) + C$$

$$\Rightarrow C = 5$$

$$\int f'(x) = f(x) = \frac{6x^3}{3} - \frac{6x^2}{2} + 5x + C_2$$

$$f(x) = 2x^3 - 3x^2 + 5x + C_2$$

$$30 = 2(1)^3 - 3(1)^2 + 5(1) + C_2$$

$$C_2 = 26$$

$$f(x) = 2x^3 - 3x^2 + 5x + 26$$

$$39) \cos \theta = \frac{2}{3}$$

$$\sin \theta = \frac{\sqrt{5}}{3}, \csc \theta = \frac{3}{\sqrt{5}}$$

$$\sec \theta = \frac{3}{2}, \tan \theta = \frac{\sqrt{5}}{2}$$

$$\cot \theta = \frac{2}{\sqrt{5}}$$

$$40) x = at^2, y = 2at$$

$$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$$

41) a) $f(x) = |x| + x$
 $g(x) = |x| - x$
 $f(x) = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$
 $g(x) = \begin{cases} -2x & , x < 0 \\ 0 & , x \geq 0 \end{cases}$
 $g \circ f = g(f(x)) = \begin{cases} 0, & x < 0 \\ 0, & x \geq 0 \end{cases}$
 $f \circ g = f(g(x)) = \begin{cases} 0, & x < 0 \\ 0, & x \geq 0 \end{cases}$
 $g \circ f = f \circ g = 0$

b) $2x^2 - xy - 3y^2 - 6x + 19y - 2 = 0$
 $a=2, b=-3, h=-1/2, g=-3, f=19/2, c=-2$
 $bg^2 \neq af^2$
 $-27 \neq 38$ (soils intersecting)
 $\tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$
 $\theta = \tan^{-1} \left| \frac{2\sqrt{1/4 + 6}}{-1} \right|$
 $\theta = \tan^{-1}(5)$

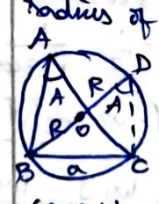
42) a) $\frac{x}{(x^2+1)(x-1)(x+2)}$
 $\frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{x+2}$
 if $x=1 \Rightarrow C = 1/6$
 if $x=-2 \Rightarrow D = 2/15$
 if $x=0 \Rightarrow B = 1/10$
 $\Rightarrow A = -3/10$
 $-\frac{3x+1}{10(x^2+1)} + \frac{1}{6(x-1)} + \frac{2}{15(x+2)}$

b) $5\hat{i} + 6\hat{j} + 7\hat{k}$
 $7\hat{i} - 8\hat{j} + 9\hat{k}$
 $3\hat{i} + 2\hat{o}\hat{j} + 5\hat{k}$
 $5\hat{i} + 6\hat{j} + 7\hat{k} = s(7\hat{i} - 8\hat{j} + 9\hat{k}) + t(3\hat{i} + 2\hat{o}\hat{j} + 5\hat{k})$
 $7s + 3t = 5$
 $-8s + 20t = 6$
 $9s + 5t = 7$
 $s = t = 1/2$, satisfies linear combination, vectors are coplanar.

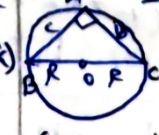
43) a) $1.2 + 2.3 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$
 $P(1) = 2 = 2$
 $P(n)$ is true
 $P(k) = 1.2 + 2.3 + \dots + k(k+1) = \frac{k(k+1)(k+2)}{3}$
 $P(k+1) = 1.2 + 2.3 + \dots + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$
 $\frac{1 \cdot k(k+1)(k+2) + (k+1)(k+2)}{3} = \frac{(k+1)(k+2)(k+3)}{3}$ //

b) $\int \frac{3x+5}{x^2+4x+7} dx$
 $3x+5 = A \frac{d}{dx}(x^2+4x+7) + B$
 $3x+5 = A(2x+4) + B$
 $A = 3/2, B = -1$
 $= \frac{3}{2} \int \frac{2x+4}{x^2+4x+7} dx - \int \frac{dx}{x^2+4x+7}$
 $= \frac{3}{2} \log|x^2+4x+7| - \int \frac{dx}{(x+2)^2+3}$
 $= \frac{3}{2} \log|x^2+4x+7| - \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x+2}{\sqrt{3}} \right) + C$

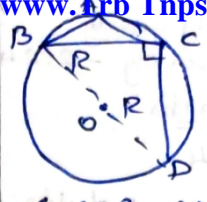
44) a) In any triangle, the lengths of the sides are proportional to the sines of opp. angles.
 In $\triangle ABC, \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$, R is circumradius of $\triangle ABC$.



Case 1) $\angle A$ is acute
 $\angle BDC = \angle BAC = A$
 $\angle BCD = 90^\circ$
 $\sin \angle BDC = \frac{BC}{BD} (\cos)$
 $\sin A = \frac{a}{2R} \Rightarrow \frac{a}{2R} = \frac{a}{\sin A} = 2R$



Case 2) $\angle A$ is right angle
 $\frac{a}{\sin A} = \frac{BC}{\sin 90^\circ}$
 $\frac{a}{\sin A} = 2R$

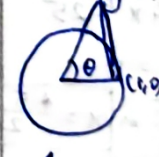


Case: 3 $\angle A$ is obtuse
 $\angle BDC + \angle BAC = 180^\circ$
 $\angle BDC = 180^\circ - A$
 $\angle BCD = 90^\circ$

$\sin \angle BDC = \frac{BC}{BD}$
 $\sin(180^\circ - A) = \frac{a}{2R}$
 $\frac{\sin A}{a} = \frac{1}{2R}$
 $\frac{a}{\sin A} = 2R$
 Similarly,
 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

b) $\int \frac{1-x}{1+x} x dx$ by $(1-x)$
 $\frac{(1-x)(1-x)}{\sqrt{(1+x)(1-x)}}$
 $\frac{1-x}{\sqrt{1-x^2}}$
 $(1-x)(1-x^2)^{-1/2}$
 $(1-x) \left[1 + \frac{1}{2}x^2 + \frac{(1/2)(1/2+1)}{2!}x^4 + \dots \right]$
 $\Rightarrow (1-x + \frac{x^2}{2}) //$

45) a) $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$
 Taking a unit circle,



$\triangle \rightarrow \text{Area} = \frac{\tan \theta}{2}$
 $\triangle \rightarrow \text{Area} = \frac{\theta}{2}$
 $\triangle \rightarrow \text{Area} = \frac{\sin \theta}{2}$

By area property,
 $\frac{\tan \theta}{2} \geq \frac{\theta}{2} \geq \frac{\sin \theta}{2}$
 $\times \text{ by } \frac{2}{\sin \theta}$
 $\frac{1}{\cos \theta} \geq \frac{\theta}{\sin \theta} \geq 1$
 $\cos \theta \leq \frac{\sin \theta}{\theta} \leq 1$

$\lim_{\theta \rightarrow 0} \frac{\cos(-\theta) - \cos \theta}{\sin(-\theta) - \sin \theta} = \frac{\cos \theta - \cos \theta}{-\sin \theta - \sin \theta} = \frac{0}{-2 \sin \theta} = 0$

Valid for θ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Taking $\lim_{\theta \rightarrow 0}$

$\lim_{\theta \rightarrow 0} \cos \theta \leq \lim_{\theta \rightarrow 0} \sin \theta \leq \lim_{\theta \rightarrow 0} 1$

$1 \leq \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \leq 1$

$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$

b) $n(s) = 36$

(i) p(getting sum 7)
 $P(A) = \frac{6}{36} = \frac{1}{6}$

(ii) p(getting sum 7 or 9)
 $P(A \cup B) = P(A) + P(B)$
 $= \frac{6}{36} + \frac{4}{36} = \frac{10}{36} = \frac{5}{18}$

(iii) p(getting sum 7 or 12)
 $= P(A \cup C) = P(A \cup C)$
 $= P(A) + P(C)$
 $= \frac{6}{36} + \frac{1}{36} = \frac{7}{36}$

46) a) $A^T = \begin{bmatrix} 4 & 5 \\ -1 & 0 \\ 2 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 2 & -1 & 1 \\ 7 & 5 & -2 \end{bmatrix}$

$A = \begin{bmatrix} 4 & -1 & 2 \\ 5 & 0 & 3 \end{bmatrix}$

(i) $(A+B)^T = A^T + B^T$

$A+B = \begin{bmatrix} 6 & -2 & 3 \\ 12 & 5 & 1 \end{bmatrix}$

$(A+B)^T = \begin{bmatrix} 6 & 12 \\ -2 & 5 \\ 3 & 1 \end{bmatrix}$

$B^T = \begin{bmatrix} 2 & 7 \\ -1 & 5 \\ 1 & -2 \end{bmatrix}$

$A^T + B^T = \begin{bmatrix} 6 & 12 \\ -2 & 5 \\ 3 & 1 \end{bmatrix}$

(ii) $(B^T)^T = B$

$(B^T)^T = \begin{bmatrix} 2 & -1 & 1 \\ 7 & 5 & -2 \end{bmatrix} = B$

b) $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$

$y_1 = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$
 $\frac{\sin^{-1} x}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$

$y_1 = 1 + \frac{x \sin^{-1} x}{\sqrt{1-x^2}}$

$(1-x^2)y_1 = 1 + xy$

$(1-x^2)y_2 + y_1(1-2x) = xy_1 + y_1$

$(1-x^2)y_2 - 3xy_1 - y = 0$

AT) a) $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = 0$

$(x-y)(y-z)(z-x)$
 $(xy+yz+zx)$

Sub $x=y$
 $|A| = \begin{vmatrix} 1 & y^2 & y^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = 0$

$(x-y)$ is a factor,
 sub $y=z$
 $|A| = \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & y^2 & y^3 \end{vmatrix} = 0$

$(y-z)$ is a factor,
 imply, $(z-x)$ is a factor.

$m = 5-3 = 2$
 $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = 0$

$k(x^2+y^2+z^2) + l(xy+yz+zx) + (x-y)(y-z)(z-x)$

put $x=0, y=1, z=2$
 $5k + 2l = 2 \rightarrow (1)$
 put $x=0, y=-1, z=1$

$2k - l = -1 \rightarrow (2)$
 solving (1) & (2) $k=0, l=1$
 $= (x-y)(y-z)(z-x)(xy+yz+zx)$

b) $P(A_1) = \frac{5}{5+3+2} = \frac{5}{10}$

$P(A_2) = \frac{3}{10}$

$P(A_3) = \frac{2}{10}$

$P(B/A_1) = 0.4$

$P(B/A_2) = 0.5$

$P(B/A_3) = 0.3$

$P(A_2/B) = \frac{P(A_2) \cdot P(B/A_2)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + P(A_3)P(B/A_3)}$

$= \frac{\frac{3}{10} \times 0.5}{\frac{5}{10} \times 0.4 + \frac{3}{10} \times 0.5 + \frac{2}{10} \times 0.3} = \frac{15}{41}$

11 PRegister No.

Time : 3.00 Hrs.

Half Yearly Examination - 2024
MATHEMATICS
PART - I

Marks : 90

20 x 1 = 20**i) Answer all the questions.**

1. If $n((A \times B) \cap (A \times C)) = 8$ and $n(B \cap C) = 2$ then $n(A)$ is a) 4 b) 6 c) 8 d) 16
2. If the determinant is positive then the nature of roots are
a) real and distinct b) real and equal c) no real roots d) none of these
3. Let $x = \{1, 2, 3, 4\}$ $y = \{a, b, c, d\}$ $f = \{(1, a) (4, b) (2, c) (3, d) (2, d)\}$. Then f is
a) an one-to-one function b) not a function c) an onto function d) a function which is not one-to-one
4. The value of $\log_{\sqrt{2}} 512$ is a) 16 b) 18 c) 9 d) 12
5. $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \dots + \cos 179^\circ =$ a) 1 b) -1 c) 0 d) 89
6. ${}^{2n}C_3 : {}^nC_3 = 11 : 1$ then n is a) 5 b) 6 c) 11 d) 7
7. The HM of two positive numbers whose AM and GM are 16, 8 respectively is a) 10 b) 6 c) 5 d) 4
8. The image of the point (2, 3) in the line $y = -x$ is a) (-3, 2) b) (-2, -3) c) (-2, 3) d) (-3, -2)
9. The value of $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots$ is a) $\frac{e^2 + 1}{2e}$ b) $\frac{(e-1)^2}{2e}$ c) $\frac{e^2 + 1}{2e}$ d) $\frac{(e+1)^2}{2e}$
10. If the point (8, -5) lies on the locus $\frac{x^2}{16} - \frac{y^2}{25} = k$, then the value of k is a) 0 b) 1 c) 2 d) 3
11. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then
a) \vec{a} and \vec{b} are parallel b) \vec{a} and \vec{b} are unit vectors c) \vec{a} and \vec{b} are perpendicular d) None of these
12. If $A = \begin{bmatrix} a & x \\ y & a \end{bmatrix}$ and if $xy = 1$, then $\det(AA^T)$ is equal to a) $(a-1)^2$ b) $(a^2 + 1)^2$ c) $a^2 - 1$ d) $(a^2 - 1)^2$
13. The unit vector parallel to the resultant of the vectors $\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$ is
a) $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{5}}$ b) $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$ c) $\frac{2\hat{i} - \hat{j} + \hat{k}}{\sqrt{5}}$ d) $\frac{2\hat{i} - \hat{j}}{\sqrt{5}}$
14. If $\lambda \hat{i} + 2\lambda \hat{j} + 2\lambda \hat{k}$ is a unit vector, then the value of λ is a) $\frac{1}{4}$ b) $\frac{1}{9}$ c) $\frac{1}{2}$ d) $\frac{1}{3}$
15. $\lim_{x \rightarrow 0} \frac{\sin 4x}{x}$ a) 4 b) $\frac{1}{4}$ c) 0 d) 1
16. If $y = \frac{1}{a-z}$ then $\frac{dz}{dy}$ is a) $(a-z)^2$ b) $-(z-a)^2$ c) $(z+a)^2$ d) $-(z+a)^2$
17. $\lim_{\theta \rightarrow 0} \frac{\sin \sqrt{\theta}}{\sqrt{\sin \theta}}$ a) 1 b) -1 c) 0 d) 2
18. $\int \sqrt{\frac{1-x}{1+x}} dx$ is a) $\sqrt{1-x^2} + \sin^{-1}x + c$ b) $\sin^{-1}x - \sqrt{1-x^2} + c$ c) $\sqrt{1-x^2} + \log|x + \sqrt{1-x^2}| + c$
d) $\log|x + \sqrt{1-x^2}| - \sqrt{1-x^2} + c$
19. $\int \tan x dx$ a) $\log \sec x + c$ b) $\sec^2 x + c$ c) $\sec x \tan x + c$ d) $\log \sin x + c$
20. Ten coins are tossed the probability of getting atleast 8 heads is a) $\frac{7}{64}$ b) $\frac{7}{32}$ c) $\frac{7}{16}$ d) $\frac{7}{128}$

PART - II**7 x 2 = 14****Note : 1) Answer any seven 2) Q.No.30 is compulsory.**

21. If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$ find $n\{(A \cup B) \times (A \cap B) \times (A \Delta B)\}$
22. Find the zeros of the polynomial function $f(x) = 4x^2 - 25$.
23. Show that $\tan(45^\circ + A) = \frac{1 + \tan A}{1 - \tan A}$
24. Find the number of ways of arranging the letters of the word BANANA.
25. Find the equation of the line passing through the point (1, 1) and (-2, 3)
26. Find $\vec{a} \cdot \vec{b}$ when $\vec{a} = \hat{i} - \hat{j} + 5\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{k}$
27. Differentiate : $y = e^{\sin x}$

28. If A and B are mutually exclusive events $P(A) = \frac{3}{8}$ and $P(B) = \frac{1}{8}$ then find (i) $P(\bar{A})$ (ii) $P(A \cup B)$
29. Calculate $\lim_{x \rightarrow 3} (x^3 - 2x + 6)$
30. $\int (6x^2 + 8) dx$ - Integrate.

PART - III

Note : i) Answer any seven 2) Q.No.40 is compulsory.

7 x 3 = 21

31. Find the range of the function $f(x) = \frac{1}{1 - 3 \cos x}$
32. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$ then prove that $xyz = 1$.
33. Find the value of n if $\frac{1}{8!} + \frac{1}{9!} = \frac{n}{10!}$
34. Write the first 6 terms of the sequences whose n^{th} term $a_n = \begin{cases} 1 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ a_{n-1} + a_{n-2}, & \text{if } n > 2 \end{cases}$
35. Find the area of the triangle whose vertices are $(-2, -3)$, $(3, 2)$ and $(-1, -8)$
36. Find a direction ratio and direction cosines of $3\hat{i} + 4\hat{j} - 6\hat{k}$
37. Compute $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}$
38. If $f''(x) = 12x - 6$ and $f(1) = 30$, $f'(1) = 5$ find $f(x)$.
39. Find the values of other five trigonometric functions for $\cos \theta = \frac{2}{3}$, θ lies in the I quadrant.
40. Find $\frac{dy}{dx}$ if $x = at^2$, $y = 2at$, $t \neq 0$.

PART - IV

Answer all questions

7 x 5 = 35

41. a) If $f, g : \mathbb{R} \rightarrow \mathbb{R}$ are defined by $f(x) = |x| + x$ and $g(x) = |x| - x$. Find $f \circ g$ and $g \circ f$. (OR)
b) Show that the equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of intersecting lines. Show further that the angle between them is $\tan^{-1}(5)$.
42. a) Resolve into partial fractions : $\frac{x}{(x^2 + 1)(x - 1)(x + 2)}$ (OR)
b) Show that the vectors $5\hat{i} + 6\hat{j} + 7\hat{k}$, $7\hat{i} - 8\hat{j} + 9\hat{k}$, $3\hat{i} + 20\hat{j} + 5\hat{k}$ are coplanar.
43. a) Using the mathematical induction, prove that, for $n \geq 1$. $1.2 + 2.3 + 3.4 + \dots + n(n + 1) = \frac{n(n + 1)(n + 2)}{3}$ (OR)
b) Evaluate $\int \frac{3x + 5}{x^2 + 4x + 7} dx$
44. a) State and prove sine formula. (OR) b) Prove that $\sqrt{\frac{1-x}{1+x}}$ is approximately equal to $1 - x + \frac{x^2}{2}$ when x is very small.
45. a) Prove that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ (OR)
b) When a pair of fair dice is rolled, what are the probabilities of getting the sum i) 7 ii) 7 or 9 iii) 7 or 12?
46. a) If $A^T = \begin{bmatrix} 4 & 5 \\ -1 & 0 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 1 \\ 7 & 5 & -2 \end{bmatrix}$ verify (i) $(A + B)^T = A^T + B^T$ ii) $(B^T)^T = B$ (OR)
b) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1 - x^2)y_2 - 3xy_1 - y = 0$
47. a) Prove that $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x - y)(y - z)(z - x)(xy + yz + zx)$ (OR)
b) The chances of A, B and C becoming manager of a certain company are 5 : 3 : 2. The probabilities that the office canteen will be improved if A, B and C become managers are 0.4, 0.5 and 0.3 respectively. If the office canteen has been improved, what is the probability that B was appointed as the manager?