CHENGALPATTU - DISTRICT

# COMMON HALF YEARLY EXAMINATION - 2024

**Standard XI** Reg.No. [ ]

## MATHEMATICS

*Time : 3.00 hrs* **Part - A** *Marks : 90*

**20 x 1 = 20**

**I. Choose the correct answer:**

1. If n(A) = 2 and $n(B \cup C) = 3$, then $n[(A \times B) \cup (A \times C)]$ is
   a) $2^3$ b) $3^2$ c) 6 d) 5
2. If a and b are the roots of the equation $x^2 - kx + 16 = 0$ and satisfy $a^2 + b^2 = 32$, then the value of k is
   a) 10 b) –8 c) –8,8 d) 6
3. The maximum value of $4\sin^2 x + 3\cos^2 x + \sin\frac{x}{2} + \cos\frac{x}{2}$ is
   a) $4+\sqrt{2}$ b) $3+\sqrt{2}$ c) 9 d) 4
4. The product of r consecutive positive integers is divisible by
   a) r ! b) (r – 1)! c) (r + 1)! d) $r^r$
5. The sum of an infinite G.P is 18. If the first term is 6, the common ratio is
   a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{1}{6}$ d) $\frac{3}{4}$
6. The y-intercept of the straight line passing through (1, 3) and perpendicular to 2x – 3y + 1 = 0 is
   a) $\frac{3}{2}$ b) $\frac{9}{2}$ c) $\frac{2}{3}$ d) $\frac{2}{9}$
7. If the points (x, –2), (5, 2), (8, 8) are collinear, then x is equal to
   a) –3 b) $\frac{1}{3}$ c) 1 d) 3
8. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + x\hat{j} + \hat{k}$, $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$ and $\vec{a}.(\vec{b} \times \vec{c}) = 70$, then x is equal to
   a) 5 b) 7 c) 26 d) 10
9. The function $f(x) = \begin{cases} \frac{x^2-1}{x^3+1} & x \neq -1 \\ P & x = -1 \end{cases}$ is not defined for x = –1. The value of f(–1) so that the function extended by this value is continuous is a) $\frac{2}{3}$ b) $-\frac{2}{3}$ c) 1 d) 0
10. If x = a sinθ and y = b cosθ, then $\frac{d^2y}{dx^2}$ is
    a) $\frac{a}{b^2}\sec^2\theta$ b) $-\frac{b}{a}\sec^2\theta$ c) $-\frac{b}{a^2}\sec^3\theta$ d) $\frac{b^2}{a^2}\sec^3\theta$
11. $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$ is
    a) $\cot(xe^x) + c$ b) $\sec(xe^x) + c$ c) $\tan(xe^x) + c$ d) $\cos(xe^x) + c$
12. If X and Y be two events such that $P(X/Y) = \frac{1}{2}$, $P(Y/X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$, then $P(X \cup Y)$ is a) $\frac{1}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{6}$ d) $\frac{2}{3}$

13. $\int e^{-4x} \cos x \, dx$ is

a) $\frac{e^{-4x}}{17}[4\cos x - \sin x] + c$
b) $\frac{e^{-4x}}{17}[-4\cos x + \sin x] + c$
c) $\frac{e^{-4x}}{17}[4\cos x + \sin x] + c$
d) $\frac{e^{-4x}}{17}[-4\cos x - \sin x] + c$

14. The unit vector parallel to the resultant of the vectors $\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$ is

a) $\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{5}}$
b) $\frac{2\hat{i}+\hat{j}}{\sqrt{5}}$
c) $\frac{2\hat{i}-\hat{j}+\hat{k}}{\sqrt{5}}$
d) $\frac{2\hat{i}-\hat{j}}{\sqrt{5}}$

15. $\lim_{x \to 0} \frac{a^x - b^x}{x} =$

a) $\log ab$
b) $\log\left(\frac{a}{b}\right)$
c) $\log\left(\frac{b}{a}\right)$
d) $\frac{a}{b}$

16. If the square of the matrix $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is the unit matrix of order 2, then $\alpha$, $\beta$ and $\gamma$ should satisfy the relation

a) $1 + \alpha^2 + \beta\gamma = 0$
b) $1 - \alpha^2 - \beta\gamma = 0$
c) $1 - \alpha^2 + \beta\gamma = 0$
d) $1 + \alpha^2 - \beta\gamma = 0$

17. The intercepts of the perpendicular bisector of the line segment joining (1,2) and (3,4) with coordinate axes are

a) 5, –5
b) 5, 5
c) 5, 3
d) 5, –4

18. The middle term in the expression of $(x + 6)^6$ is

a) $10x^3y^3$
b) $10x^2y^2$
c) $20x^3y^3$
d) $20x^2y^3$

19. Value of $\cos\left(67\frac{1}{2}^{\circ}\right)$ is

a) $\sqrt{2-\sqrt{2}}$
b) $\frac{\sqrt{2-\sqrt{2}}}{2}$
c) $\frac{\sqrt{2+\sqrt{2}}}{2}$
d) $\sqrt{2+\sqrt{2}}$

20. $\lim_{x \to 0} x^2 \sin\frac{1}{x}$ is

a) $\infty$
b) $-\infty$
c) 0
d) Limit not exist

## Part - B

**II. Answer any 7 questions. (Q.No.30 is compulsory)** **7 x 2 = 14**

21. Two sets have m and k elements. If the total number of subsets of the first set is 112 more than that of the second set, find the values of m and k

22. Solve: $\frac{1}{5}[10x - 2] < 1$

23. Convert (i) $18^{\circ}$ to radians (ii) $\frac{\pi}{5}$ radian to degrees

24. Find the distinct permutations of the letters of the word MISSISSIPPI?

25. Write the first 6 terms of the sequences whose $n^{th}$ term $a_n$ is given below.

$$a_n = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even} \end{cases}$$

26. If $\lambda = -2$, determine the value of $\begin{vmatrix} 0 & 2\lambda & 1 \\ \lambda^2 & 0 & 3\lambda^2+1 \\ -1 & 6\lambda-1 & 0 \end{vmatrix}$

27. Show that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $3\hat{i}-4\hat{j}-4\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ form a right-angled triangle.

28. Differentiate $y = (x^2 + 4x + 6)^5$

29. Suppose a fair die is rolled, find the probability of getting
(i) an even number (ii) multiple of three

30. Find the equations of the straight lines, making y intercept of 7 and angle between the line and the y-axis is $30^o$.

## Part - C

**III. Answer any 7 questions. (Q.No.40 is compulsory)** **7 x 3 = 21**

31. For any vector $\vec{a}$ prove that $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2 = 2|\vec{a}|^2$

32. Find the range of function $f(x) = \frac{1}{1-3\cos x}$

33. Resolve the following rational expressions into partial fractions : $\frac{3x+1}{(x-2)(x+1)}$

34. Prove that $\tan\left(\frac{\pi}{4}+\theta\right) - \tan\left(\frac{\pi}{4}-\theta\right) = 2\tan 2\theta$

35. Prove that ${}^{2n}C_n = \frac{2^n \times 1\times 3\times \ldots\ldots \times(2n-1)}{n!}$

36. Prove that : $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}^2 = \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix}$

37. Draw the function $f'(x)$ if $f(x) = 2x^2 - 5x + 3$

38. Integrate : (i) $\frac{x^{24}}{x^{25}}$ ii) $(1 + x^2)^{-1}$ iii) $(1-x^2)^{-1/2}$

39. A problem in mathematics is given to three students whose changes of solving it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$. What is the probability that exactly one of them will solve it?

40. Evaluate the following limits : $\lim_{x\to\infty} x\left[3^{1/x} + 1 - \cos\left(\frac{1}{x}\right) - e^{1/x}\right]$

## Part - D

**IV. Answer all the questions.** 7 x 5 = 35

41. a) If R→R is defined by $f(x) = 3x - 5$, prove that f is a bijection and find its inverse.

*(OR)*

b) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

42. a) Prove that $\log_{10} 2 + 16\log_{10}\frac{16}{15} + 12\log_{10}\frac{25}{24} + 7\log_{10}\frac{81}{80} = 1$ *(OR)*

b) By the principle of mathematical induction, prove that, for all integers $n \geq 1$

$1^2 + 2^2 + 3^2 + ........ + n^2 = \frac{n(n+1)(2n+1)}{6}$

43. a) Prove that $\frac{\cot(180^\circ+\theta)\sin(90^\circ-\theta)\cos(-\theta)}{\sin(270^\circ+\theta)\tan(-\theta)\operatorname{cosec}(360^\circ+\theta)} = \cos^2\theta\cot\theta$ *(OR)*

b) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large

44. a) Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines Find the distance between them. *(OR)*

b) If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1 - x^2)y_2 - 3xy_1 - y = 0$

45. a) Prove that $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ *(OR)*

b) A construction company employs 2 executive engineer, Engineer-1 does the work for 60% of jobs of the company. Engineer-2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer-1 does the work is 0.03, where as the probability of an error in the work engineer-2 is 0.04%. Suppose a serious error occurs in the work, which engineer would you guess did the work?

46. a) A tree is growing so that, after t-years its height is increasing at a rate of $\frac{18}{\sqrt{t}}$ cm per year. Assume that when t = 0, the height is 5 cm. (i) Find the height of the tree after 4 years (ii) After how many years will the height be 149 cm? *(OR)*

b) If $A + B + C = \pi$, prove that $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A\cos B\cos C$

47. a) Integrate the following with respect to x : $\frac{2x-3}{x^2+4x-12}$ *(OR)*

b) Show that the points whose position vectors $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar.

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23/12/24

# ST. JOSEPH'S HR. SEC. SCHOOL - CHENGALPATTU
## COMMON HALF YEARLY EXAMINATION - 2024
STD - 11 MATHEMATICS - KEY MARKS : 90

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I

| | | |
|---|---|---|
| 1 | c | 6 |
| 2 | c | $-8, 8$ |
| 3 | a | $4+\sqrt{2}$ |
| 4 | a | $r!$ |
| 5 | b | $2/3$ |
| 6 | b | $9/2$ |
| 7 | d | 3 |
| 8 | c | 26 |
| 9 | b | $-2/3$ |
| 10 | c | $-\frac{b}{a^2}\sec^3\theta$ |
| 11 | c | $\tan(xe^x)+c$ |
| 12 | d | $2/3$ |
| 13 | b | $\frac{e^{-4x}}{17}[-4\cos x+\sin x]+c$ |
| 14 | d | $\frac{2i-j}{\sqrt{5}}$ |
| 15 | b | $\log(a/b)$ |
| 16 | b | $1-\alpha^2-\beta\gamma=0$ |
| 17 | b | $5, 5$ |
| 18 | | MA |
| 19 | b | $\frac{\sqrt{2-\sqrt{2}}}{2}$ |
| 20 | c | 0 |

II

21. $2^m-2^k=112 \Rightarrow 2^k(2^{m-k}-1)=2^4\times 7$
$k=4$, $m=7$

22. $-5<10x-2<5$
$-5+2<10x-2+2<5+2 \Rightarrow -3<10x<7$
$\frac{-3}{10}<x<\frac{7}{10}$

23. i. $18^0=\frac{\pi}{10}$ radian
ii. $\frac{\pi}{5}=\frac{180}{5}=36^0$

24. $=\frac{11!}{4!\,4!\,2!}=34650$

25. $a_1=2$, $a_2=2$, $a_3=4$, $a_4=4$
$a_5=6$, $a_6=6$

26. $=\begin{vmatrix}0 & -4 & 1\\ 4 & 0 & 13\\ -1 & -13 & 0\end{vmatrix}=0 \Rightarrow 52-52=0$

27. $|\vec{a}|=\sqrt{6}$, $|\vec{b}|=\sqrt{41}$, $|\vec{c}|=\sqrt{35}$
$AB^2+AC^2=BC^2 \Rightarrow 6+35=41 \Rightarrow 41=41$

28. $\frac{dy}{dx}=5(x^2+4x+6)^4(2x+4)$

29. $n(S)=6$, $n(A)=3$, $n(B)=2$
(i) $P(A)=\frac{n(A)}{n(S)}=\frac{1}{2}$
(ii) $P(B)=\frac{n(B)}{n(S)}=\frac{1}{3}$

30. $m_1=\tan 60^0=\sqrt{3}$, $m_2=\tan 120^0=-\sqrt{3}$
$y=m_1x+b$, $y=m_2x+b$
$y=\sqrt{3}x+7$, $y=-\sqrt{3}x+7$

III

31. $|a\times i|^2=a_3^2+a_2^2$, $|a\times j|^2=a_1^2+a_3^2$, $|a\times k|^2=a_2^2+a_1^2$
$=a_3^2+a_2^2+a_1^2+a_3^2+a_2^2+a_1^2$
$=2(a_1^2+a_2^2+a_3^2)$
$=2|\vec{a}|^2$

32. $-1\le\cos x\le 1$
$-3\le -3\cos x\le 3$
$1-3\le 1-3\cos x\le 1+3 \Rightarrow -\frac{1}{2}\ge\frac{1}{1-3\cos x}\ge\frac{1}{4}$
Range f is $(-\infty,-\frac{1}{2}]\cup[\frac{1}{4},\infty)$

33. $=\frac{A}{x-2}+\frac{B}{x+1}$
$3x+1=A(x+1)+B(x-2)$ —(1)
$x=2 \Rightarrow A=7/3$
$x=-1 \Rightarrow B=2/3$
$\frac{3x+1}{(x-2)(x+1)}=\frac{7}{3(x-2)}+\frac{2}{3(x+1)}$

34. $=\frac{1+\tan\theta}{1-\tan\theta}-\frac{1-\tan\theta}{1+\tan\theta}$
$=\frac{(1+\tan\theta)^2-(1-\tan\theta)^2}{(1-\tan\theta)(1+\tan\theta)}=\frac{4\tan\theta}{1-\tan^2\theta}$
$=2\cdot\frac{2\tan\theta}{1-\tan^2\theta}=2\tan 2\theta$

35. $2nC_n=\frac{2n!}{n!\,n!}=\frac{2n(2n-1)(2n-2)\cdots 4\cdot 3\cdot 2\cdot 1}{n!\,n!}$
$=\frac{(2n\cdot(2n-2)\cdots 4\cdot 2)((2n-1)\cdots 3\cdot 1)}{n!\,n!}$
$=\frac{2^n(n(n-1)(n-2)\cdots 2\cdot 1)(1\cdot 3\cdots(2n-3))}{n!\,n!}$
$=\frac{2^n(1\cdot 3\cdot 5\cdots(2n-1))}{n!}$

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36. LHS $= \begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} \times \begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} = \begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} \times \begin{vmatrix} 1 & x & x \\ -x & 1 & -x \\ -x & -x & -1 \end{vmatrix}$

$= \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix} = RHS$

b. $= \begin{vmatrix} a & -b & 0 \\ 0 & b & -c \\ 1 & 1 & 1+c \end{vmatrix}$ $R_1 - R_2$, $R_2 - R_3$

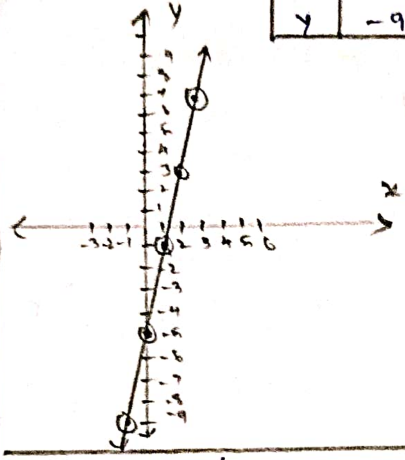
$= a[b(1+c)+c] + b[0+c]$

$= abc + ab + bc + ac$

$= abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

37. $f(x) = 4x-5$

| x | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| y | -9 | -5 | -1 | 3 | 7 |



42 a. LHS $= \log 2 + \log\frac{(16)^{16}}{(15)^{16}} + \log\frac{(25)^{12}}{(24)^{12}} + \log\frac{(81)^7}{(80)^7}$

$= \log\left[2 \times \frac{(16)^{16}}{3^{16}\times 5^{16}} \times \frac{5^{12}\times 5^{12}}{4^{12}\times 6^{12}} \times \frac{3^{28}}{16^7 \times 5^7}\right]$

$= \log\left[\frac{2\times 2^{36}\times 5^8 \times 3^{12}}{2^{24}\times 2^{12}\times 3^{12}\times 5^7}\right] = \log[2\times 5]$

$= \log_{10} 10 = 1 = RHS$

b. $P(n) = 1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$n=1$ P(1) : LHS : 1, RHS : 1 LHS = RHS

P(1) is true

$n=k$ $P(k) = 1^2+2^2+3^2+\dots+k^2 = \frac{k(k+1)(2k+1)}{6}$

P(k) is true

S.T. $P(k+1) = 1^2+2^2+3^2+\dots+k^2+(k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$P(k+1) = \frac{(k+1)(k+2)(2k+3)}{6}$

P(k) is true also P(k+1) is true.

$P(n) = 1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}, \forall\ n \geq 1$

38. (i) $y = \frac{x^{24}}{x^{25}} = \int \frac{1}{x}dx = \log x + c$

ii. $\int (1+x^2)^{-1} dx = \int \frac{1}{1+x^2}dx = \tan^{-1}x + c$

iii. $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + c$

39. $P(A) = 1/3$, $P(A') = 2/3$, $P(C) = 1/5$

$P(B) = 1/4$, $P(B') = 3/4$, $P'(C) = 4/5$

$P(A\cap B'\cap C'$ or $A'\cap B\cap C'$ or $A'\cap B'\cap C)$

$= \frac{1}{3}\cdot\frac{3}{4}\cdot\frac{4}{5} + \frac{2}{3}\cdot\frac{1}{4}\cdot\frac{4}{5} + \frac{2}{3}\cdot\frac{3}{4}\cdot\frac{1}{5}$

$= \frac{26}{60} = \frac{13}{30}$

43 a. LHS $= \frac{\cot\theta\cos\theta\cos\theta}{(-\cos\theta)(-\tan\theta)\operatorname{cosec}\theta} = \frac{\cot\theta\cos\theta}{\tan\theta\operatorname{cosec}\theta}$

$= \frac{\cot\theta}{\frac{\sin\theta}{\cos\theta}} \cdot \frac{\cos\theta}{\frac{1}{\sin\theta}} = \cot\theta\cos\theta\cdot\cos\theta = \cot\theta\cos^2\theta$

40. $y = 1/x$ $x\to\infty$, $y\to 0$

$= \lim_{y\to 0} \frac{1}{y}(3^y + 1 - \cos y - e^y)$

$= \lim_{y\to 0}\left[\frac{3^y-1}{y} + \frac{1-\cos y}{y} - \frac{e^y - 1}{y}\right]$

$= \lim_{y\to 0}\left[\left(\frac{3^y-1}{y}\right) + \frac{\sin^2 y/2}{y/2} - \left(\frac{e^y-1}{y}\right)\right]$

$= \lim_{y\to 0}\left(\frac{3^y-1}{y}\right) + \lim_{y\to 0}\left(\frac{\sin y/2}{y/2}\right)\lim_{y\to 0}\sin\frac{y}{2} - \lim\frac{e^y - 1}{y}$

$= \log 3 + 1(0) - 1 = \log 3 - 1$

b. $\sqrt[3]{x^3+7} = (x^3+7)^{1/3} = \left[x^3\left(1+\frac{7}{x^3}\right)\right]^{1/3}$

$= x\left(1 + \frac{1}{3}\times\frac{7}{x^3} + \frac{\frac{1}{3}(\frac{1}{3}-1)}{2!}\left(\frac{7}{x^3}\right)^2 + \dots\right)$

$= x + \frac{7}{3}\times\frac{1}{x^2} - \frac{49}{9}\times\frac{1}{x^5} + \dots$

$\sqrt[3]{x^3+4} = (x^3+4)^{1/3} = \left[x^3(1+4/x^3)\right]^{1/3}$

$= x\left(1 + \frac{1}{3}\times\frac{4}{x^3} + \frac{\frac{1}{3}(\frac{1}{3}-1)}{2!}\left(\frac{4}{x^3}\right)^2 + \dots\right)$

$= x + \frac{4}{3}\times\frac{1}{x^2} - \frac{16}{9}\times\frac{1}{x^5}$

$\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4} = x + \frac{7}{3x^2} - \dots - x - \frac{4}{3}\frac{1}{x^2}$

$= \frac{1}{x^2}$ (app)

IV 41. a. $y = 3x-5$, $x = \frac{y+5}{3}$, $g(y) = \frac{y+5}{3}$

$(g\circ f)(x) = g(3x-5) = \frac{3x-5+5}{3} = x = I_x$

$(f\circ g)(x) = f\left(\frac{y+5}{3}\right) = 3\left(\frac{y+5}{3}\right) - 5 = y = I_y$

$f^{-1}(y) = \frac{y+5}{3} \Rightarrow f^{-1}(x) = \frac{x+5}{3}$

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**44 a.** $a=9,\ h=-12,\ b=16$

$ab-h^2 = 9(16)-(-12)^2 = 0,\ h^2=ab$

$9x^2-24xy+16y^2 = (3x-4y)^2$

$9x^2-24xy+16y^2-12x+16y-12 = (3x-4y+l)(3x-4y+m)$

co-eff. of $x$: $m+l=-4$

co-eff. of $y$: $m+l=-4$ $\qquad lm=-12$

$m^2+4m-12=0$

$(m+6)(m-2)=0$

$3x-4y+2=0$

$3x-4y-6=0$

D.B. $\dfrac{|c_1-c_2|}{\sqrt{a^2+b^2}} = \dfrac{2+6}{\sqrt{3^2+(-4)^2}} = 8/5$

**b.** $y = \dfrac{\sin^{-1}x}{\sqrt{1-x^2}} \Rightarrow y\sqrt{1-x^2} = \sin^{-1}x$

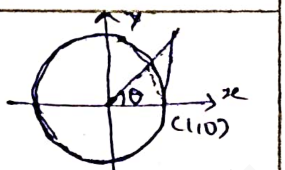
$y\left(\dfrac{1}{2\sqrt{1-x^2}}(-2x)\right) + \left(\sqrt{1-x^2}\right)y_1 = \dfrac{1}{\sqrt{1-x^2}}$

$\div \sqrt{1-x^2} \Rightarrow -xy+(1-x^2)y_1 = 1$

$-[xy_1+y]+(1-x^2)y_2+y_1(-2x)=0$

$(1-x^2)y_2-3xy_1-y=0$

**45 a.** $\lim\limits_{\theta\to 0}\dfrac{\sin\theta}{\theta}=1$




$\dfrac{\tan\theta}{2} \ge \dfrac{\theta}{2} \ge \dfrac{\sin\theta}{2}$

$\dfrac{1}{\cos\theta} \ge \dfrac{\theta}{\sin\theta} \ge 1 \Rightarrow \cos\theta \le \dfrac{\sin\theta}{\theta} \le 1$

$\cos(-\theta)=\cos\theta,\quad \dfrac{\sin(-\theta)}{-\theta} = \dfrac{\sin\theta}{\theta},\quad \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\lim\limits_{\theta\to 0}\cos\theta = 1,\quad \lim\limits_{\theta\to 0}(1)=1,\ \Rightarrow \lim\limits_{\theta\to 0}\dfrac{\sin\theta}{\theta}=1$

**b.** $P(A_1)=0.60,\quad P(B/A_1)=0.03$

$P(A_2)=0.40,\quad P(B/A_2)=0.04$

$P(A_1/B) = \dfrac{P(A_1)P(B/A_1)}{P(A_1)P(B/A_1)+P(A_2)P(B/A_2)} = 9/17$

$P(A_2/B) = \dfrac{P(A_2)P(B/A_2)}{P(A_1)P(B/A_1)+P(A_2)P(B/A_2)} = \dfrac{8}{17}$

$P(A_1/B) > P(A_2/B)$

**46 a.** $\dfrac{dh}{dt} = \dfrac{18}{\sqrt{t}} = 18t^{-1/2}$

$h = \int 18t^{-1/2}dt = 36\sqrt{t}+c$

$t=0,\ h=5,\ \Rightarrow c=5 \Rightarrow h=36\sqrt{t}+5$

i. after 4 years $\Rightarrow h = 36\sqrt{4}+5 = 77$ cm

ii. $h=149$ cm

$h=36\sqrt{t}+5 \Rightarrow 149 = 36\sqrt{t}+5$

$\sqrt{t}=4 \Rightarrow t=16$

**b.** $\cos^2A+\cos^2B+\cos^2C = \frac{1}{2}[2\cos^2A+2\cos^2B+2\cos^2C]$

$= \frac{1}{2}[(1+\cos2A)+(1+\cos2B)+(1+\cos2C)]$

$= \frac{3}{2}+\frac{1}{2}[2\cos(A+B)\cos(A-B)+(2\cos^2C-1)]$

$= \frac{3}{2}+\frac{1}{2}[-2\cos C\cos(A-B)+2\cos^2C-1]$

$= \frac{3}{2}-\frac{1}{2}+\frac{1}{2}[-2\cos C(\cos(A-B)-\cos C)]$

$= 1-\cos C[\cos(A-B)+\cos(A+B)]$

$= 1-\cos C[2\cos A\cos B]$

$= 1-2\cos A\cos B\cos C$

**47 a.** $I = \int\dfrac{2x-3}{x^2+4x-12}dx$

$2x-3 = A(2x+4)+B$

$2A=2 \Rightarrow \boxed{A=1},\ \boxed{B=-7}$

$I = \int\dfrac{(2x+4)-7}{x^2+4x-12}dx = \int\dfrac{2x+4}{x^2+4x-12}dx - 7\int\dfrac{1}{x^2+4x-12}dx$

$I = \log|x^2+4x-12| - 7I_1$

$I_1 = \int\dfrac{1}{x^2+4x-12}dx = \int\dfrac{1}{(x+2)^2-(4)^2}dx$

$= \dfrac{1}{2(4)}\log\left|\dfrac{x+2-4}{x+2+4}\right| = \dfrac{1}{8}\log\left|\dfrac{x-2}{x+6}\right|$

$\therefore I = \log|x^2+4x-12| - \dfrac{7}{8}\log\left|\dfrac{x-2}{x+6}\right| + c$

**b.** $\vec{OA} = 4\hat{i}+5\hat{j}+\hat{k}$

$\vec{OB} = -\hat{j}-\hat{k}$

$\vec{OC} = 3\hat{i}+9\hat{j}+4\hat{k}$

$\vec{OD} = -4\hat{i}+4\hat{j}+4\hat{k}$

$\vec{AB} = -4\hat{i}-6\hat{j}-2\hat{k}$

$\vec{AC} = -\hat{i}+4\hat{j}+3\hat{k}$

$\vec{AD} = -8\hat{i}-\hat{j}+3\hat{k}$

$= \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$

$= -4(12+3)+6(-3+24)-2(1+32)$

$= -4(15)+6(21)-2(33)$

$= -60+126-66$

$= -126+126$

$= 0 \Rightarrow [\vec{AB}\ \vec{AC}\ \vec{AD}]=0$

$\therefore$ points A, B, C, D are coplanar.

**OR**

$a = mb+nc$

$-4\hat{i}-6\hat{j}-2\hat{k} = m(-\hat{i}+4\hat{j}+3\hat{k}) + n(-8\hat{i}-\hat{j}+3\hat{k})$

$m+8n=4$ — (1)

$4m-n=-6$ — (2)

$3m+3n=-2$ — (3)

(1) & (2) $\Rightarrow m = \dfrac{-44}{33} = -4/3$

$n = \dfrac{16}{3\times 8} = 2/3$

(3) $\Rightarrow 3(-4/3)+3(2/3) = \dfrac{-12}{3}+\dfrac{6}{3} = -2$ RHS

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