XI STD – PHYSICS

IMPORTANT PROBLEMS

UNIT – 1 – Nature of physical world and measurement

1. **From a point on the ground, the top of a tree is seen to have an angle of elevation 60°. The distance between the tree and a point is 50 m. Calculate the height of the tree?**

Solution:

Angle $\theta = 60^0$

The distance between the tree and a point $x = 50$ m

Height of the tree (h) = ?

For triangulation method $\tan\theta = \frac{h}{x}$

$h = x \tan\theta$

$= 50 \times \tan 60^0$

$= 50 \times 1.732$

$h = 8.86$ m

The height of the tree is 8.86m

2. **A RADAR signal is beamed towards a planet and its echo is received 7 minutes later. If the distance between the planet and the Earth is 6.3 × 1010 m. Calculate the speed of the signal?**

Solution:

The distance of the planet from the Earth $d = 6.3 \times 10^{10}$ m

Time t = 7 minutes = 7 × 60 s. the speed of signal v = ?

The speed of signal

$v = \frac{2d}{t} = \frac{2 \times 6.3 \times 10^{10}}{7 \times 60} = 3 \times 10^8$ ms-1

3. **Two resistances R1 = (100 ± 3) W, R2 = (150 ± 2) W, are connected in series. What is their equivalent resistance?**

Solution:

R1 = 100 ± Ω ; R2 = 150 ± 2Ω

Equivalent resistance R = ?

Equivalent resistance R = R1+R2

$= (100 \pm 3150) + (\pm 2)$

$= (100 + 150) \pm (3 + 2)$

$R = (250 \pm 5)\ \Omega$

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4. **The temperatures of two bodies measured by a thermometer are t_1 = (20 + 0.5)°C, t_2 = (50 ± 0.5)°C. Calculate the temperature difference and the error therein.**

Solution:

t_1 = (20 ±0.5)°C t_2 = (50 ±0.5)°C

temperature difference t=?

t = t_2 - t_1 = (50 ±0.5) - (20 ±0.5)°C (Using equation1.4)

= (50 – 20) ± (0.5 + 0.5)

t = (30 ±1)° C

5. **A physical quantity x is given by x = $\frac{a^3b^3}{c\sqrt{d}}$. If the percentage errors of measurement in a, b, c and d are 4%, 2%, 3% and 1% respectively, then calculate the percentage error in the calculation of x.**

Solution:

Given x = $\frac{a^3b^3}{c\sqrt{d}}$

The percentage error in x is given by

$\frac{\Delta x}{x}$ x 100 = 2 $\frac{\Delta a}{a}$ x 100 + 3 $\frac{\Delta b}{b}$ x 100 + $\frac{\Delta c}{c}$ x 100 + $\frac{1}{2}\frac{\Delta d}{d}$ x 100

= (2 × 4%) + (3 × 2%) + (1 × 3%) + (½ ×1%)

= 8% +6%+3% +0.5%

The percentage error is x = 17.5%

6. **State the number of significant figures in the following**

i) 600800 **iv) 5213.0**

ii) 400 **v) 2.65 × 1024m**

iii) 0.007 **vi) 0.0006032**

Solution:

i) four ii) one iii) one

iv) five v) three vi) four

7. **Round off the following numbers as indicated**
i) 18.35 up to 3 digits
ii) 19.45 up to 3 digits
iii) 101.55 × 10^6 up to 4 digits
iv) 248337 up to digits 3 digits
v) 12.653 up to 3 digits.

Solution:

i) 18.4 ii) 19.4 iii) 101.6 × 106

iv) 248000 v) 12.7

8. Convert 76 cm of mercury pressure into Nm−2 using the method of dimensions.

Solution:

In cgs system 76 cm of mercury pressure = 76 × 13.6 × 980 dyne cm^{-2}

The dimensional formula of pressure P is $[ML^{-1} T^{-2}]$

$P_1 [M_1^a L_1^b T_1^c] = P_2 [M_2^a L_2^b T_2^c]$

We have $P_2 = P_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

$M_1 = 1g, M_2 = 1$ kg

$L_1 = 1$ cm, $L_2 = 1$ m

$T_1 = 1$ s, $T_2 = 1s$

As a = 1, b = -1, c = -2

Then

$$P_2 = 76 \times 13.6 \times 980 \left[\frac{1g}{1kg}\right]^1 \left[\frac{1cm}{1\,m}\right]^{-1} \left[\frac{1s}{1s}\right]^{-2}$$

$$= 76 \times 13.6 \times 980 \left[\frac{10^{-3}kg}{1\,kg}\right]^1 \left[\frac{10^{-2}m}{1\,m}\right]^{-1} \left[\frac{1s}{1s}\right]^{-2}$$

$$= 76 \times 13.6 \times 980 \times (10^{-3}) \times 10^2$$

$P_2 = 1.01 \times 10^5\ Nm^{-2}$.

9. If the value of universal gravitational constant in SI is $6.6 \times 10^{-11}\ Nm^2\ kg^{-2}$, then find its value in CGS System?

Solution

Let G_{SI} be the gravitational constant in the SI system and G_{cgs} in the cgs system. Then

$G_{SI} = 6.6 \times 10^{-11}\ Nm^2\ Kg^{-2}$

$G_{cgs} = ?$

$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

$G_{cgs} = G_{SI} \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

$M_1 = 1kg, M_2 = 1$ g

$L_1 = 1$ m, $L_2 = 1$ cm

$T_1 = 1$ s, $T_2 = 1s$

The dimensional formula for G is $M^{-1} L^3 T^{-2}$

a = -1 b = 3 and c = -2

$$G_{cgs} = 6.6 \times 10^{-11} \left[\frac{1kg}{1g}\right]^{-1} \left[\frac{1m}{1\,cm}\right]^3 \left[\frac{1s}{1s}\right]^{-2}$$

$= 6.6 \times 10^{-11}\left[\frac{1kg}{10^{-3}kg}\right]^{-1}\left[\frac{1\,m}{10^{-2}m}\right]^{3}\left[\frac{1s}{1s}\right]^{-2}$

$= 6.6 \times 10^{-11} \times 10^{-3} \times 10^{6} \times 1$

$G_{cgs} = 6.6 \times 10^{-8}$ dyne $cm^2\, g^{-2}$

10. Check the correctness of the equation $\frac{1}{2}mv^2 = mgh$ using dimensional analysis method.

Solution

Dimensional formula for

$\frac{1}{2}mv^2 = [M]\,[LT^{-2}] = [ML^2T^{-2}]$

Dimensional formula for

$mgh = [M]\,[LT^{-2}]\,[L] = [ML^2T^{-2}]$

$[ML^2T^{-2}] = [ML^2T^{-2}]$

Both sides are dimensionally the same, hence the equations $\frac{1}{2}mv^2 = mgh$ is dimensionally correct.

11. Obtain an expression for the time period T of a simple pendulum. The time period T depends on (i) mass 'm' of the bob (ii) length 'l' of the pendulum and (iii) acceleration due to gravity g at the place where the pendulum is suspended. (Constant k = 2π) i.e

Solution

$T \propto m^a\, l^b\, g^c$

$T = km^a\, l^b\, g^c$

Here k is the dimensionless constant. Rewriting the above equation with dimensions

$[T^1] = [M^a]\,[L^b]\,[LT^{-2}]^c$

$[M^0\, L^0\, T^1] = [M^a\, L^{b+c}\, T^{-2c}]$

Comparing the powers of M, L and T on both sides, a=0, b+c=0, -2c=1

Solving for a, b and c a = 0, b = 1/2, and c = −1/2

From the above equation $T = km^0\, l^{1/2}\, g^{-1/2}$

$T = k\left(\frac{l}{g}\right)^{1/2} = k\sqrt{l/g}$

Experimentally k = 2π, hence $T\,g = 2\pi\sqrt{l/g}$

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UNIT – 2 – Kinematics

1. **Two vectors $\vec{A}$ and $\vec{B}$ are given in the component form as $\vec{A} = 5\hat{i} + 7\hat{j} - 4\hat{k}$ and $\vec{B} = 6\hat{i} + 3\hat{j} + 2\hat{k}$. Find $\vec{A} + \vec{B}$, $\vec{B} + \vec{A}$, $\vec{A} - \vec{B}$, $\vec{B} - \vec{A}$.**

Solution

$\vec{A} + \vec{B} = (5\hat{i} + 7\hat{j} - 4\hat{k}) + (6\hat{i} + 3\hat{j} + 2\hat{k})$

$= 11\hat{i} + 10\hat{j} - 2\hat{k}$

$\vec{B} + \vec{A} = (6\hat{i} + 3\hat{j} + 2\hat{k}) + (5\hat{i} + 7\hat{j} - 4\hat{k})$

$= (6 + 5)\hat{i} + (3 + 7)\hat{j} + (2 - 4)\hat{k}$

$= 11\hat{i} + 10\hat{j} - 2\hat{k}$

$\vec{A} - \vec{B} = (5\hat{i} + 7\hat{j} - 4\hat{k}) - (6\hat{i} + 3\hat{j} + 2\hat{k})$

$= -\hat{i} + 4\hat{j} - 6\hat{k}$

$\vec{B} - \vec{A} = (6\hat{i} + 3\hat{j} + 2\hat{k}) - (5\hat{i} + 7\hat{j} - 4\hat{k})$

$= \hat{i} - 4\hat{j} + 6\hat{k}$

2. **Given two vectors $\vec{A} = 2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = \hat{i} + 3\hat{j} + 6\hat{k}$, Find the product $\vec{A}\cdot\vec{B}$, and the magnitudes of $\vec{A}$ and $\vec{B}$. What is the angle between them?**

Solution

$\vec{A}\cdot\vec{B} = 2 + 12 + 30 = 44$

Magnitude $A = \sqrt{4 + 16 + 25} = \sqrt{45}$ units

Magnitude $B = \sqrt{1 + 9 + 36} = \sqrt{46}$ units

The angle between the two vectors is given by

$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$

$= \cos^{-1}\left(\frac{44}{\sqrt{45 \times 46}}\right) = \cos^{-1}\left(\frac{44}{45.9}\right)$

$= \cos^{-1}(0.967)$

$\theta \cong 15^0$.

3. **Check whether the following vectors are orthogonal.**

(i) $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 4\hat{i} - 5\hat{j}$

(ii) $\vec{C} = 5\hat{i} + 2\hat{j}$ and $\vec{D} = 2\hat{i} - 5\hat{j}$

Solution

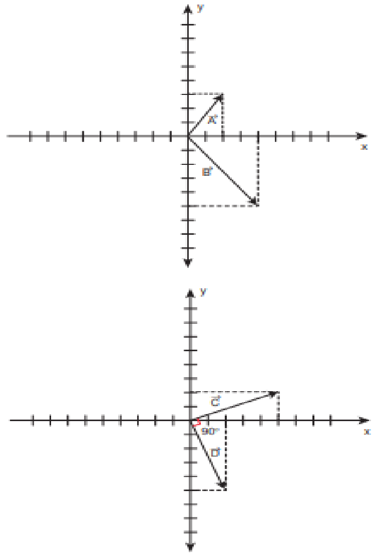
$\vec{A}\cdot\vec{B} = 8 - 15 = -7 \neq 0$

Hence $\vec{A}$ and $\vec{B}$ are not orthogonal to each other.

$\vec{C} \cdot \vec{D} = 10 - 10 = 0$

Hence, $\vec{C}$ and $\vec{D}$ are orthogonal to each other.

It is also possible to geometrically show that the vectors $\vec{C}$ and $\vec{D}$ are orthogonal to each other. This is shown in the following Figure



In physics, the work done by a force $\vec{F}$ to move an object through a small displacement $\overrightarrow{dr}$ is defined as,

$W = \vec{F} \cdot \overrightarrow{dr}$

$W = F\, dr \cos\theta$

The work done is basically a scalar product between the force vector and the displacement vector. Apart from work done, there are other physical quantities which are also defined through scalar products.

4. **Two vectors are given as $\vec{r} = 2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ and $\vec{F} = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$. Find the resultant vector $\vec{\tau} = \vec{r} \times \vec{F}$.**

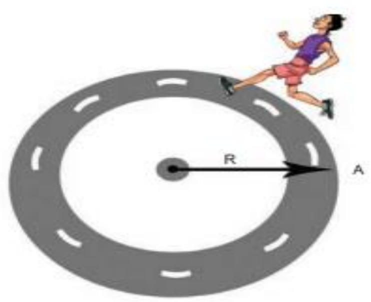
<u>***Solution***</u>

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 5 \\ 3 & -2 & 4 \end{vmatrix}$$

$\vec{\tau} = (12-(-10))\hat{\imath} + (15-8)\,\hat{\jmath} + (-4-9)\,\hat{k}$

$\vec{\tau} = 22\hat{\imath} + 7\hat{\jmath} - 13\hat{k}$

5. **An athlete covers 3 rounds on a circular track of radius 50 m. Calculate the total distance and displacement travelled by him.**



Solution

The total distance the athlete covered =3x circumference of track

Distance = 3 ×2π x 50 m

= 300π m (or)

Distance ≈ 300 x 3.14 ≈ 942 m

The displacement is zero, since the athlete reaches the same point A after three rounds from where he started.

6. The velocity of three particles A, B, C are given below. Which particle travels at the greatest speed?

$\overline{V_A} = 3\hat{\imath} - 5\hat{\jmath} + 2\hat{k}$

$\overline{V_B} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\overline{V_C} = 5\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

Solution

We know that speed is the magnitude of the velocity vector. Hence,

Speed of A = $|\overline{V_A}| = \sqrt{(3)^2 + (-5)^2 + (2)^2}$

$= \sqrt{9 + 25 + 4} = \sqrt{38}$ ms^{-1}

Speed of B = $|\overline{V_B}| = \sqrt{(1)^2 + (2)^2 + (3)^2}$

$= \sqrt{1 + 4 + 9} = \sqrt{14}$ ms^{-1}

Speed of C = $|\overline{V_C}| = \sqrt{(5)^2 + (3)^2 + (4)^2}$

$= \sqrt{25 + 9 + 16} = \sqrt{50} ms^{-1}$

The particle C has the greatest speed. $\sqrt{50} > \sqrt{38} > \sqrt{14}$

7. Consider two masses of 10 g and 1 kg moving with the same speed 10 m s-1. Calculate the magnitude of the momentum.

Solution

We use p = mv For the mass of 10 g, m = 0.01 kg

p = 0.01 × 10 = 0.1 kg m s-1

For the mass of 1 kg

p = 1 × 10 = 10 kg m s-1

Thus even though both the masses have the same speed, the momentum of the heavier mass is 100 times greater than that of the lighter mass.

8. A particle moves along the x-axis in such a way that its coordinates x varies with time 't' according to the equation $x = 2 - 5t + 6t^2$. What is the initial velocity of the particle?

Solution

$x = 2 - 5t + 6t^2$

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(2 - 5t + 6t^2)$

Or $v = -5 + 12t$

For initial velocity, $t = 0$

$\therefore$ Initial velocity = $-5 ms^{-1}$

The negative sign implies that at t = 0 the velocity of the particle is along negative x direction.

Average speed = total path length / total time period.

9. Consider two trains A and B moving along parallel tracks with the same velocity in the same direction. Let the velocity of each train be 50 km h^{-1} due east. Calculate the relative velocities of the trains.

Solution

Relative velocity of B with respect to A,

$v_{BA} = v_B - v_A$

$= 50 \text{ km } h^{-1} + (-50) \text{ km } h^{-1}$

$= 0 \text{ km } h^{-1}$

Similarly, relative velocity of A with respect to B i.e., v_{AB} is also zero.

Thus each train will appear to be at rest with respect to the other.

10. How long will a boy sitting near the window of a train travelling at 36 km h^{-1} see a train passing by in the opposite direction with a speed of 18 km h^{-1}. The length of the slow - moving train is 90 m.

Solution

The relative velocity of the slow-moving train with respect to the boy is = (36 + 18) km h^{-1} = 54 km h^{-1} = 54 × 5 18 m s^{-1} = 15 m s^{-1}

Since the boy will watch the full length of the other train, to find the time taken to watch the full train:

We have, $15 = \frac{90}{t}$

Or $t = \frac{90}{15} = 6s.$

11. An iron ball and a feather are both falling from a height of 10 m.

a) What are the time taken by the iron ball and feather to reach the ground?

b) What are the velocities of iron ball and feather when they reach the ground?

(Ignore air resistance and take g = 10 m s^{-2})

Solution

Since kinematic equations are independent of mass of the object, according to equation (2.8) the time taken by both iron ball and feather to reach the ground are the same. This is given by

$$T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 10}{10}} = \sqrt{2} \text{ s} \approx 1.414 \text{ s}$$

Thus, both feather and iron ball reach ground at the same time.

By following equation (2.19) both iron ball and feather reach the Earth with the same speed. It is given by

$$V = \sqrt{2gh} = \sqrt{2 \times 10 \times 10} = \sqrt{200} \text{ ms}^{-1} \approx 1.414 \text{ s}$$

12. A train was moving at the rate of 54 km h^{-1} when brakes were applied. It came to rest within a distance of 225 m. Calculate the retardation produced in the train.

Solution

The final velocity of the particle v = 0 The initial velocity of the particle

$u = 54 \times \frac{5}{18} \text{ m s}^{-1} = 15 \text{ m s}^{-1}$

$s = 225$ m

Retardation is always against the velocity of the particle.

$V^2 = u^2 - 2aS$

$0 = (15)^2 - 2a(225)$

$450a = 225$

$a = \frac{225}{450} \text{ ms}^{-2} = 0.5 \text{ ms}^{-2}$

Hence, retardation = 0.5 m s^{-2}

13. Suppose an object is thrown with initial speed 10 m s^{-1} at an angle π/4 with the horizontal, what is the range covered? Suppose the same object is thrown similarly in the Moon, will there be any change in the range? If yes, what is the change? (The acceleration due to gravity in the Moon $g_{moon} = \frac{1}{6} g$)

Solution

In projectile motion, the range of particle is given by,

$$R = \frac{u^2 \sin 2\theta}{g}$$

$\theta = \pi/4 \quad u = v_0 = 10 \text{m s}^{-1}$

$\therefore R_{earth} = \frac{10^2 \sin \pi/2}{9.8} = 100/9.8$

$R_{earth} = 10.20$ m (Approximately 10m)

If the same object is thrown in the Moon, the range will increase because in the Moon, the acceleration due to gravity is smaller than g on Earth,

$g_{moon} = \frac{1}{6} g$

$R_{moon} = \frac{u^2 \sin 2\theta}{g_{moon}} = \frac{v_0 \sin 2\theta}{g/6}$

$\therefore R_{moon} = 6R_{earth}$

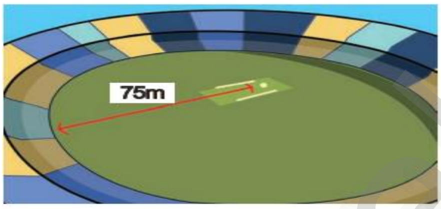
$R_{moon} = 6 \times 10.20 = 61.20$ m (Approximately 60m)

The range attained on the Moon is approximately six times that on Earth.

14. In the cricket game, a batsman strikes the ball such that it moves with the speed 30 m s^{-1} at an angle 300 with the horizontal as shown in the figure. The boundary line of the

cricket ground is located at a distance of 75 m from the batsman? Will the ball go for a six? (Neglect the air resistance and take acceleration due to gravity g = 10 m s^{-2}).

Solution



The motion of the cricket ball in air is essentially a projectile motion. As we have already seen, the range (horizontal distance) of the projectile motion is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

The initial speed u = 30 m s^{-1}

The projection angle θ = 30^0

The horizontal distance travelled by the cricket ball

$$R = \frac{30^2 \times \sin 60^0}{10}$$

$$= \frac{900 \times \frac{\sqrt{3}}{2}}{10} = 77.94 \text{ m}$$

This distance is greater than the distance of the boundary line. Hence the ball will cross this line and go for a six.

15.A particle is in circular motion with an acceleration α = 0.2 rad s^{-2} .

(a) What is the angular displacement made by the particle after 5 s?

(b) What is the angular velocity at t = 5 s?.

Assume the initial angular velocity is zero

Solution

Since the initial angular velocity is zero (ω_0 = 0).

The angular displacement made by the particle is given by

$$\theta = \omega_0 t + \frac{1}{t} \alpha t^2$$

$$\theta = \frac{1}{2} \text{ x } 2 \text{ x } 10^{-1} \text{ x } 25 = 2.5 \text{ rad}$$

In terms of degree

$$\theta = 2.5 \text{ x } 57.27^0 \approx 143^0$$

1. **If two objects of masses 2.5 kg and 100 kg experience the same force 5 N, what is the acceleration experienced by each of them?**

Solution

From Newton's second law (in magnitude form), F = ma

For the object of mass 2.5 kg, the acceleration is $a = \frac{F}{m} = \frac{5}{2.5} = 2 \text{ m s}^{-2}$

For the object of mass 100 kg, the acceleration is $a = \frac{F}{m} = \frac{5}{100} = 0.05 \text{ m s}^{-2}$

2. **The position vector of a particle is given by $3t\hat{i} + 5t^2\hat{j} + 7\hat{k}$. Find the direction in which the particle experiences net force?**

Solution

Velocity of the particle,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3t)\,\hat{i} + \frac{d}{dt}(5t^2)\,\hat{j} + \frac{d}{dt}(7)\,\hat{k}$$

$$\frac{\overrightarrow{dr}}{dt} = 3\hat{i} + 10t\,\hat{j}$$

Acceleration of the particle

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = 10\hat{j}$$

3. **A particle of mass 2 kg experiences two forces, $\overrightarrow{F_1} = 5\hat{i} + 8\hat{j} + 7\hat{k}$ and , $\overrightarrow{F_2} = 3\hat{i} - 4\hat{j}\ 3\hat{k}$. What is the acceleration of the particle?**

Solution

We use Newton's second law, $\vec{F}_{net} = m\vec{a}$ where $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$. From the above equations the acceleration is $\vec{a} = \frac{\overrightarrow{F_{net}}}{m}$, where

$$\vec{F}_{net} = (5+3)\,\hat{i} + (8-4)\,\hat{j} + (7+3)\,\hat{k}$$

$$\vec{F}_{net} = 8\hat{i} + 4\hat{j} + 10\hat{k}$$

$$\vec{a} = \left(\frac{8}{2}\right)\hat{i} + \left(\frac{4}{2}\right)\hat{j} + \left(\frac{10}{2}\right)\hat{k}$$

$$\vec{a} = 4\hat{i} + 2\hat{j} + 5\hat{k}$$

4. **An object of mass 10 kg moving with a speed of 15 m s^{-1} hits the wall and comes to rest within**
   a) 0.03 second
   b) 10 second
   Calculate the impulse and average force acting on the object in both the cases.

Solution

Initial momentum of the object $P_i = 10 \times 15 = 150$ k gm s^{-1}

Final momentum of the object $P_f = 0$

$\Delta p = 150 - 0 = 150$ kg ms^{-1}

(a) Impulse J = Δp = 150 Ns

(b) Impulse J = Δp = 150 Ns

(a) Average force $F_{avg} = \frac{\Delta p}{\Delta t} = \frac{150}{0.03} = 5000$ N

(b) Average force $F_{avg} = \frac{150}{10} = 15$ N

We see that, impulse is the same in both cases, but the average force is different.

5. **If a stone of mass 0.25 kg tied to a string executes uniform circular motion with a speed of 2 m s^{-1} of radius 3 m, what is the magnitude of tensional force acting on the stone?**

Solution

$$F_{cp} = \frac{mv^2}{r}$$

$$F_{cp} = \frac{\frac{1}{4}\times(2)^2}{3} = 0.333 \text{ N}$$

6. **The Moon orbits the Earth once in 27.3 days in an almost circular orbit. Calculate the centripetal acceleration experienced by the Moon? (Radius of the Earth is 6.4 × 10^6 m)**

Solution

The centripetal acceleration is given by a = $\frac{v^2}{r}$. This expression explicitly depends on Moon's speed which is non trivial. We can work with the formula

$$\omega^2 R_m = a_m$$

a_m is centripetal acceleration of the Moon due to Earth's gravity.

ω is angular velocity.

R_m is the distance between Earth and the Moon, which is 60 times the radius of the Earth.

R_m = 60 R = 60 x 6.4 x 10^6 = 384 x 10^6 m .

As we know the angular velocity $\omega = \frac{2\pi}{T}$ and T = 27.3 days = 27.3 × 24 × 60 × 60 second = 2.358 × 10^6 sec

By substituting these values in the formula for acceleration.

$$a_m = \frac{(4\pi^2)(384\times10^6)}{(2.358\times10^6)^2} = 0.00272 \text{ ms}^{-2}$$

The centripetal acceleration of Moon towards the Earth is 0.00272 m s^{-2}.

7. **Consider a circular leveled road of radius 10 m having coefficient of static friction 0.81. Three cars (A, B and C) are travelling with speed 7 m s^{-1} , 8 m s^{-1} and 10 ms^{-1} respectively. Which car will skid when it moves in the circular level road? (g =10 m s^{-2})**

Solution

From the safe turn condition the speed of the vehicle (v) must be less than or equal to $\sqrt{u_s rg}$

$$V \le \sqrt{u_s rg}$$

$$\sqrt{u_s rg} = \sqrt{0.81\times10\times10} = 9 \text{ ms}^{-1}$$

For Car C, $\sqrt{u_s rg}$ is less than v

The speed of car A, B and C are 7 m s^{-1} , 8 m s^{-1} and 10 m s^{-1} respectively. The cars A and B will have safe turns. But the car C has speed 10 m s^{-1} while it turns which exceeds the safe turning speed. Hence, the car C will skid.

8. **Consider a circular road of radius 20 meter banked at an angle of 15 degree. With what speed a car has to move on the turn so that it will have safe turn?**

Solution

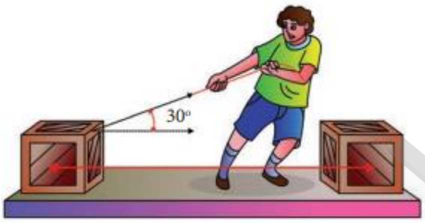
$$V = \sqrt{(rg\ tan\theta)} = \sqrt{20\times9.8\times tan\ 15^0}$$

$= \sqrt{20 \times 9.8 \times 0.26} = 7.1\ ms^{-1}$

The safe speed for the car on this road is 7.1 m s^{-1}

UNIT – 4 – Work, Energy and Power

1. A box is pulled with a force of 25 N to produce a displacement of 15 m. If the angle between the force and displacement is 30o , find the work done by the force.



Solution

Force, F = 25 N

Displacement, dr = 15 m

Angle between F and dr, θ = 30°

Work done, W= F dr cos θ

$W = 25 \times 15 \text{ x } \cos 30^0 = 25 \text{ x } 15 \text{ x } \frac{\sqrt{3}}{2}$

W = 324. 76J.

2. Two objects of masses 2 kg and 4 kg are moving with the same momentum of 20 kg ms^{-1}.
(a) Will they have same kinetic energy?
(b) Will they have same speed?

Solution

(a) The kinetic energy of the mass is given by $KE = \frac{P^2}{2m}$

For the object of mass 2 kg, kinetic energy is $KE_1 = \frac{(20)^2}{2 \times 2} = \frac{400}{4} = 100\ J$

For the object of mass 4 kg , kinetic energy is $KE_2 = \frac{(20)^2}{2 \times 4} = \frac{400}{8} = 50\ J$

Note that $KE_1 \neq KE_2$ i.e., even though both are having the same momentum, the kinetic energy of both masses is not the same. The kinetic energy of the heavier object has lesser kinetic energy than smaller mass. It is because the kinetic energy is inversely proportional to the mass ($KE \propto \frac{1}{m}$) for a given momentum.

(b) As the momentum, p = mv, the two objects will not have same speed.

3. **Water in a bucket tied with rope is whirled around in a vertical circle of radius 0.5 m. Calculate the minimum velocity at the lowest point so that the water does not spill from it in the course of motion. (g = 10 ms^{-2})**

<u>*Solution*</u>

Radius of circle r = 0.5 m

The required speed at the highest point $v_2 = \sqrt{gr} = \sqrt{10 \times 0.5} = \sqrt{5}$ ms^{-1}. The speed at lowest point $v_1 = \sqrt{5gr} = \sqrt{5} \times \sqrt{gr} = \sqrt{5} \times \sqrt{5} = 5$ ms^{-1}.

4. **Calculate the energy consumed in electrical units when a 75 W fan is used for 8 hours daily for one month (30 days).**

<u>*Solution*</u>

Power, P = 75 W

Time of usage, t = 8 hour × 30 days = 240 hours

Electrical energy consumed is the product of power and time of usage.

Electrical energy = power × time of usage = P × t

= 75 watt x 240 hour

=18000 watt hour

=18 kilowatt hour =18kWh

1 electrical unit = 1 kWh

Electrical energy =18 unit

5. **A vehicle of mass 1250 kg is driven with an acceleration 0.2 ms^{-2} along a straight level road against an external resistive force 500 N. Calculate the power delivered by the vehicle's engine if the velocity of the vehicle is 30 ms $^{-1}$.**

<u>*Solution*</u>

The vehicle's engine has to do work against resistive force and make vechile to move with an acceleration. Therefore, power delivered by the vehicle engine is

P = (resistive force + mass × acceleration) (velocity)

$P = \vec{F}_{tot} \cdot \vec{v} = (F_{resistive} + F)\vec{V}$

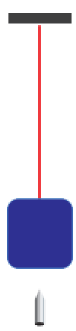
$P = \vec{F}_{tot} \cdot \vec{v} = (F_{resistive} + ma)\vec{V}$

= (500 N + (1250 kg) x (0.2 ms^{-2})) (30ms^{1}) = 22.5 kW

6. **A bullet of mass 50 g is fired from below into a suspended object of mass 450 g. The object rises through a height of 1.8 m with bullet remaining inside the object. Find the speed of the bullet. Take g = 10 ms^{-2}.**

<u>*Solution*</u>

m_1 = 50 g = 0.05 kg; m_2 = 450 g = 0.45 kg



The speed of the bullet is u_1 . The second body is at rest (u_2 = 0). Let the common velocity of the bullet and the object after the bullet is embedded into the object is v.

$$V = \frac{m_1u_1+m_2u_2}{(m_1+m_2)}$$

$$= \frac{0.05u_1+(0.45\times0)}{(0.05+0.45)} = \frac{0.05}{0.50} u_1$$

The combined velocity is the initial velocity for the vertical upward motion of the combined bullet and the object. From second equation of motion,

$$V = \sqrt{2gh}$$

$$V = \sqrt{2\times10\times1.8} = \sqrt{36}$$

$$V = 6ms^{-1}.$$

Substituting this in the above equation, the value of u1 is

$$6 = \frac{0.05}{0.50} u_1 \text{ or } u_1 = \frac{0.50}{0.05} \text{ x } 6 = 10 \text{ x } 6$$

$$U_1 = 60 \text{ ms}^{-1}$$

7. Show that the ratio of velocities of equal masses in an inelastic collision when one of the masses is stationary is $\frac{V_1}{V_2} = \frac{1-e}{1+e}$.

Solution

$$\mathbf{e} = \frac{\text{velocity of separation (after collision)}}{\text{velocity of approach (before collision)}}$$

$$= \frac{(v_2-v_1)}{(u_1-u_2)} = \frac{(v_2-v_1)}{(u_1-0)} = \frac{(v_2-v_1)}{u_1}$$

$\Rightarrow v_2 - v_1 = eu_1$ ------ (1)

From the law of conservation of linear momentum,

$m u_1 = m v_1 + m v_2 \Rightarrow u_1 = v_1 + v_2$ ---------- (2)

Using the equation (2) for u_1 in (1), we get

$$v_2 - v_1 = e (v_1 + v_2)$$

On simplification, we get

$$\frac{v_1}{v_2} = \frac{1-e}{1+e}$$

UNIT – 5 – Motion of system of particles and right bodies

1. A force of $(4\hat{\imath} - 3\hat{\jmath} + 5\hat{k})$ N is applied at a point whose position vector $(7\hat{\imath} + 4\hat{\jmath} - 2\hat{k})$ m. Find the torque of force about the origin.

Solution

$$\vec{r} = 7\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$$

$\vec{F}$ = **4$\hat{\imath}$ - 3 $\hat{\jmath}$ + 5$\hat{k}$**

Torque, $\vec{\tau} = \vec{r} \times \vec{F}$

$$\vec{\tau} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 7 & 4 & -2 \\ 4 & -3 & 5 \end{vmatrix}$$

$\vec{\tau}$ = **$\hat{\imath}$ (20-6) - $\hat{\jmath}$ (35+8) + $\hat{k}$ (-21-16)**

$\vec{\tau}$ = (14$\hat{\imath}$ **-43 $\hat{\jmath}$ -37$\hat{k}$) N m**

2. A cyclist while negotiating a circular path with speed 20 m s^{-1} is found to bend an angle by 30^{o} with vertical. What is the radius of the circular path? (given, g = 10 m s^{-2})

Solution

Speed of the cyclist, v = 20 m s^{-1}

Angle of bending with vertical, θ = 30^{o}

Equation for angle of bending, $\tan\theta = \frac{v^2}{rg}$

Rewriting the above equation for radius $r = \frac{v^2}{\tan\theta\, g}$

Substituting,

$$r = \frac{(20)^2}{(\tan 30^0) \times 10} = \frac{20 \times 20}{(\tan 30^0) \times 10}$$

$$= \frac{400}{\left(\frac{1}{\sqrt{3}}\right) \times 10}$$

r = ($\sqrt{3}$) x 40 = 1.732 x 40

r = 69.28 m

3. **A jester in a circus is standing with his arms extended on a turn table rotating with angular velocity ω. He brings his arms closer to his body so that his moment of inertia is reduced to one third of the original value. Find his new angular velocity. [Given: There is no external torque on the turn table in the given situation.]**

Solution

Let the moment of inertia of the jester with his arms extended be I. As there is no external torque acting on the jester and the turn table, his total angular momentum is conserved. We can write the equation,

$$I_i\, \omega_i = I_f\, \omega_f$$

$$I_i\, \omega_i = \frac{1}{3} I_f\, \omega_f \qquad \left(\because I_f = \frac{1}{3} I_i\right)$$

$$\omega_f = 3\omega_f$$

The above result tells that the final angular velocity is three times that of initial angular velocity.

4. Find the rotational kinetic energy of a ring of mass 9 kg and radius 3 m rotating with 240 rpm about an axis passing through its centre and perpendicular to its plane. (rpm is a unit of speed of rotation which means revolutions per minute)

Solution

The rotational kinetic energy is, KE = $\frac{1}{2}$ I ω^2

The moment of inertia of the ring is, I = MR^2

I = 9 x 3^2 = 9 x 9 = 81 kg m^2

The angular speed of the ring is,

ω = 240 rpm = $\frac{240\times 2\pi}{60}$ rad s^{-1}

$KE = \frac{1}{2} \times 81 \times \left(\frac{240 \times 2\pi}{60}\right)^2 = \frac{1}{2} \times 81 \times (8\pi)^2$

KE = $\frac{1}{2}$ x 81 x 64 x $(\pi)^2$ = 2592 x $(\pi)^2$

KE ≈ 25920J $\because (\pi)^2 \approx 10$

KE = 25.920 kJ.

5. Four round objects namely a ring, a disc, a hollow sphere and a solid sphere with same radius R start to roll down an incline at the same time. Find out which object will reach the bottom first.

Solution

For all the four objects namely the ring, disc, hollow sphere and solid sphere, the radii of gyration K are R, $\sqrt{\frac{1}{2}}$R, $\sqrt{\frac{2}{3}}$R, $\sqrt{\frac{2}{5}}$R. With numerical values the radius of gyration K are 1R, 0.707R, 0.816R, 0.632R respectively. The expression for time taken for rolling has the radius of gyration K in the numerator as per equation 5.63.

$$t = \sqrt{\frac{2h\left(1+\frac{K^2}{R^2}\right)}{g\sin^2\theta}}$$

The one with least value of radius of gyration K will take the shortest time to reach the bottom of the inclined plane. The order of objects reaching the bottom is first, solid sphere; second, disc; third, hollow sphere and last, ring.

UNIT – 6 – Gravitation

1. Calculate the value of g in the following two cases:
(a) If a mango of mass ½ kg falls from a tree from a height of 15 meters, what is the acceleration due to gravity when it begins to fall?

Solution

$g' = g\left(1 - 2\frac{h}{R_e}\right)$

$g' = 9.8\left(1 - \frac{2\times 15}{6400 \times 10^3}\right)$

g' = 9.8 (1 – 0.469 x 10^{-5})

But 1 – 0.00000469 ≅ 1

Therefore g' = g

(b) Consider a satellite orbiting the Earth in a circular orbit of radius 1600 km above the surface of the Earth. What is the acceleration experienced by the satellite due to Earth's gravitational force?

Solution

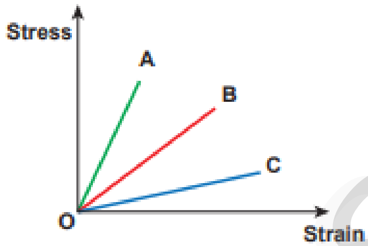
$g' = g\left(1 - 2\frac{h}{R_e}\right)$

$$g' = g\left(1 - \frac{2\times1600\times10^3}{6400\times10^3}\right)$$
$$g' = g\left(1 - \frac{2}{4}\right)$$
$$\mathbf{g' = g}\ \left(1 - \frac{1}{2}\right) = g/2$$

The above two examples show that the acceleration due to gravity is a constant near the surface of the Earth.

UNIT – 7 – Properties of matter

1. **Within the elastic limit, the stretching strain produced in wires A, B, and C due to stress is shown in the figure. Assume the load applied are the same and discuss the elastic property of the material.**



Write down the elastic modulus in ascending order.

Solution

Here, the elastic modulus is Young modulus and due to stretching, stress is tensile stress and strain is tensile strain.

Within the elastic limit, stress is proportional to strain (obey Hooke's law). Therefore, it shows a straight line behaviour. So, Young modulus can be computed by taking slope of these straight lines. Hence, calculating the slope for the straight line, we get

Slope of A > Slope of B > Slope of C Which implies,

Young modulus of C < Young modulus of B < Young modulus of A

Notice that larger the slope, lesser the strain (fractional change in length). So, the material is much stiffer. Hence, the elasticity of wire A is greater than wire B which is greater than C. From this example, we have understood that Young's modulus measures the resistance of solid to a change in its length.

2. **A wire 10 m long has a cross-sectional area 1.25×10^{-4} m^2 . It is subjected to a load of 5 kg. If Young's modulus of the material is 4×10^{10} N m^{-2}, calculate the elongation produced in the wire. Take g = 10 ms^{-2}.**

Solution

We know that $\frac{F}{A} = Y \times \frac{\Delta L}{L}$

$$\Delta L = \left(\frac{F}{A}\right)\left(\frac{L}{Y}\right)$$

$$= \left(\frac{50}{1.25\times10^{-4}}\right)\left(\frac{10}{4\times10^{10}}\right) = 10^{-4}\text{ m}$$

3. A metallic cube of side 100 cm is subjected to a uniform force acting normal to the whole surface of the cube. The pressure is 106 pascal. If the volume changes by 1.5 × 10^{-5} m^3 , calculate the bulk modulus of the material.

Solution

By definition, $K = \frac{\frac{F}{A}}{\frac{\Delta V}{V}} = P\,\frac{\Delta V}{V}$

$K = \frac{10^6\times1}{1.5\times10^{-5}} = 6.67 \times 10^{10}\text{ Nm}^{-2}$.

4. A metal cube of side 0.20 m is subjected to a shearing force of 4000 N. The top surface is displaced through 0.50 cm with respect to the bottom. Calculate the shear modulus of elasticity of the metal.

Solution

Here, L = 0.20 m, F = 4000 N, x = 0.50 cm = 0.005 m and Area A = L2 = 0.04 m2

Therefore,

$\eta_R = \frac{F}{A} \times \frac{L}{x} = \frac{4000}{0.04} \times \frac{0.20}{0.005} = 4 \times 10^6\text{ N m}^{-2}$.

5. A wire of length 2 m with the area of cross-section 10^{-6} m^2 is used to suspend a load of 980 N. Calculate i) the stress developed in the wire ii) the strain and iii) the energy stored. Given: Y=12 × 10^{10}N m^{-2}.

Solution

(i) Stress $= \frac{F}{A} = \frac{980}{10^{-6}} = 98 \times 10^7\text{ N m}^{-2}$

(ii) Strain $= \frac{stress}{Y} = \frac{98 \times 10^7}{12 \times 10^{10}} = 8.17 \times 10^{-3}$

(iii) Since volume = 2 x $10^{-6}$$m^3$,

Energy $= \frac{1}{2}$ (stress x stain) x volume

$= \frac{1}{2}(98 \times 10^7) \times (8.17 \times 10^{-3}) \times 2 \times 10^{-6} = 8$ J.

6. A solid sphere has a radius of 1.5 cm and a mass of 0.038 kg. Calculate the specific gravity or relative density of the sphere.

Solution

Radius of the sphere R = 1.5 cm

mass m = 0.038 kg

Volume of the sphere $V = \frac{4}{3}\pi R^3$

$= \frac{4}{3} \times (3.14) \times (1.5 \times 10^{-2})^3$

$= 1.413 \times 10^{-5} m^3$

Therefore, density

$\rho = \frac{m}{V} = \frac{0.038kg}{1.413 \times 10^{-5}\, m^3}$

Hence, the specific gravity of the sphere

$= \frac{2690}{1000} = 2.69$

7. **Two pistons of a hydraulic lift have diameters of 60 cm and 5 cm. What is the force exerted by the larger piston when 50 N is placed on the smaller piston?**

<u>*Solution*</u>

Since, the diameter of the pistons are given, we can calculate the radius of the piston

$r = \frac{D}{2}$

Area of smaller piston, $A_1 = \pi \left(\frac{5}{2}\right)^2 = \pi\ (2.5)^2$

Area of larger piston, $A_2 = \pi \left(\frac{60}{2}\right)^2 = \pi\ (30)^2$

$F_2 = \frac{A_2}{A_1} \times F_1 = (50\ N) \times \left(\frac{30}{2.5}\right)^2 = 7200\ N$

This means that with the force of 50 N, the force of 7200 N can be lifted.

8. **A metal plate of area $2.5\times10^{-4}\ m^2$ is placed on a 0.25×10^{-3} m thick layer of castor oil. If a force of 2.5 N is needed to move the plate with a velocity $3\times10^{-2}\ m\ s^{-1}$, calculate the coefficient of viscosity of castor oil.**

Given: $A=2.5\times10^{-4}\ m^2$, $dx = 0.25\times10^{-3}m$, F=2.5 N and $dv = 3\times10^{-2}\ m\ s^{-1}$.

<u>*Solution*</u>

$F = \eta A \frac{dv}{dx}$

In magnitude, $\eta = \frac{F\ dx}{A\ dv}$

$= \frac{(2.5\ N)}{(2.5\times10^{-4}m^2} \frac{(0.25\times10^{-3}m)}{(3\times10^{-2}ms^{-1})}$

$= 0.083 \times 10^3\ N\ m^{-2}\ s.$

9. **Let 2.4×10^4 J of work is done to increase the area of a film of soap bubble from 50 cm^2 to 100 cm^2. Calculate the value of surface tension of soap solution.**

<u>*Solution*</u>

A soap bubble has two free surfaces, therefore increase in surface area

$\Delta A = A_2 - A_1 = 2\ (100 - 50) \times 10^{-4}\ m^2 = 100 \times 10^{-4}\ m^2.$

Since, work done $W = T \times \Delta A \Rightarrow T = \frac{W}{\Delta A}$

$= \frac{2.4\times10^{-4} J}{100\times10^{-4}m^2} = 2.4 \times 10^{-2}\ Nm^{-1}.$

10. **If excess pressure is balanced by a column of oil (with specific gravity 0.8) 4 mm high, where R = 2.0 cm, find the surface tension of the soap bubble.**

<u>*Solution*</u>

The excess of pressure inside the soap bubble is $\Delta P = P_2 - P_1 = \frac{4T}{R}$

But $\Delta P = P_2 - P_1 = \rho gh \Rightarrow \rho gh = \frac{4T}{R}$

$\Rightarrow$ Surface tension,

$T = \frac{\rho ghR}{4} = \frac{(800)(9.8)(4\times10^{-3})(2\times10^{-2})}{4}$

$T = 15.68 \times 10^{-2}\ N\ m^{-1}.$

11. **Water rises in a capillary tube to a height of 2.0cm. How much will the water rise through another capillary tube whose radius is one-third of the first tube?**

<u>*Solution*</u>

From equation (7.34), we have

$h \propto \frac{1}{r} \Rightarrow hr$ = constant

Consider two capillary tubes with radius r_1 and r_2 which on placing in a liquid, capillary rises to height h_1 and h_2 , respectively.

Then, $h_1 r_1 = h_2 r_2$ = constant

$\Rightarrow h_2 = \frac{h_1 r_1}{r_2} = \frac{(2\times10^{-2}m)\times r}{\frac{r}{3}} \Rightarrow h_2 = 6 \times 10^{-2}$ m

12. In a normal adult, the average speed of the blood through the aorta (radius r = 0.8 cm) is 0.33 ms^{-1}. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius 0.4 cm. Calculate the speed of the blood through the arteries.

Solution

$a_1 v_1 = 30 a_2 v_2 \Rightarrow \pi r_1^2 v_1 = 30 \pi r_2^2 v_2$

$v_2 = \frac{1}{30}\left(\frac{r_1}{r_2}\right)^2 v_1$

$\Rightarrow v_2 = \frac{1}{30} \times \left(\frac{0.8\times10^{-2}m}{0.4\times10^{-2}m}\right)^2 \times (0.33\ ms^{-1})$

$V_2 = 0.044 ms^{-1}$.

UNIT – 8 – Heat and Thermodynamics

1. A student comes to school by a bicycle whose tire is filled with air at a pressure 240 kPa at 27°C. She travels 8 km to reach the school and the temperature of the bicycle tire increases to 39°C. What is the change in pressure in the tire when the student reaches school?

Solution

We can take air molecules in the tire as an ideal gas. The number of molecules and the volume of tire remain constant. So the air molecules at 27°C satisfies the ideal gas equation $P_1 V_1 = NkT_1$ and at 39°C it satisfies $P_2 V_2 = NkT_2$

But we know

$V_1 = V_2 = V$

$\frac{P_1 V}{P_2 V} = \frac{NkT_1}{NkT_2}$

$\frac{P_1}{P_2} = \frac{T_1}{T_2}$

$P_2 = \frac{T_2}{T_1} P_1$

$P_2 = \frac{312\ K}{300\ K} \times 240 \times 10^3$ Pa = 249.6 kPa.

2. Eiffel tower is made up of iron and its height is roughly 300 m. During winter season (January) in France the temperature is 2°C and in hot summer its average temperature 25°C. Calculate the change in height of Eiffel tower between summer and winter. The linear thermal expansion coefficient for iron $\alpha = 10 \times 10^{-6}$ per °C.

Solution

$\frac{\Delta L}{L_0} = \alpha_L \, \Delta T$

$\Delta L = \alpha_L L_0 \; \Delta T$

ΔL = 10 x 10^{-6} x 300 x 23 = 0.069m = 69 mm.

3. **The power radiated by a black body A is E_A and the maximum energy radiated was at the wavelength λ_A. The power radiated by another black body B is E_B = N E_A and the radiated energy was at the maximum wavelength, ½ λ_A. What is the value of N?**

Solution

According to Wien's displacement law λ_{max} T = constant for both object A and B

$\lambda_A T_A = \lambda_B T_B$. Here $\lambda_B = \frac{1}{2}\lambda_A$

$\frac{T_A}{T_B} = \frac{\lambda_A}{\lambda_B} = \frac{\lambda_A}{\left(\frac{1}{2}\right)\lambda_A} = 2$

$T_B = 2T_A$

From Stefan – Boltzmann law

$\frac{E_B}{E_A} = \left(\frac{T_B}{T_A}\right)^4 = (2)^4 = 16 = N$

Object B has emitted at lower wavelength compared to A. So the object B would have emitted more energetic radiation than A.

4. **A person does 30 kJ work on 2 kg of water by stirring using a paddle wheel. While stirring, around 5 kcal of heat is released from water through its container to the surface and surroundings by thermal conduction and radiation. What is the change in internal energy of the system?**

Solution

Work done on the system (by the person while stirring), W = -30 kJ = −30,000J

Heat flowing out of the system, Q = −5 kcal = −5 × 4184 J =−20920 J

Using First law of thermodynamics

$\Delta U = Q - W$

ΔU = -20,920 J-(-30,000)J

ΔU = -20,920 J+30,000 J = 9080 J

Here, the heat lost is less than the work done on the system, so the change in internal energy is positive.

5. **Jogging every day is good for health. Assume that when you jog a work of 500 kJ is done and 230 kJ of heat is given off. What is the change in internal energy of your body?**

Solution

Work done by the system (body),

W = +500 kJ

Heat released from the system (body),

Q = –230 kJ

The change in internal energy of a body

= ΔU= – 230 kJ – 500 kJ = – 730 kJ

6. **We often have the experience of pumping air into bicycle tyre using hand pump. Consider the air inside the pump as a thermodynamic system having volume V at atmospheric pressure and room temperature, 27°C. Assume that the nozzle of the tyre is blocked and you push the pump to a volume 1/4 of V. Calculate the final temperature of air in the pump? (For air , since the nozzle is blocked air will not flow into tyre and it can be treated as an adiabatic compression). Take γ for air = 1.4**

Solution

Here, the process is adiabatic compression. The volume is given and temperature is to be found. we can use the equation (8.38)

$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$.

T_i = 300 K (273 + 27^{0}C = 300 K)

$V_i = V$ & $V_f = \frac{V}{4}$

$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1}$ = 300 K × $4^{1.4-1}$ = 300 K × 1.741

T_2 ≈ = 522 K or 249°C

This temperature is higher than the boiling point of water. So it is very dangerous to touch the nozzle of blocked pump when you pump air.

7. **500 g of water is heated from 30°C to 60°C. Ignoring the slight expansion of water, calculate the change in internal energy of the water? (specific heat of water 4184 J/kg.K)**

Solution

When the water is heated from 30°C to 60°C, there is only a slight change in its volume. So we can treat this process as isochoric. In an isochoric process the work done by the system is zero. The given heat supplied is used to increase only the internal energy.

$\Delta U = Q = ms_v \Delta T$

The mass of water = 500 g =0.5 kg

The change in temperature = 30K

The heat Q = 0.5×4184×30 = 62.76 kJ

8. **During a cyclic process, a heat engine absorbs 500 J of heat from a hot reservoir, does work and ejects an amount of heat 300 J into the surroundings (cold reservoir). Calculate the efficiency of the heat engine?**

Solution

The efficiency of heat engine is given by

$\eta = 1 - \frac{Q_L}{Q_H}$

$\eta = 1 - \frac{300}{500} = 1 - \frac{3}{5}$

$\eta = 1 - 0.6 = 0.4$

The heat engine has 40% efficiency, implying that this heat engine converts only 40% of the input heat into work.

9. **(a) A steam engine boiler is maintained at 250°C and water is converted into steam. This steam is used to do work and heat is ejected to the surrounding air at temperature 300K. Calculate the maximum efficiency it can have?**

Solution

The steam engine is not a Carnot engine, because all the process involved in the steam engine are not perfectly reversible. But we can calculate the maximum possible efficiency of the steam engine by considering it as a Carnot engine.

$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{300K}{523\ K} = 0.43$

The steam engine can have maximum possible 43% of efficiency, implying this steam engine can convert 43% of input heat into useful work and remaining 57% is ejected as heat. In practice the efficiency is even less than 43%.

10. **There are two Carnot engines A and B operating in two different temperature regions. For Engine A the temperatures of the two reservoirs are 150°C and 100°C. For engine B the temperatures of the reservoirs are 350°C and 300°C. Which engine has lesser efficiency?**

Solution

The efficiency for engine A = $1 - \frac{373}{423} = 0.11$.

Engine A has 11% efficiency

The efficiency for engine B = $1 - \frac{573}{623} = 0.08$

Engine B has only 8% efficiency.

11. A refrigerator has COP of 3. How much work must be supplied to the refrigerator in order to remove 200 J of heat from its interion?

Solution

$COP = \beta = \frac{Q_L}{W}$

$W = \frac{Q_L}{COP} = \frac{200}{3} = 66.67$ J.

UNIT – 9 – Kinetic Theory of Gases

1. **A room contains oxygen and hydrogen molecules in the ratio 3:1. The temperature of the room is 27°C. The molar mass of O_2 is 32 g mol^{-1} and of H_2 is 2 g mol^{-1}. The value of gas constant R is 8.32 J mol^{-1} K^{-1}**

Calculate

(a) rms speed of oxygen and hydrogen molecule

(b) Average kinetic energy per oxygen molecule and per hydrogen molecule

(c) Ratio of average kinetic energy of oxygen molecules and hydrogen molecules

Solution

(a) Absolute Temperature T=27°C =27+273=300 K.

Gas constant R=8.32 J mol^{-1} k^{-1}

For Oxygen molecule: Molar mass M=32 g =32 x 10^{-3} kg mol^{-1}

rms speed $V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.32 \times 300}{32 \times 10^{-3}}}$

$= 483.73\ ms^{-1} \approx 484\ m\ s^{-1}$.

For Hydrogen molecule:

Molar mass M = 2×10^{-3} kg mol^{-1}

rms speed $V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.32 \times 300}{2 \times 10^{-3}}}$

$= 1934\ ms^{-1} \approx 1.93\ km\ s^{-1}$

Note that the rms speed is inversely proportional to M and the molar mass of oxygen is 16 times higher than molar mass of hydrogen. It implies that the rms speed of hydrogen is 4 times greater than rms speed of oxygen at the same temperature. $\frac{1934}{484} \approx 4$.

(b) The average kinetic energy per molecule is $\frac{3}{2}kT$. It depends only on absolute temperature of the gas and is independent of the nature of molecules. Since both the gas molecules are at the same temperature, they have the same average kinetic energy per molecule. k is Boltzmaan constant.

$$\frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21}\ J$$

(c) Average kinetic energy of total oxygen molecules $= \frac{3}{2} N_o kT$ where N_o - number of oxygen molecules in the room Average kinetic energy of total hydrogen molecules $= \frac{3}{2} N_H kT$ where N_H- number of hydrogen molecules in the room.

It is given that the number of oxygen molecules is 3 times more than number of hydrogen molecules in the room. So the ratio of average kinetic energy of oxygen molecules with average kinetic energy of hydrogen molecules is 3:1.

2. **Ten particles are moving at the speed of 2, 3, 4, 5, 5, 5, 6, 6, 7 and 9 m s-1. Calculate rms speed, average speed and most probable speed.**

Solution

The average speed

$$\bar{v} = \frac{2+3+4+5+5+5+6+6+7+9}{10} = 5.2\ ms^{-1}$$

To find the rms speed, first calculate the mean square speed $\overline{v^2}$

$$\overline{v^2} = \frac{2^2+3^2+4^2+5^2+5^2+5^2+6^2+6^2+7^2+9^2}{10} = 30.6\ m^2\ s^{-2}.$$

The rms speed

$$V_{rms} = \sqrt{\overline{v^2}} = \sqrt{30.6} = 5.53\ m\ s^{-1}.$$

The most probable speed is 5 m s^{-1} because three of the particles have that speed.

3. **Calculate the rms speed, average speed and the most probable speed of 1 mole of hydrogen molecules at 300 K. Neglect the mass of electron.**

Solution

The hydrogen atom has one proton and one electron. The mass of electron is negligible compared to the mass of proton.

Mass of one proton = 1.67×10^{-27}kg.

One hydrogen molecule = 2 hydrogen

atoms = 2 × 1.67× 10^{-27}kg.

The average speed

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}} = 1.60\sqrt{\frac{kT}{m}}$$

$$= 1.60\sqrt{\frac{(1.38\times10^{-23})\times(300)}{2(1.67\times10^{-27})}} = 1.78 \times 10^{3}\ ms^{-1}$$

(Boltzmann Constant k = 1.38 × 10^{-23} J K^{-1})

The rms Speed

$$V_{rms} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}$$

$$= 1.73\sqrt{\frac{(1.38\times10^{-23})\times(300)}{2(1.67\times10^{-27})}} = 1.93 \times 10^{3}\ ms^{-1}$$

Most probable speed $v_{mp} = \sqrt{\frac{2kT}{m}} = 1.41\sqrt{\frac{kT}{m}}$

$$= 1.41\sqrt{\frac{(1.38\times10^{-23})\times(300)}{2(1.67\times10^{-27})}} = 1.57 \times 10^{3}\ ms^{-1}$$

Note that $v_{rms} > \bar{v} > v_{mp}$

4. Find the adiabatic exponent γ for mixture of μ1 moles of monoatomic gas and μ2 moles of a diatomic gas at normal temperature (27°C) .

Solution

The specific heat of one mole of a monoatomic gas $C_v = \frac{3}{2}R$

For μ_1 mole, $C_v = \frac{3}{2}\mu_1 R$ $C_p = \frac{5}{2}\mu_1 R$

The specific heat of one mole of a diatomic gas

$$C_v = \frac{5}{2}R$$

For μ_1 mole, $C_v = \frac{5}{2}\mu_2 R$ $C_p = \frac{7}{2}\mu_2 R$

The specific heat of the mixture at constant volume

$$C_v = \frac{3}{2}\mu_1 R + \frac{5}{2}\mu_2 R$$

The specific heat of the mixture at constant pressure

$$C_p = \frac{5}{2}\mu_1 R + \frac{7}{2}\mu_2 R$$

The adiabatic exponent $\gamma = \frac{C_p}{C_v} = \frac{5\mu_1+7\mu_2}{3\mu_1+5\mu_2}$.

5. An oxygen molecule is travelling in air at 300 K and 1 atm, and the diameter of oxygen molecule is 1.2 × 10^{-10}m. Calculate the mean free path of oxygen molecule.

Solution

From (9.26) $\lambda = \frac{1}{\sqrt{2}\pi n d^2}$

We have to find the number density n By using ideal gas law

n = $\frac{N}{V} = \frac{P}{KT} = \frac{101.3 \times 10^3}{1.381 \times 10^{-23} \times 300}$ = 2.449 x 10^{25} molecules / m^3

$\lambda = \frac{1}{\sqrt{2} \times \pi \times 2.449 \times 10^{25} \times (1.2 \times 10^{-10})^2} = \frac{1}{15.65 \times 10^5}$

λ = 0.63 x 10^{-6} m.

UNIT – 10 – Oscillations

1. **Consider a particle undergoing simple harmonic motion. The velocity of the particle at position x_1 is v_1 and velocity of the particle at position x_2 is v_2 . Show that the ratio of time period and amplitude is**

$$\frac{T}{A} = 2\pi \sqrt{\frac{x_2^2 - x_1^2}{v_1^2 x_2^2 - v_2^2 x_1^2}}$$

Solution

Using equation (10.8)

$v = \omega\sqrt{A^2 - x^2} \Rightarrow v^2 = \omega^2 (A^2 - x^2)$

Therefore, at position x_1,

$$v_1^2 = \omega^2(A^2 - x_1^2) \qquad (1)$$

Similarly, at position x_2,

$$v_2^2 = \omega^2(A^2 - x_2^2) \qquad (2)$$

Subtrating (2) from (1), we get

$$v_1^2 - v_2^2 = \omega^2(A^2 - x_1^2) - \omega^2(A^2 - x_2^2)$$

$$= \omega^2(x_2^2 - x_1^2)$$

$$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}} \Rightarrow T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}} \qquad (3)$$

Dividing (1) and (2), we get

$$\frac{v_1^2}{v_2^2} = \frac{\omega^2\left(A^2 - x_1^2\right)}{\omega^2\left(A^2 - x_2^2\right)} \Rightarrow A = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}} \qquad (4)$$

Dividing equation (3) and equation (4), we have

$$\frac{T}{A} = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 x_2^2 - v_2^2 x_1^2}}$$

2. A nurse measured the average heart beats of a patient and reported to the doctor in terms of time period as 0.8 s. Express the heart beat of the patient in terms of number of beats measured per minute.

Solution

Let the number of heart beats measured be f. Since the time period is inversely proportional to the heart beat, then

$$f = \frac{1}{T} = \frac{1}{0.8} = 1.25 s^{-1}$$

One minute is 60 second,

$$(1 \text{ second} = \frac{1}{60} \text{ minute} \Rightarrow 1\ s^{-1} = 60\ min^{-1})$$

$f = 1.25\ s^{-1} \Rightarrow f = 1.25 \times 60\ min^{-1}$ = 75 beats per minute.

3. Calculate the amplitude, angular frequency, frequency, time period and initial phase for the simple harmonic oscillation given below

a. $y = 0.3 \sin (40\pi t + 1.1)$

b. $y = 2 \cos (\pi t)$

c. $y = 3 \sin (2\pi t - 1.5)$

Solution

Simple harmonic oscillation equation is
$y = A\sin(\omega t + \varphi_0)$ or $y = A\cos(\omega t + \varphi_0)$

a. For the wave, $y = 0.3\sin(40\pi t + 1.1)$

Amplitude is $A = 0.3$ unit

Angular frequency $\omega = 40\pi$ rad s^{-1}

Frequency $f = \frac{\omega}{2\pi} = \frac{40\pi}{2\pi} = 20\,Hz$

Time period $T = \frac{1}{f} = \frac{1}{20} = 0.05\,s$

Initial phase is $\varphi_0 = 1.1$ rad

b. For the wave, $y = 2\cos(\pi t)$

Amplitude is $A = 2$ unit

Angular frequency $\omega = \pi$ rad s^{-1}

Frequency $f = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = 0.5\,Hz$

Time period $T = \frac{1}{f} = \frac{1}{0.5} = 2\,s$

Initial phase is $\varphi_0 = 0$ rad

For the wave, $y = 3\sin(2\pi t + 1.5)$

Amplitude is $A = 3$ unit

Angular frequency $\omega = 2\pi$ rad s^{-1}

Frequency $f = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 1\,Hz$

Time period $T = \frac{1}{f} = \frac{1}{1} = 1\,s$

Initial phase is $\varphi_0 = 1.5$ rad

4. Consider two springs whose force constants are 1 N m^{-1} and 2 N m^{-1} which are connected in series. Calculate the effective spring constant (k_s) and comment on k_s.

Solution

$k_1 = 1$ N m^{-1}, $k_2 = 2$ N m^{-1}

$k_s = \frac{k_1 k_2}{k_1 + k_2}$ N m^{-1}

$k_s = \frac{1\times2}{1+2} = \frac{2}{3}$ N m^{-1}

$k_s < k_1$ and $k_s < k_2$

Therefore, the effective spring constant is lesser than both k_1 and k_2 .

5. **Consider two springs with force constants 1 N m^{-1} and 2 N m^{-1} connected in parallel. Calculate the effective spring constant (k_p) and comment on k_p .**

Solution

$k_1 = 1$ N m^{-1}, $k_2 = 2$ N m^{-1}

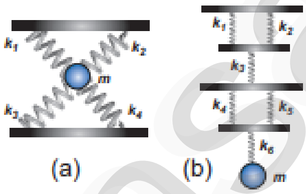
$k_p = k_1 + k_2$ N m^{-1}

$k_p = 1 + 2 = 3$ N m^{-1}

$k_p > k_1$ and $k_p > k_2$

Therefore, the effective spring constant is greater than both k_1 and k_2 .

6. **Calculate the equivalent spring constant for the following systems and also compute if all the spring constants are equal:**



Solution

a. Since k_1 and k_2 are parallel, $k_u = k_1 + k_2$
Similarly, k_3 and k_4 are parallel, therefore, $k_d = k_3 + k_4$
But k_u and k_d are in series,

therefore, $k_{eq} = \frac{k_u k_d}{k_u + k_d}$

If all the spring constants are equal then, $k_1 = k_2 = k_3 = k_4 = k$
Which means, $k_u = 2k$ and $k_d = 2k$

Hence, $k_{eq} = \frac{4k^2}{4k} = k$

b. Since k_1 and k_2 are parallel, $k_A = k_1 + k_2$
Similarly, k_4 and k_5 are parallel, therefore, $k_B = k_4 + k_5$
But k_A, k_3, k_B, and k_6 are in series,

therefore, $\frac{1}{k_{eq}} = \frac{1}{k_A} + \frac{1}{k_3} + \frac{1}{k_B} + \frac{1}{k_6}$

If all the spring constants are equal then, $k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = k$
which means, $k_A = 2k$ and $k_B = 2k$

$$\frac{1}{k_{eq}} = \frac{1}{2k} + \frac{1}{k} + \frac{1}{2k} + \frac{1}{k} = \frac{3}{k}$$

$$k_{eq} = \frac{k}{3}$$

7. **If the length of the simple pendulum is increased by 44% from its original length, calculate the percentage increase in time period of the pendulum.**

<u>***Solution***</u>

Since

$T \propto \sqrt{l}$

Therefore,

$T = \text{Constant}\sqrt{l}$

$\frac{T_f}{T_i} = \sqrt{\frac{l+\frac{44}{100}l}{l}} = \sqrt{1.44} = 1.2$

Therefore, $T_f = 1.2\ T_i = T_i + 20\%\ T_i$

8. **Compute the position of an oscillating particle when its kinetic energy and potential energy are equal.**

<u>***Solution***</u>

Since the kinetic energy and potential energy of the oscillating particle are equal,

$\frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} m\omega^2\ x^2$

$A^2 - x^2 = x^2$

$2x^2 = A^2$

$\Rightarrow x \pm \frac{A}{\sqrt{2}}.$

UNIT – 11 – Waves

1. **The average range of frequencies at which human beings can hear sound waves varies from 20 Hz to 20 kHz. Calculate the wavelength of the sound wave in these limits. (Assume the speed of sound to be 340 m s^{-1}.**

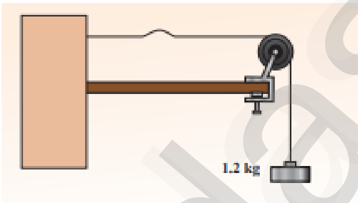
Solution

$$\lambda_1 = \frac{v}{f_1} = \frac{340}{20} = 17 \text{ m}$$

$$\lambda_2 = \frac{v}{f_2} = \frac{340}{20\times10^3} = 0.017 \text{ m}$$

Therefore, the audible wavelength region is from 0.017 m to 17 m when the velocity of sound in that region is 340 m s^{-1}.

2. **Calculate the velocity of the travelling pulse as shown in the figure below. The linear mass density of pulse is 0.25 kg m^{-1}. Further, compute the time taken by the travelling pulse to cover a distance of 30 cm on the string.**



Solution

The tension in the string is T = m g = 1.2 × 9.8 = 11.76 N

The mass per unit length is μ = 0.25 kg m^{-1}

Therefore, velocity of the wave pulse is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{11.76}{0.25}} = 6.858 \text{ m s}^{-1} = 6.8 \text{ m s}^{-1}$$

The time taken by the pulse to cover the distance of 30 cm is

$$t = \frac{d}{v} = \frac{30\times10^{-2}}{6.8} = 0.044\text{s} = 44 \text{ ms where, ms = millisecond.}$$

3. **Calculate the speed of sound in a steel rod whose Young's modulus Y = 2 × 10^{11} N m^{-2} and ρ = 7800 kg m^{-3}.**

Solution

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2\times10^{11}}{7800}} = \sqrt{0.2564 \times 10^8}$$

= 0.506 x 10^4 ms^{-1} = 5 x 10^3 ms^{-1}.

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Therefore, longitudinal waves travel faster in a solid than in a liquid or a gas. Now you may understand why a shepherd checks before crossing railway track by keeping his ears on the rails to safeguard his cattle.

4. **An increase in pressure of 100 kPa causes a certain volume of water to decrease by 0.005% of its original volume. (a) Calculate the bulk modulus of water? (b) Compute the speed of sound (compressional waves) in water?**

Solution

(a) Bulk modules

$$B = V\left|\frac{\Delta P}{\Delta V}\right| = \frac{100 \times 10^3}{0.005 \times 10^{-2}}$$

$$= \frac{100\times10^3}{5\times10^{-5}} = 2000 \text{ MPa, where}$$

MPa is a mega pascal

(b) Speed of sound in water is $\sqrt{\frac{K}{P}} = \sqrt{\frac{2000 \times 10^6}{1000}}$ = 1414 ms^{-1}.

5. **Suppose a man stands at a distance from a cliff and claps his hands. He receives an echo from the cliff after 4 second. Calculate the distance between the man and the cliff. Assume the speed of sound to be 343 m s^{-1}.**

Solution

The time taken by the sound to come back as echo is 2t = 4 ⇒ t = 2 s

∴The distance is d = vt =(343 m s^{-1})(2 s) = 686 m.

6. **The wavelength of two sine waves are λ_1 = 1m and λ_2 = 6m. Calculate the corresponding wave numbers.**

Solution

$k_1 = \frac{2\pi}{1}$ = 6.28 rad m^{-1}

$k_2 = \frac{6\pi}{1}$ = 1.05 rad m^{-1}.

7. **A mobile phone tower transmits a wave signal of frequency 900MHz. Calculate the length of the waves transmitted from the mobile phone tower.**

Solution

Frequency, f = 900MHz = 900 x10^6 Hz

The speed of wave is $c = 3 \times 10^8$ m s^{-1}

$$\lambda = \frac{v}{f} = \frac{3\times10^8}{900\times10^6} = 0.33 \text{ m}$$

8. Consider two sound waves with wavelengths 5 m and 6 m. If these two waves propagate in a gas with velocity 330 ms^{-1}. Calculate the number of beats per second.

Solution

Given λ_1 = 5m and λ_2 = 6m

Velocity of sound waves in a gas is v = 330 ms^{-1}

The relation between wavelength and velocity is $v = \lambda f \Rightarrow f = \frac{v}{\lambda}$

The frequency corresponding to wavelength

λ_1 is $f_1 = \frac{v}{\lambda_1} = \frac{330}{5} = 66$ Hz

The frequency corresponding to wavelength

λ_2 is $f_2 = \frac{v}{\lambda_2} = \frac{330}{6} = 55$ Hz

The number of beats per second is

$|f_1 - f_2| = |66 - 55| = 11$ beats per sec.

9. Two vibrating tuning forks produce waves whose equation is given by $y_1 = 5\sin(240\pi t)$ and $y_2 = 4\sin(244\pi t)$. Compute the number of beats per second.

Solution

Given $y_1 = 5\sin(240\pi t)$ and $y_2 = 4\sin(244\pi t)$

Comparing with $y = A\sin(2\pi f_1 t)$, we get

$2\pi f_1 = 240\pi \Rightarrow f_1 = 120$Hz

$2\pi f_2 = 244\pi \Rightarrow f_2 = 122$Hz

The number of beats produced is $|f_1 - f_2| = |120 - 122| = |-2| = 2$ beats per sec.

10. Compute the distance between anti-node and neighbouring node.

Solution

For n^{th} mode, the distance between antinode and neighbouring node is

$$\Delta x_n = \left(\frac{2n+1}{2}\right)\frac{\lambda}{2} = n\frac{\lambda}{2} = \frac{\lambda}{4}$$

11. Let f be the fundamental frequency of the string. If the string is divided into three segments l_1, l_2 and l_3 such that the fundamental frequencies of each segments be f_1, f_2 and f_3, respectively. Show that

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

Solution

For a fixed tension T and mass density μ, frequency is inversely proportional to the string length i.e.

$$f \propto \frac{1}{l} \Rightarrow f = \frac{v}{2l} \Rightarrow l = \frac{v}{2f}$$

For the first length segment

$$f_1 = \frac{v}{2l_1} \Rightarrow l_1 = \frac{v}{2f_1}$$

For the second length segment

$$f_2 = \frac{v}{2l_2} \Rightarrow l_2 = \frac{v}{2f_2}$$

For the third length segment

$$f_3 = \frac{v}{2l_3} \Rightarrow l_3 = \frac{v}{2f_3}$$

Therefore, the total length

$$l = l_1 + l_2 + l_3$$

$$\frac{v}{2f} = \frac{v}{2f_1} + \frac{v}{2f_2} + \frac{v}{2f} \Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

12. Consider a string in a guitar whose length is 80 cm and a mass of 0.32 g with tension 80 N is plucked. Compute the first four lowest frequencies produced when it is plucked.

Solution

The velocity of the wave

$$v = \sqrt{\frac{T}{\mu}}$$

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The length of the string, $L = 80$ cm $= 0.8$ m
The mass of the string, m $= 0.32$ g
$= 0.32 \times 10^{-3}$ kg
Therefore, the linear mass density,

$$\mu = \frac{0.32 \times 10^{-3}}{0.8} = 0.4 \times 10^{-3} kg\, m^{-1}$$

The tension in the string, $T = 80\ N$

$$v = \sqrt{\frac{80}{0.4 \times 10^{-3}}} = 447.2 \text{ m s}^{-1}$$

The wavelength corresponding to the fundamental frequency f_1 is $\lambda_1 = 2L = 2 \times 0.8 = 1.6$ m

The fundamental frequency f_1 corresponding to the wavelength λ_1

$$f_1 = \frac{v}{\lambda_1} = \frac{447.2}{1.6} = 279.5\, Hz$$

Similarly, the frequency corresponding to the second harmonics, third harmonics and fourth harmonics are

$$f_2 = 2f_1 = 559 \text{ Hz}$$

$$f_3 = 3f_1 = 838.5 \text{ Hz}$$

$$f_4 = 4f_1 = 1118 \text{ Hz}$$

13. A baby cries on seeing a dog and the cry is detected at a distance of 3.0 m such that the intensity of sound at this distance is 10^{-2} W m^{-2}. Calculate the intensity of the baby's cry at a distance 6.0 m.

Solution

I_1 is the intensity of sound detected at a distance 3.0 m and it is given as 10^{-2} W m^{-2}. Let I_2 be the intensity of sound detected at a distance 6.0 m. Then,

$r_1 = 3.0$ m, $r_2 = 6.0$ m

and since, $I \propto \frac{1}{r^2}$

the power output does not depend on the observer and depends on the baby.

Therefore,

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

$$I_2 = I_1 \frac{r_1^2}{r_2^2}$$

$I_2 = 0.25 \times 10^{-2}$ W m^{-2}.

9. The sound level from a musical instrument playing is 50 dB. If three identical musical instruments are played together then compute the total intensity. Calculate the intensity of the sound from each instrument as the threshold of hearing is 10^{-12} W m^{-2}.

Solution

$\Delta L = 10 \log_{10}\left(\frac{I_1}{I_0}\right) = 50$ dB

$\log_{10}\left(\frac{I_1}{I_0}\right) = 5$ dB

$\frac{I_1}{I_0} = 10^5 \Rightarrow I_1 = 10^5 I_0 = 10^5 \times 10^{-12}$ Wm^{-2}

$I_1 = 10^{-7}$ Wm^{-2}

Since three musical instruments are played, therefore, $I_{total} = 3I_1 = 3 \times 10^{-7}$ Wm^{-2}.

10. If a flute sounds a note with 450Hz, what are the frequencies of the second, third, and fourth harmonics of this pitch? If the clarinet sounds with a same note as 450Hz, then what are the frequencies of the lowest three harmonics produced?

Solution

For a flute which is an open pipe, we have

Second harmonics $f_2 = 2 f_1 = 900$ Hz

Third harmonics $f_3 = 3 f_1 = 1350$ Hz

Fourth harmonics $f_4 = 4 f_1 = 1800$ Hz

For a clarinet which is a closed pipe, we have

Second harmonics $f_2 = 3 f_1 = 1350$ Hz

Third harmonics $f_3 = 5 f_1 = 2250$ Hz

Fourth harmonics $f_4 = 7 f_1 = 3150$ Hz

11. If the third harmonics of a closed organ pipe is equal to the fundamental frequency of an open organ pipe, compute the length of the open organ pipe if the length of the closed organ pipe is 30 cm.

Solution

Let l_2 be the length of the open organ pipe, with l_1 =30 cm the length of the closed organ pipe.

It is given that the third harmonic of closed organ pipe is equal to the fundamental frequency of open organ pipe.

The third harmonic of a closed organ pipe is

$$f_2 = \frac{v}{\lambda_2} = \frac{3v}{4l_1} = 3f_1$$

The fundamental frequency of open organ pipe is

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2l_2}$$

Therefore

$$\frac{v}{2l_2} = \frac{3v}{4l_1} \Rightarrow l_2 = \frac{2l_1}{3} = 20 \text{ cm}$$

12. A frequency generator with fixed frequency of 343 Hz is allowed to vibrate above a 1.0 m high tube. A pump is switched on to fill the water slowly in the tube. In order to get resonance, what must be the minimum height of the water?
(speed of sound in air is 343 m s^{-1})

Solution

The wavelength, $\lambda = \frac{c}{f}$

$$\lambda = \frac{343\ ms^{-1}}{343\ Hz} = 1.0\ \text{m}$$

Let the length of the resonant columns be L_1, L_2 and L_3.

The first resonance occurs at length L_1

$L_1 = \frac{\lambda}{4} = \frac{1}{4} = 0.25$ m

The second resonance occurs at length L_2

$L_2 = \frac{3\lambda}{4} = \frac{3}{4} = 0.75$ m

The third resonance occurs at length

$L_3 = \frac{5\lambda}{4} = \frac{5}{4} = 1.25$ m

and so on.

Since total length of the tube is 1.0 m the third and other higher resonances do not occur. Therefore, the minimum height of water H_{min} for resonance is,

$H_{min} = 1.0\ \text{m} - 0.75\ \text{m} = 0.25\ \text{m}$

13. A student performed an experiment to determine the speed of sound in air using the resonance column method. The length of the air column that resonates in the fundamental mode with a tuning fork is 0.2 m. If the length is varied such that the same tuning fork resonates with the first overtone at 0.7 m. Calculate the end correction.

Solution

End correction

$$e = \frac{L_2 - 3L_1}{2} = \frac{0.7 - 3(0.2)}{2} = 0.05\text{m}.$$

14. Consider a tuning fork which is used to produce resonance in an air column. A resonance air column is a glass tube whose length can be adjusted by a variable piston. At room temperature, the two successive resonances observed are at 20 cm and 85 cm of the column length. If the frequency of the length is 256 Hz, compute the velocity of the sound in air at room temperature.

Solution

Given two successive length (resonance) to be L_1 = 20 cm and L_2 = 85 cm

The frequency is f = 256 Hz

$v = f\lambda = 2f\,\Delta L = 2f\,(L_2 - L_1)$

$= 2 \times 256 \times (85 - 20) \times 10^{-2}$ m s^{-1}

$v = 332.8$ m s^{-1}

15. A sound of frequency 1500 Hz is emitted by a source which moves away from an observer and moves towards a cliff at a speed of 6 ms^{-1}.

(a) Calculate the frequency of the sound which is coming directly from the source.

(b) Compute the frequency of sound heard by the observer reflected off the cliff. Assume the speed of sound in air is 330 m s^{-1}.

Solution

(a) Source is moving away and observer is stationary, therefore, the frequency of sound heard directly from source is

$$f' = \left[\frac{v}{v+v_s}\right] f = \left[\frac{330}{330+6}\right] \times 1500 = 1473\,\text{Hz}$$

(b) Sound is reflected from the cliff and reaches observer, therefore,

$$f' = \left[\frac{v}{v-v_s}\right] f = \left[\frac{330}{330-6}\right] \times 1500 = 1528\,\text{Hz}$$

16. An observer observes two moving trains, one reaching the station and other leaving the station with equal speeds of 8 m s^{-1}. If each train sounds its whistles with frequency 240 Hz, then calculate the number of beats heard by the observer.

Solution

Observer is stationary

(i) Source (train) is moving towards an observer:

The observed frequency due to train arriving station is

$$f_{in} = \left[\frac{v}{v - v_s}\right] f = \left[\frac{330}{330 - 8}\right] \times 240 = 246\,\text{Hz}$$

(ii) Source (train) is moving away from an observer:

The observed frequency due to train leaving station is

$$f_{out} = \left[\frac{v}{v + v_s}\right] f = \left[\frac{330}{330 + 8}\right] \times 240 = 234\,\text{Hz}$$

So the number of beats = $| f_{in} - f_{out} |$ = (246–234) = 12

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