Class : 12

Register Number

COMMON HALF YEARLY EXAMINATION - 2024-25
BUSINESS MATHEMATICS
AND STATISTICS

Time Allowed : 3.00 Hours]

PART – I

[Max. Marks : 90

$20 \times 1 = 20$

1. Answer all the questions by choosing the correct answer from the given 4 alternatives

2. Write question number, correct option and corresponding answer

3. Each question carries 1 mark

1. If $T = \begin{matrix} A \\ B \end{matrix} \begin{pmatrix} 0.7 & 0.3 \\ 0.6 & x \end{pmatrix}$ is a transition probability matrix, then the value of x is

   (a) 0.2   (b) 0.3   (c) 0.4   (d) 0.7

2. If $\rho(A) \neq \rho(A, B)$, then the system is

   (a) Consistent and has infinitely many solutions   (b) Consistent and has a unique solution

   (c) inconsistent   (d) consistent

3. $\int \frac{2x^3}{4+x^4} dx$ is

   (a) $\log|4 + x^4| + c$ (b) $\frac{1}{2}\log|4 + x^4| + c$ (c) $\frac{1}{4}\log|4 + x^4| + c$ (d) $\log\left|\frac{2x^3}{4+x^4}\right| + c$

4. For a demand function p, if $\int \frac{dp}{p} = k \int \frac{dx}{x}$ then k is equal to

   (a) η_d (b) $- \eta_d$ (c) $\frac{-1}{\eta_d}$ (d) $\frac{1}{\eta_d}$

5. The profit of a function p(x) is maximum when

   (a) MC – MR = 0   (b) MC=0   (c) MR=0   (d) MC+MR=0

6. The differential equation $\left(\frac{dx}{dy}\right)^3 + 2y^{\frac{1}{2}} = x$ is

   (a) of order 2 and degree 1   (b) of order 1 and degree 3

   (c) of order 1 and degree 6   (d) of order 1 and degree 2

7. For the given points (x_0, y_0) and (x_1, y_1) the Lagrange's formula is

   (a) $y(x) = \frac{x-x_1}{x_0-x_1}y_0 + \frac{x-x_0}{x_1-x_0}y_1$ (b) $y(x) = \frac{x_1-x}{x_0-x_1}y_0 + \frac{x-x_0}{x_1-x_0}y_1$

   (c) $y(x) = \frac{x-x_1}{x_0-x_1}y_1 + \frac{x-x_0}{x_1-x_0}y_0$ (d) $y(x) = \frac{x_1-x}{x_0-x_1}y_1 + \frac{x-x_0}{x_1-x_0}y_0$

8. If h=1, then $\Delta(x^2) =$

   (a) 2x   (b) 2x –1   (c) 2x + 1   (d) 1

9. Given E(X) = 5 and E(Y) = –2, then E(X – Y) is

   (a) 3   (b) 5   (c) 7   (d) -2

10. A variable which can assume finite or countably infinite number of values is known as

    (a) continuous   (b) discrete   (c) qualitative   (d) none of them

11. The parameters of the normal distribution $f(x) = \left(\frac{1}{\sqrt{72\pi}}\right)\frac{e^{-(a-10)^2}}{72}$ $-\infty < x < \infty$

    (a) (10,6)   (b) (10,36)   (c) (6,10)   (d) (36,10)

12. Let z be a standard normal variable. If the area to the right of z is 0.8413, then the value of z must be:

    (a) 1.00   (b) –1.00   (c) 0.00   (d) –0.41

13. Type II error is

    (a) Accept H_0 when it is wrong   (b) Accept H_0 when it is true

    (c) Reject H_0 when it is true   (d) Reject H_0 when it is false

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14. A finite subset of statistical individuals in a population is called ....   (d) census

   (a) a sample   (b) a population   (c) universe

15. The components of a time series which is attached to short term fluctuation is

   (a) Secular trend   (b) Seasonal variations   (c) Cyclic variation   (d) Irregular variation

16. R is calculated using

   (a) $x_{max} - x_{min}$ (b) $x_{min} - x_{max}$ (c) $\bar{x}_{max} - \bar{x}_{min}$ (d) $\bar{\bar{x}}_{max} - \bar{\bar{x}}_{min}$

17. Solution for transportation problem using _____ method is nearer to an optimal solution.

   (a) NWCM   (b) LCM   (c) VAM   (d) Row Minima

18. A type of decision –making environment is

   (a) certainty   (b) uncertainty   (c) risk   (d) all of the above

19. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} - \sin x \, dx$

   (a) $\frac{1-\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1+\sqrt{3}}{2}$

20. The integrating factor of $\frac{dy}{dx} - \frac{y}{\cot x} = x$ is

   (a) cos x   (b) sin x   (c) sec x   (d) tan x

PART – II

                                                        7 x 2 = 14

   1.   **Answer any 7 questions**

   2.   **Each question carries 2 marks**

   3.   **Question number 30 is compulsory**

21. Integrate with respect to x : $\frac{6x+7}{\sqrt{3x^2+7x-1}}$

22. Find the rank of the matrix $\begin{pmatrix} 0 & -1 & 5 \\ 2 & 4 & -6 \\ 1 & 1 & 5 \end{pmatrix}$

23. Calculate the producer's surplus at x = 5 for the supply function p = 7 + x .

24. Solve $(D^2 - 3D - 4)y = 0$

25. A person wants to invest in one of three alternative investment plans: Stock, Bonds and Debentures. It is assumed that the person wishes to invest all of the funds in a plan. The pay-off matrix based on three potential economic conditions is given in the following table:

Alternative	Economic conditions		
	High growth(Rs.)	Normal growth(Rs.)	Slow growth (Rs.)s
Stocks	10000	7000	3000
Bonds	8000	6000	1000
Debentures	6000	6000	6000

Determine the best investment plan using each of following criteria i) Maxmin ii) Minimax

26. Prove that if E(X) = 0, then $V(X) = E(X^2)$.

27. Write down the conditions in which the Normal distribution is a limiting case of binomial distribution.

28. Define critical region.

29. Define seasonal index.

30. Find the missing term

x	0	1	2
f(x)	3	-	5

1. Answer any 7 questions
2. Each question carries 3 marks　　　　　PART – III
3. Question number 40 is compulsory

$7 \times 3 = 21$

31. A commodity was produced by using 3 units of labour and 2 units of capital, the total cost is ₹62. If the commodity had been produced by using 4 units of labour and one unit of capital, the cost is ₹56. Using determinant method find the cost per unit of labour and capital?

32. Integrate with respect to x : $\dfrac{1}{\sqrt{x^2+6x+13}}$

33. What do you mean by balanced transportation problem?

34. Solve : $\dfrac{dy}{dx} + \dfrac{3x^2}{1+x^3} y = \dfrac{1+x^2}{1+x^3}$

35. Given $y_3 = 2 , y_4 = -6 , y_5 = 8 , y_6 = 9$ and $y_7 = 17$ Calculate $\Delta^4 y_3$

36. The number of miles an automobile tire lasts before it reaches a critical point in tread wear can be represented by a p.d.f. $f(x) = \begin{cases} \frac{1}{30} e^{-\frac{x}{30}}, & \text{for } x > 0 \\ 0, & \text{for } x \le 0 \end{cases}$ Find the expected number of miles (in thousands) a tire would last until it reaches the critical tread wear point.

37. The mortality rate for a certain disease is 7 in 1000. What is the probability for just 2 deaths on account of this disease in a group of 400? Given $(e^{-2.8} = 0.06)$

38. In a sample of 400 population from a village 230 are found to be eaters of vegetarian items and the rest non-vegetarian items. Compute the standard error assuming that both vegetarian and non-vegetarian foods are equally popular in that village?

39. The following data gives the readings for 8 samples of size 6 each in the production of a certain product. Find the control limits using mean chart.

Sample	1	2	3	4	5	6
Mean	300	342	351	319	326	333
Range	25	37	20	28	30	22

Given for n = 6, $A_2 = 0.483$,

40. Find the Area under the curve $y = \dfrac{1}{1+x^2}$ about x-axis $x = -1$ to $x = 1$.

PART – IV

1. Answer all the questions
2. Each question carries 5 marks

$7 \times 5 = 35$

41. a) Consider the following transportation problem

	D_1	D_2	D_3	D_4	Availability
O_1	5	8	3	6	30
O_2	4	5	7	4	50
O_3	6	2	4	6	20
Requirement	30	40	20	10	

Determine initial basic feasible solution by VAM

(OR)

b) Suppose that the quantity demanded $Q_d = 29 - 2p - 5\dfrac{dp}{dt} + \dfrac{d^2 p}{dt^2}$ and quantity supplied $Q_s = 5 + 4p$ where p is the price. Find the equilibrium price for market clearance.

42. a) Using integration find the area of the circle whose center is at the origin and the radius is a units.

(OR)

b) Evaluate the integral as the limit of the sum : $\int_1^3 (2x + 3)dx$

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43. a) The following table shows the number of salesmen working for a certain concern:

Year	1992	1993	1994	1995	1996
No.of salesman	46	48	42	56	52

Use the method of least squares to fit a straight line and estimate the number of salesmen in 1997.

(OR)

b) The amount of bread (in hundreds of pounds) x that a certain bakery is able to sell in a day is found to be a numerical valued random phenomenon, with a probability function specified by the probability density function f (x) is given by $f(x) = \begin{cases} Ax & \text{,for } 0 \leq x < 10 \\ A(20-x), & \text{for } 10 \leq x < 20 \\ 0, & \text{otherwise} \end{cases}$

(a) Find the value of A.

(b) What is the probability that the number of pounds of bread that will be sold tomorrow is

    (i) More than 10 pounds,   (ii) Less than 10 pounds, and   (iii) Between 5 and 15 pounds?

44. a) Using the following data, construct Fisher's Ideal Index Number and Show that it satisfies Factor Reversal Test and Time Reversal Test ?

Commodities	Price		Quantity	
	Base Year	Current Year	Base Year	Current Year
Wheat	6	10	50	56
Ghee	2	2	100	120
Firewood	4	6	60	60
Sugar	10	12	30	24
Cloth	8	12	40	36

(OR)

b) If 18% of the bolts produced by a machine are defective, determine the probability that out of the 4 bolts chosen at random

    (i) exactly one will be defective   (ii) none will be defective   (iii) atmost 2 will be defective

45. a) The demand equation for a products is $x = \sqrt{100 - p}$ and the supply equation is $x = \frac{P}{2} - 10$. Determine the consumer's surplus and producer's surplus, under market equilibrium.

(OR)

b) Investigate for what values of 'a' and 'b' the following system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + az = b$ have (i) no solution   (ii) a unique solution   (iii) an infinite number of solutions.

46. a) Use Lagrange's formula and estimate from the following data the number of workers getting income not exceeding Rs. 26 per month.

Income not exceeding (₹)	15	25	30	35
No.of workers	36	40	45	48

(OR)

b) The mean breaking strength of cables supplied by a manufacturer is 1,800 with a standard deviation 100. By a new technique in the manufacturing process it is claimed that the breaking strength of the cables has increased. In order to test this claim a sample of 50 cables is tested. It is found that the mean breaking strength is 1,850. Can you support the claim at 0.01 level of significance.

47. a) Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin^3 x}}{\sqrt{\sin^3 x} + \sqrt{\cos^3 x}} dx$

(OR)

b) Solve : $x^3 dx = y^2 dy - xy^2 dx - yx^2 dy$.

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