Chengalpattu District
12 - Business Mathematics and Statistics - Key Answer
Part - I

Q.No.	Option	Answer
1.	c)	2
2.	c)	0
3.	c)	9log|x – 3| – log|x + 1| + c
4.	b)	$\frac{\sqrt{\pi}}{2}$
5.	a)	$\frac{1}{2}$ sq. units
6.	c)	9
7.	b)	of order 1 and degree 3
8.	d)	x dx + y dy = 0
9.	a)	1 + Δ
10.	a)	linear
11.	c)	probability density function
12.	a)	zero
13.	c)	Poisson
14.	d)	all of the above statements are true
15.	b)	finite subset
16.	c)	-2.33
17.	d)	Family budget method formula
18.	b)	Family budget method formula
19.	a)	top left corner
20.	c)	1 or 0

Part - II

21.

$$\left(\begin{array}{ccc|} 3 & -2 & 6 \\ 0 & 0 & -2 \end{array}\right) R_2 \to R_2 - 2R_1$$

$$\therefore \rho([A, B]) = 2, \quad \rho(A) = 1$$

$$\rho(A) \neq \rho([A, B])$$

∴The given system is inconsistent and has no solution.

22.

$$MC = 300x^{\frac{2}{5}}$$

$$C = \int 300x^{\frac{2}{5}} dx + k = 300\frac{x^{\frac{7}{5}}}{\frac{7}{5}} + k$$

fixed cost = 0, $k = 0$

$$C = \frac{1500}{7}x^{\frac{7}{5}}$$

$$\text{Average cost} = \frac{C}{x} = \frac{1500}{7}x^{\frac{2}{5}}$$

23. Equation of the straight line passing through the origin (0, 0) is

$y = mx$ .... (1)

$$\frac{dy}{dx} = m(1) \Rightarrow m = \frac{dy}{dx}$$

$$y = x\frac{dy}{dx}$$

24.

$$\Delta^2\left(\frac{1}{x}\right) = \Delta\left(\Delta\left(\frac{1}{x}\right)\right)$$

Now $\Delta\left[\frac{1}{x}\right] = \frac{1}{x+1} - \frac{1}{x}$

$$\Delta^2\left(\frac{1}{x}\right) = \Delta\left(\frac{1}{1+x} - \frac{1}{x}\right)$$

$$= \Delta\left(\frac{1}{1+x}\right) - \Delta\left(\frac{1}{x}\right)$$

Similarly $\Delta^2\left(\frac{1}{x}\right) = \frac{2}{x(x+1)(x+2)}$

25.

$P_X(x)=\frac{1}{6}$, for $x = 1,2,3,4,5$ and 6

$$E(X)=\sum_x x\,P_X(x)$$

$$=\left(1\times\frac{1}{6}\right)+\left(2\times\frac{1}{6}\right)+\left(3\times\frac{1}{6}\right)+\left(4\times\frac{1}{6}\right)+\left(5\times\frac{1}{6}\right)+\left(6\times\frac{1}{6}\right)$$

$$=\frac{1}{6}(1+2+3+4+5+6)=\frac{7}{2}=3.5$$

26.

1. Binomial distribution is symmetrical if p = q = 0.5. It is skew symmetric if p ≠ q. It is positively skewed if p < 0.5 and it is negatively skewed if p > 0.5

2. For Binomial distribution, variance is less than mean Variance npq = (np)q < np

27. The hypothesis which is complementary to the null hypothesis is called as the alternative hypothesis, denoted by H_1

28.

3-yearly moving Total(₹)	3-yearly moving averages(₹)
----	----
45510	15170
52010	17336.667
63040	21013.333
79470	26490
94050	31350
102450	34150
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29.

Minimize $Z=\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\,x_{ij}$

Subject to the constrains

$$\sum_{i=1}^{n} x_{ij}=1,\ j=1,2,.....n$$

$$\sum_{j=1}^{n} x_{ij}=1,\ i=1,2,.....n \quad \text{and } x_{ij}=0$$

(or) 1 for all *i,j*

30.

Successive derivatives	Repeated integrals
Take $u=x^2-2x+5$	and $dv=e^{-x}dx$
	$v=-e^{-x}$
$u'=2x-2$	$v_1=e^{-x}$
$u''=2$	$v_2=-e^{-x}$

$$\int\left(x^2-2x+5\right)e^{-x}dx$$

$$=\int u\,dv$$

$$=uv-u'v_1+u''v_2-u'''v_3+\dots$$

$$=\left(x^2-2x+5\right)\left(-e^{-x}\right)-(2x-2)e^{-x}+2\left(-e^{-x}\right)+c$$

$$=e^{-x}\left(-x^2-5\right)+c$$

Part - III

31.

Matrix A	Elementary Transformation
$A=\begin{pmatrix}0&1&2&1\\1&2&3&2\\3&1&1&3\end{pmatrix}$	
$A\sim\begin{pmatrix}1&2&3&2\\0&1&2&1\\3&1&1&3\end{pmatrix}$	$R_1\leftrightarrow R_2$
$\sim\begin{pmatrix}1&2&3&2\\0&1&2&1\\0&-5&-8&-3\end{pmatrix}$	$R_3\to R_3-3R_1$
$\sim\begin{pmatrix}1&2&3&2\\0&1&2&1\\0&0&2&2\end{pmatrix}$	$R_3\to R_3+5R_2$

$\therefore \rho(A)=3.$

32.

By rationalisation,

$$\frac{1}{x-\sqrt{x^2-1}}=\frac{1}{x-\sqrt{x^2-1}}\times\frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}=x+\sqrt{x^2-1}$$

$$\int\frac{1}{x-\sqrt{x^2-1}}dx$$

$$=\int\left[x+\sqrt{x^2-1}\right]dx$$

$$=\int x\,dx+\int\sqrt{x^2-1}\,dx$$

$$=\frac{x^2}{2}+\frac{x}{2}\sqrt{x^2-1}-\frac{1}{2}\log\left|x+\sqrt{x^2-1}\right|+c$$

33.

$$= 2\int_0^a y\,dx$$

$$= 2\int_0^a \sqrt{4ax}\,dx$$

$$= 2(2\sqrt{a})\int_0^a x^{\frac{1}{2}}\,dx$$

$$= 4\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$$

$$= (4\sqrt{a})\frac{2}{3}a^{\frac{3}{2}} = \frac{8}{3}a^2 \text{ sq.units}$$

34.

$$(D^2 - 4D + 5)y = 0$$

The auxiliary equation is

$$m^2 - 4m + 5 = 0$$

$$\Rightarrow (m-2)^2 - 4 + 5 = 0$$

$$(m-2)^2 = -1$$

$$m - 2 = \pm\sqrt{-1}$$

m = $2 \pm i$, it is if the form $\alpha \pm i\beta$

$\therefore$ C.F= $e^{2x}[A\cos x + B\sin x]$

The general solution is

$$y = e^{2x}[A\cos x + B\sin x]$$

35.

$$\Delta^4 y_0 = 0, \therefore (E-1)^4 y_0 = 0$$

$$(E^4 - 4E^3 + 6E^2 - 4E + 1)y_0 = 0$$

$$E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4Ey_0 + y_0 = 0$$

$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

$$81 - 4y_3 + 6(9) - 4(3) + 1 = 0$$

$$81 - 4y_3 + 54 - 12 + 1 = 0$$

$$4y_3 = 124$$

$$\Rightarrow y_3 = 31$$

36.

(i) $E(a) = a$, where 'a' is a constant

(ii) $E(aX) = aE(X)$

(iii) $E(aX+b) = aE(X)+b$, where 'a' and 'b' are constants.

(iv) If $X \geq 0, then\ E(X) \geq 0$

(v) $V(a) = 0$

(vi) If X is random variable, then $V(aX+b) = a^2V(X)$

37.

p = 5/100 = 0.05 and n = 120

$\Rightarrow \lambda = np = (0.05)(120) = 6$

$$= \frac{e^{-6}6^0}{0!} = 0.0025$$

38.

Sample proportion p = $\frac{36}{600}$ = 0.06

Population proportion P = 4% = 0.04

Q = 1 – P = 1 – 0.04 = 0.96

$$S.E = \sqrt{\frac{PQ}{N}} = \sqrt{\frac{(0.04)(0.96)}{600}}$$

$$= \sqrt{0.000064} = 0.008$$

39.

Minimum payoff	Maximum payoff
2000	8000
3500	5000
4000	5000

Max (2000, 3500, 4000) = 4000.
crop C as the best crop.
Min (8000, 5000, 5000) = 5000
crop B and crop C as the best crop.

40.

Commodities	Price: Base year (p_0)	Price: Current year (p_1)	Quantity (q_0)	p_0q_0	p_1q_0
Rice	32	48	25	800	1200
Sugar	25	42	10	250	420
Oil	54	85	6	324	510
Coffee	250	460	1	250	460
Tea	175	275	2	350	550
			Total	1974	3140

Cost of Living Index Number

$$= \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 = \frac{3140}{1974} \times 100 = 159.0679$$

Part - IV

41. a)

$x = 8,\ p_0 = 36$

$$CS = \int_0^8 (100 - x^2)\,dx - (8)(36)$$

$$= \left(100x - \frac{x^3}{3}\right)_0^8 - 288$$

$$= 800 - \frac{512}{3} - 288 = \frac{1024}{3}$$

$$= \frac{1024}{3} \text{ units}$$

$$PS = 8\,(36) - \int_0^8 2(x+10)\,dx$$

$$= 288 - 2\left[\frac{x^2}{2} + 10x\right]_0^8$$

$$= 288 - 2\left[\frac{64}{2} + 80\right]$$

= 288 - 224 = 64

41. b)

Augmented matrix $[A,B]$	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & a & b \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & a-1 & b-6 \end{pmatrix}$	$R_2 \to R_2 - R_1,$ $R_3 \to R_3 - R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & a-3 & b-10 \end{pmatrix}$	$R_3 \to R_3 - R_2$

Case (i) For no solution:

The system possesses no solution only when $\rho(A) \neq \rho([A,B])$ which is possible only when $a - 3 = 0$ and $b - 10 \neq 0$

Hence for $a = 3,\ b \neq 10$, the system possesses no solution.

Case (ii) For a unique solution:

The system possesses a unique solution only when $\rho(A) = \rho([A,B])$ = number of unknowns.

i.e when $\rho(A) = \rho([A,B]) = 3$

Hence for $a \neq 3$ and $b \in R$, the system possesses a unique solution.

Case (iii) For an infinite number of solutions:

The system possesses an infinite number of solutions only when

$\rho(A) = \rho([A,B]) <$ number of unknowns

i,e when $\rho(A) = \rho([A,B]) = 2 < 3$ (number of unknowns) which is possible only when $a - 3 = 0,\quad b - 10 = 0$

Hence for $a = 3,\ b = 10$, the system possesses infinite number of solutions.

42. a)

$\int_1^3 (2x+3)\,dx \qquad \int_a^b f(x)\,dx = \lim_{\substack{n\to\infty \\ h\to 0}} \sum_{r=1}^{n} h\, f(a+rh)$

Here $a = 1,\ b = 3,\ h = \frac{3-1}{n} = \frac{2}{n} \qquad f(x) = 2x + 3$

$$f(a+rh) = f\left(1 + \frac{2r}{n}\right)$$

$$= 2\left(1 + \frac{2r}{n}\right) + 3 = \left(5 + \frac{4r}{n}\right)$$

$$\int_1^3 f(x)\,dx = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{2}{n}\left(5 + \frac{4r}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{10}{n} + \frac{8r}{n^2}\right)$$

$$= \lim_{n\to\infty} \left[\frac{10}{n}\sum_{r=1}^{n} 1 + \frac{8}{n^2}\sum_{r=1}^{n} r\right]$$

$$= \lim_{n\to\infty} \left[\frac{10}{n} n + \frac{8}{n^2}\frac{n(n+1)}{2}\right]$$

$$= 10 + 4\lim_{n\to\infty}\left(1 + \frac{1}{n}\right) = 10 + 4 = 14$$

42 b)

$n = 4$	$p = \frac{1}{2}$	$p + q = 1$ $q = 1 - p$ $q = 1 - \frac{1}{2} = \frac{1}{2}$

$P(X=x) = p(x) = {}^nC_x\, p^x q^{n-x}$

(i) Probability that atleast one boy

$P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - \left[4C0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4-0}\right]$$

$$= 1 - \left[4C0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4}\right]$$

$$= 1 - \left[(1)(1)\left(\frac{1}{16}\right)\right]$$

$$= 1 - 0.0625$$

$$= 0.9375$$

So out of 750 families the number of families would be expected to have atleast one boy is $= 750 \times 0.9375 = 703.12 \cong 703$

(ii) Probability that atmost 2 girls = P(2G, 2B) + P(1G, 3B) + P(0G, 4B)

$= P(X = 2) + P(X = 3) + P(X = 4)$

$$= 4C2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} + 4C3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{4-3} + 4C4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{4-4}$$

$$= {}^4C_2\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^4 + {}^4C_4\left(\frac{1}{2}\right)^4$$

$$= \frac{1}{16}[6 + 4 + 1] = \frac{11}{16}$$

Thus out of 750 families, the number of families would be expected to have atmost 2 girls is $= 750 \times \frac{11}{16} = 515.625 \cong 516$

(iii) Probability that children of both sexes = P(1B, 3G) + P(2B, 2G) + P(3B, 1G)

$= P(X = 1) + P(X = 2) + P(X = 3)$

$$= 4C1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{4-1} + 4C2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} + 4C3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{4-3}$$

$$= {}^4C_1\left(\frac{1}{2}\right)^4 + {}^4C_2\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^4$$

$$= \frac{1}{16}[4 + 6 + 4] = \frac{14}{16} = \frac{7}{8}$$

Thus out of 750 families, the number of families would be expected to have children of both sexes is $= 750 \times \frac{7}{8} = 656.25 \cong 656$

43 a)

$$T = \begin{matrix} & A & B \\ A & 0.6 & 0.4 \\ B & 0.2 & 0.8 \end{matrix}$$

The current position of A and B in the market is

$$\begin{pmatrix} A & B \\ 0.5 & 0.5 \end{pmatrix}$$

After one week

The shares of A and B are given by

$$\begin{pmatrix} 0.5 & 0.5 \end{pmatrix} \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = \begin{pmatrix} 0.3+0.1 & 0.2+0.4 \end{pmatrix} = \begin{pmatrix} 0.4 & 0.6 \end{pmatrix}$$

After one week

The shares of A and B are given by

$$\begin{pmatrix} 0.5 & 0.5 \end{pmatrix} \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = \begin{pmatrix} 0.3+0.1 & 0.2+0.4 \end{pmatrix} = \begin{pmatrix} 0.4 & 0.6 \end{pmatrix}$$

A = 40%, B=60%

After two weeks

The shares of A and B are given by

$$\begin{pmatrix} 0.4 & 0.6 \end{pmatrix} \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = \begin{pmatrix} 0.24+0.12 & 0.16+0.48 \end{pmatrix} = \begin{pmatrix} 0.36 & 0.64 \end{pmatrix}$$

A = 36 %, B = 64%

$$\begin{pmatrix} A & B \end{pmatrix} \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = \begin{pmatrix} A & B \end{pmatrix}$$

(0.6A + 0.2B 0.4A + 0.8B) = (A B)
Share A is 33% and share of B is 67%.

43 b)

	D_1	D_2	D_3	D_4	
O_1	(1) 2	(5) 3	11	7	6
O_2	1	0	6	(1) 1	1
O_3	(6) 5	8	(3) 15	(1) 9	10
	7	5	3	2	

Transportation schedule:
O1 → D1, O1 → D2, O2 → D4, O3 → D1, O3 → D3, O3 → D4

Total cost = (1 × 2) + (5 × 3) + (1 × 1) + (6 × 5) + (3 × 15) + (1 × 9)
= 2 + 15 + 1 + 30 + 45 + 9
= 102

44 a)

(a) greater than 72 inches

When $X = 72$

$$Z = \frac{(X-\mu)}{\sigma} = \frac{72-68}{3} = \frac{4}{3} = 1.3333$$

P(X > 72) = P(Z > 1.33)
= 0.5 – P(0 < Z < 1.33)
= 0.5 – 0.4082
= 0.0918
= 500 × 0.0918 = 45.9 ~ 46

(b) less than or equal to 64 inches

When $X = 64$

$$Z = \frac{(X-\mu)}{\sigma} = \frac{64-68}{3} = \frac{-4}{3} = -1.3333$$

P(X ≤ 64) = P(Z ≤ -1.33)
= 0.5 - P(-1.33 < Z < 0)
= 0.5 - P(0< Z < 1.33)
= 0.0918
=0.0918 × 500 = 46

(c) between 65 and 71 inches.

When $X = 65$

$$Z = \frac{(X-\mu)}{\sigma} = \frac{65-68}{3} = \frac{-3}{3} = -1$$

When $X = 65$

$$Z = \frac{(X-\mu)}{\sigma} = \frac{71-68}{3} = \frac{3}{3} = 1$$

P(65< X < 71) = P(-1 < Z < 1)
= P(-1 < Z < 0) + P(0 < Z < 1)
= P(0 < Z < 1) + P(0 < Z < 1)
= 2 x P(0 < Z < 1)
= 2 (0.3413)
= 0.6826
= 0.6826 x 500 = 341.3 ~ 342.

44 b)

p_0q_0	p_0q_1	p_1q_0	p_1q_1
300	336	500	560
200	240	200	240
240	240	360	360
500	240	600	288
320	288	480	432
1560	1344	2140	1880

Fisher's ideal index

$$= \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}} \times 100$$

$$= \sqrt{\frac{2140}{1560} \times \frac{1880}{1344}} \times 100$$

$$= \sqrt{\frac{40,23,200}{20,96,640}} \times 100$$

$$= \sqrt{1.92} \times 100$$

$$= 1.385 \times 100 = 138.5$$

$$P_{01}^{F} = 138.5$$

Time reversal test:

To prove $P_{01} \times P_{10} = 1$

$$P_{01} \times P_{10} = \sqrt{\frac{\sum p_1q_0 \times \sum p_1q_1}{\sum p_0q_0 \times \sum p_0q_1}} \times \sqrt{\frac{\sum p_0q_1}{\sum p_1q_1} \times \frac{\sum p_0q_0}{\sum p_1q_0}}$$

$$= \sqrt{\frac{2140}{1560} \times \frac{1880}{1344} \times \frac{1344}{1880} \times \frac{1560}{2140}}$$

$$P_{01} \times P_{10} = 1$$

Time reversal test is satisfied.

Factor Reversal Test:

To prove $P_{01} \times Q_{01} = \frac{\sum p_1q_1}{\sum p_0q_0}$

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum p_1q_0 \times \sum p_1q_1}{\sum p_0q_0 \times \sum p_0q_1}} \times \sqrt{\frac{\sum q_1p_0 \times \sum q_1p_1}{\sum q_0p_0 \times \sum q_0p_1}}$$

$$= \sqrt{\frac{2140}{1560} \times \frac{1880}{1344} \times \frac{1344}{1560} \times \frac{1880}{2140}}$$

$$= \sqrt{\frac{1880 \times 1880}{1560 \times 1560}}$$

$$= \frac{1880}{1560} = \frac{\sum p_1q_1}{\sum p_0q_0}$$

Factor Reversal Test is satisfied.

45 a)

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
0	1			
		1		
1	2		8	
		9		6
2	11		14	
		23		
3	34			

Now $x_n = 3, h = 1, x = 2.8$

2.8 = 3 + n(1)

n = 2.8 – 3 = -0.2

$$y_{(x=x_n+nh)} = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \dots$$

$$y = 34 + \frac{(0.2)}{1!}(23) + \frac{(-0.2)(-0.2+1)}{2!}(14) + \frac{(-0.2)(-0.2+1)(-0.2+2)}{3!}(6)$$

y = 34 – 4.6 – 1.12 – 0.288

y = 27.992

45 b)

Sample Number	Observations			$\bar{X}$	R
1	32	36	42	36.67	10
2	28	32	40	33.33	12
3	39	52	28	39.67	24
4	50	42	31	41	19
5	42	45	34	40.33	11
6	50	29	21	33.33	29
7	44	52	35	43.67	17
8	22	35	44	33.67	22
	Total			301.67	144

$$\bar{\bar{X}} = \frac{\sum \bar{X}}{8} = \frac{301.67}{8} = 37.71$$

$$\bar{R} = \frac{\sum R}{8} = \frac{144}{8} = 18$$

The control limits for $\bar{X}$ chart is

$UCL = \bar{\bar{X}} + A_2\bar{R}$

$= 37.71 + (1.023)(18) = 56.12$

$CL = \bar{\bar{X}} = 37.71$

$LCL = \bar{\bar{X}} - A_2\bar{R}$

$= 37.71 - (1.023)(18) = 19.296$

The control limits for R chart is

$UCL = D_4\bar{R} = (2.574)(18) = 46.33$

$CL = \bar{R} = 18$

$LCL = D_3\bar{R} = 0(18) = 0$

46 a)	$\int_{-\infty}^{\infty} f(x)dx=1$ $\int_{-\infty}^{\infty} ke^{-\|x\|}dx=1$ $k\int_{-\infty}^{\infty} e^{-\|x\|}dx=1$ $2k\int_{0}^{\infty} e^{-x}dx=1$ $(\because\ e^{-\|x\|}$ is an even function) $2k\left[\frac{e^{-x}}{-1}\right]_0^{\infty}=1$ $k=\frac{1}{2}$ Mean of the random variable is $E(X)=\int_{-\infty}^{\infty} x\,f(x)\,dx$ $E(X)=\int_{-\infty}^{\infty} x\,k\,e^{-\|x\|}dx$ $(\because\ xe^{-\|x\|}$ is an odd function of x) $=\frac{1}{2}\int_{-\infty}^{\infty} x\,e^{-\|x\|}dx$ $=0$ $E(X^2)=\int_{-\infty}^{\infty} x^2 f(x)\,dx$ $=\int_{-\infty}^{\infty} x^2 k\,e^{-\|x\|}dx$ $=\frac{1}{2}\int_{-\infty}^{\infty} x^2 e^{-\|x\|}dx$ $=\int_{0}^{\infty} x^2 e^{-x}dx$ $(\because\ e^{-\|x\|}$ is an even function) $=\Gamma(3)\left(\because \Gamma(\alpha)=\int_0^{\infty} x^{\alpha-1}e^{-x}dx, \alpha>0; \Gamma(n)=(n-1)!\right)$ $=2$ $V(X)=E(X^2)-[E(X)]^2$ $=2-[0]^2$ $=2$	46 b)	$x\frac{dC}{dx} = \frac{3}{x}-C$ $\frac{dC}{dx} = \frac{3}{x^2}-\frac{C}{x}$ $\frac{dC}{dx}+\frac{C}{x} = \frac{3}{x^2}$ $\frac{dC}{dx}+\frac{1}{x}C = \frac{3}{x^2}$ Here, $P=\frac{1}{x}, Q=\frac{3}{x^2}$ $\int Pdx = \int\frac{1}{x}dx = \log x$ I.F $= e^{\int pdx} = e^{\log x} = x$ The Solution is $C(I.F) = \int Q(I.F)dx + k$ $Cx = \int\frac{3}{x^2}xdx + k$ $=3\int\frac{1}{x}dx+k$ $Cx = 3\log x + k$ (1) Given $C = 2$ When $x=1$ (1) $\Rightarrow$ $2\times1=k \Rightarrow k=2$ The relationship between C and x is $Cx = 3\log x + 2$

47 a)	Let $I = \int_2^5 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{7-x}} dx \quad \ldots(1)$ $I = \int_2^5 \frac{\sqrt{2+5-x}}{\sqrt{2+5-x} + \sqrt{7-(2+5-x)}} dx$ $\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$ $I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-x} + \sqrt{x}} dx \ldots(2)$ $(1)+(2) \Rightarrow$ $2I = \int_2^5 \left[\frac{\sqrt{x}}{\sqrt{x} + \sqrt{7-x}} + \frac{\sqrt{7-x}}{\sqrt{7-x} + \sqrt{x}}\right] dx$ $= \int_2^5 \left[\frac{\sqrt{x} + \sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}}\right] dx$ $= \int_2^5 dx = [x]_2^5 = 3$ $\therefore \quad I = \frac{3}{2}$	47 b)	Sample size n = 50 workers Total wages Σx = 2550 Sample mean $\overline{x} = \frac{total\ wages}{n} = \frac{\Sigma x}{n}$ $= \frac{2550}{50} = 51$ units Population mean μ = 52 Population variance σ^2 = 25 Population SD σ = 5 Under the null hypothesis $H_0 : \mu = 52$ Against the alternative hypothesis $H_1 : \mu \neq 52$ (Two tail) Level of significance μ = 0.01 Test statistic $Z = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$ $Z = \frac{51-52}{\frac{5}{\sqrt{50}}} = \frac{-1}{0.7071} = -1.4142$ Since alternative hypothesis is of two tailed test we can take $\|Z\| = 1.4142$ Critical value at 1% level of significance is $Z_{\alpha/2} = 2.58$ Inference: Since the calculated value is less than table value i.e., $Z < Z_{\alpha/2}$ at 1% level of

COMMON HALF YEARLY EXAMINATION - 2024

Standard XII

Reg.No.

BUSINESS MATHEMATICS AND STATISTICS

Time : 3.00 hrs *Marks : 90*

I. Choose the correct answer: 20 x 1 = 20

1. If $A = \begin{pmatrix} 2 & 0 \\ 0 & 8 \end{pmatrix}$ then $\rho(A)$ is ______. a) 0 b) 1 c) 2 d) n
2. In a transition probability matrix, all the entries are greater than or equal to ______.
   a) 2 b) 1 c) 0 d) 3
3. $\int \left[\frac{9}{x-3} - \frac{1}{x+1}\right] dx$ a) $\log|x-3| - \log|x+1| + c$ b) $\log|x-3| + \log|x+1| + c$
   c) $9\log|x-3| - \log|x+1| + c$ d) $9\log|x-3| + \log|x+1| + c$
4. $\Gamma\left(\frac{3}{2}\right)$ a) $\sqrt{\pi}$ b) $\frac{\sqrt{\pi}}{2}$ c) $2\sqrt{\pi}$ d) $\frac{3}{2}$
5. Area bounded by the curve $y = \frac{1}{x^2}$ between the limits 1 and 2 is
   a) 1/2 sq.units b) 1 sq.units c) 2 sq.units d) 1/4 sq.units
6. If MR and MC denote the marginal revenue and marginal cost and $MR - MC = 36x - 3x^2 - 81$, then the maximum profit at x is equal to a) 3 b) 6 c) 9 d) 5
7. The differential equation $\left(\frac{dx}{dy}\right)^3 + 2y^{\frac{1}{2}} = x$ is a) of order 2 and degree 1
   b) of order 1 and degree 3 c) of order 1 and degree 6 d) of order 1 and degree 2
8. The differential equation of $x^2 + y^2 = a^2$ a) $xdy + ydx = 0$ b) $ydx - xdy = 0$
   c) $xdx - ydx = 0$ d) $xdx + ydy = 0$
9. $E \equiv$ a) $1 + \Delta$ b) $1 - \Delta$ c) $1 + \nabla$ d) $1 - \nabla$
10. The first two terms in Newton's method will give the ______ interpolation.
    a) linear b) parabolic c) minimum d) maximum
11. A formula or equation used to represent the probability distribution of a continuous random variable is called a) probability distribution b) distribution function
    c) probability density function d) mathematical expectation
12. If c is a constant in a continuous probability distribution, then $p(x = c)$ is always equal to
    a) zero b) one c) negative d) does not exist
13. In a parametric distribution the mean is equal to variance is :
    a) binomial b) normal c) poisson d) all the above
14. Which of the following statements is/are true regarding the normal distribution curve?
    a) it is symmetrical and bell shaped curve
    b) it is asymptotic in that each end approaches the horizontal axis but never reaches it
    c) its mean, median and mode are located at the same point
    d) all of the above statements are true.
15. A ______ of statistical individuals in a population is called a sample.
    a) Infinite set b) finite subset c) finite set d) entire set
16. For testing $H_0 : \mu = \mu_0$ against $H_1 : \mu < \mu_0$ what is the critical value at $\alpha = 0.01$?
    a) 1.645 b) −1.645 c) −2.33 d) 2.33
17. Consumer price index are obtained by: a) Paasche's formula
    b) Fisher's ideal formula c) Marshall Edgeworth formula d) Family budget method formula

18. Another name of consumer's price index number is:
 a) Whole–sale price index number b) Cost of living index
 c) Sensitive d) Composite
19. North–West Corner refers to ______.
 a) top left corner b) top right corner c) bottom right corner d) bottom left corner
20. In an assignment problem the value of decision variable x_{ij} is ______.
 a) 1 b) 0 c) 1 or 0 d) none of them

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. Show that the equations 3x – 2y = 6, 6x – 4y = 10 are inconsistent
22. The marginal cost function is $MC = 300\, x^{\frac{2}{5}}$ and fixed cost is zero. Find out the total cost and average cost functions.
23. Find the differential equation of the family of all straight lines passing through the origin.
24. Evaluate $\Delta^2\left(\frac{1}{x}\right)$ by taking '1' as the interval of differencing.
25. If $P(x) = \begin{cases} \frac{x}{20} & x = 0,1,2,3,4,5 \\ 0 & \text{Otherwise} \end{cases}$. Find (i) $P(X < 3)$ and (ii) $P(2 < X \le 4)$
26. Mention the properties of Binominal distribution
27. Define alternative hypothesis.
28. The following figures relates to the profits of a commercial concern for 8 years

Year	1986	1987	1988	1989	1990	1991	1992	1993
Profit (₹)	14,520	15,470	15,520	21,020	26,500	31,950	35,600	34,900

 Find the trend of profits by the method of three yearly moving averages.
29. Give mathematical form of assignment problem.
30. Evaluate : $\int (x^2 - 2x + 5)e^{-x}\, dx$

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. Find the rank of the matrix $A = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 3 & 1 & 1 & 3 \end{pmatrix}$
32. Evaluate : $\int \frac{1}{x - \sqrt{x^2 - 1}}\, dx$
33. Calculate the area bounded by the parabola $y^2 = 4ax$ and its latusrectum.
34. Solve : $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$
35. Find the missing entry in the following table

x	0	1	2	3	4
y_x	1	3	9	–	81

36. What are the properties of Mathematical expectation?
37. It is given that 5% of the electric bulbs manufactured by a company are defective. Using poisson distribution find the probability that a sample of 120 bulbs will contain no defective bulb. (e^{-6} = 0.0025)

38. A wholesaler in apples claims that only 4% of the apples supplied by him are defective. A random sample of 600 apples contained 36 defective apples. Calculate the standard error concerning of good apples.

39. A farmer wants to decide which of the three crops he should plant on his 100–acre farm. The profit from each is dependent on the rainfall during the growing season. The farmer has categorized the amount of rainfall as high medium and low. His estimated profit for each is shown in the table.

Rain fall	Estimated Conditional Profit (Rs.)		
	Crop A	Crop B	Crop C
High	8000	3500	5000
Medium	4500	4500	5000
Low	2000	5000	4000

If the farmer wishes to plant only crop, decide which should be his best crop using (i) Maximin (ii)Minimax

40. Calculate the cost of living index number by consumer price index number for the year 2016 with respect to base year 2011 of the following data.

Commodities	Price		Quantity
	Base year	Current year	
Rice	32	48	25
Sugar	25	42	10
Oil	54	85	6
Coffee	250	460	1
Tea	175	275	2

IV. Answer all the questions. **7 x 5 = 35**

41. a) The demand equation for a products is $x = \sqrt{100 - p}$ and the supply equation is $x = \frac{p}{2} - 10$. Determine the consumer's surplus and producer's surplus, under market equilibrium. *(OR)*

b) Investigate for what values of 'a' and 'b' the following system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + az = b$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

42. a) Evaluate the integral as the limit of a sum $\int_1^3 (2x+3)dx$ *(OR)*

b) Out of 750 families with 4 children each, how many families would be expected to have (i) atleast one boy (ii) atmost 2 girls (iii) and children of both sexes? Assume equal probabilities for boys and girls.

43. a) Two products A and B currently share the market with shares 50% and 50% each respectively. Each week some brand switching takes place. Of those who bought A the previous week, 60% buy it again whereas 40% switch over to B. Of those who bought B the previous week, 80% buy it again where as 20% switch over to A. Find their shares after one week and after two weeks. If the price war continues, when is the equilibrium reached? *(OR)*

b) Explain Vogel's approximation method by obtaining initial feasible solution of the following transportation problem

	D_1	D_2	D_3	D_4	Supply
O_1	2	3	11	7	6
O_2	1	0	6	1	1
O_3	5	8	15	9	10
Demand	7	5	3	2	

44\. a) If the heights of 500 students are normally distributed with mean 68.0 inches and standard deviation 3.0 inches, how many students have height a) greater than 72 inches (b) less than or equal to 64 inches c) between 65 and 71 inches.

Z	1	1.33
Area	0.3413	0.4082

(OR)

b) Using the following data, construct Fisher's Ideal index and show how it satisfies Factor Reversal Test and Time Reversal Test?

Commodity	Price in Rupees per unit		Number of units	
	Base year	Current year	Base year	Current year
A	6	10	50	56
B	2	2	100	120
C	4	6	60	60
D	10	12	50	24
E	8	12	40	36

45\. a) Find f (2.8) from the following table.

x	0	1	2	3
f(x)	1	2	11	34

(OR)

b) Construct $\overline{X}$ and R charts for the following data:

Sample Number	Observations		
1	32	36	42
2	28	32	40
3	39	52	28
4	50	42	31
5	42	45	34
6	50	29	21
7	44	52	35
8	22	35	44

Given for ($n = 3$, $A_2 = 0.58$, $D_3 = 0$ and $D_4 = 2.115$)

46\. a) The probability density function of a random variable X is $f(x) = ke^{-|x|}, -\infty < x < \infty$ Find the value of k and also find mean and variance for the random variable.

(OR)

b) A firm has found that the cost C of producing x tons of certain product by the equation $x\frac{dc}{dx} = 3 - C$ and $c = 2$ when $x = 1$ Find the relationship between C and x.

47\. a) Evaluate : $\int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}} dx$

(OR)

b) The wages of the factory workers are assumed to be normally distributed with mean and variance 25. A random sample of 50 workers gives the total wages equal to ₹2,550. Test the hypothesis m = 52, against the alternative hypothesis m = 49 at 1% level of significance.

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