SECOND MIDTERM TEST NOVEMBER 2024

12 TH STANDARD

TIME : 1.30 Hrs

MATHEMATICS

Marks : 45

PART 1

I. Choose the best answer   **10x1=10**

1. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

   (1) $t = 0$ (2) $t = \frac{1}{3}$ (3) $t = 1$ (4) $t = 3$

2. The number given by the Rolle's theorem for the function $x^3 - 3x^2, x \in [0,3]$ is

   (1) 1   (2) $\sqrt{2}$ (3) $\frac{3}{2}$ (4) 2

3. The point of inflection of the curve $y = (x - 1)^3$ is

   (1) (0,0)   (2) (0,1)   (3) (1,0)   (4) (1,1)

4. A random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is

   (1) 6   (2) 4   (3) 3   (4) 2

5. Suppose that X takes on one of the values 0,1 and 2. If for some constant k, $P(X = i) = kP(X = i - 1)$ for $i = 1,2$ and $P(X = 0) = \frac{1}{7}$. Then the value of k is

   (1) 1   (2) 2   (3) 3   (4) 4

6. If in 6 trials, X is a binomial variate which follows the relation $9P(X = 4) = P(X = 2)$, then the probability of success is

   (1) 0.125   (2) 0.25   (3) 0.375   (4) 0.75

7. The operation $*$ defined by $a * b = \frac{ab}{7}$ is not a binary operation on

   (1) $\mathbb{Q}^+$ (2) $\mathbb{Z}$ (3) $\mathbb{R}$ (4) $\mathbb{C}$

8. If a compound statement involves 3 simple statements, then the number of rows in the truth table is

   (1) 9   (2) 8   (3) 6   (4) 3

9. The instantaneous rate of change at t=2 for the function f(t) = te^{-2t}+5

   (1) $-5e^{-4}$ (2) $5e^{-4}$ (3) $-3e^{-4}$ (4) $3e^{-4}$

10. Let X be a random variable with Var(X)=4. Then Var(X/2) is

    (1) 1   (2) 2   (3) 0   (4) none of these

II. Answer any 3 of the following .Q.No 15 compulsory   **3x2=6**

11. If the volume of a cube of side length x is $V = x^3$. Find the rate of change of the volume with respect to x when $x = 5$ units.

12. Find the values in the interval (1,2) of the mean value theorem satisfied by the function $f(x) = x - x^2$ for $1 \le x \le 2$.

13. On $\mathbb{Z}$, define $*$ by $(m * n) = m^n + n^m: \forall m, n \in \mathbb{Z}$. Is $*$ binary on $\mathbb{Z}$?   *TTK-12-MAT-EM-1*

14. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

15. In set of integers * is defined by a*b =a+b-1, find the identity element.

III. Answer any 3 of the following. Q.No 20 compulsory   3x3=9

16. use l'Hôpital Rule evaluate $\lim\limits_{x \to \infty} e^{-x}\sqrt{x}$

17. Find the smallest possible value of $x^2 + y^2$ given that $x + y = 10$.

18. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.

19. If $X \sim B(n,p)$ such that $4P(X = 4) = P(x = 2)$ and $n = 6$. Find the distribution, mean and standard deviation.

20. Using mean value theorem prove that for, $a > 0, b > 0, |e^{-a} - e^{-b}| < |a - b|$.

IV. Answer the following   4x5=20

21. If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally if, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$. (OR)

A random variable X has the following probability mass function:

X	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii)$P(2 \leq X < 5)$ (iii)$P(3 < X)$

22. A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.(OR)Let A be $Q - \{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is $*$ a binary on A. If so, examine the closure, commutative, associative, the existence of identity and existence of inverse properties.

23. A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws.
(i)Find the probability mass function (ii)Find the cumulative distribution function
(iii) Find $P(3 \leq X < 6)$ (iv) Find$P(X \geq 4)$.(OR)
Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

24. The probability density function of the random variable X is given by
$f(x) = \begin{cases} 16xe^{-4x} & \text{for } x > 0 \\ 0, & \text{for } x \leq 0 \end{cases}$. Find the mean and variance of X.(OR)

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subset $A = \{1,3,4,5,9\}$of the set of remainders $\{0,1,2,3,4,5,6,7,8,9,10\}$.

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TTK-12-MAT-EM-2