Register No.

12 R

Half Yearly Examination - 2024
MATHEMATICS

Time : 3.00 Hrs. Marks : 90

PART - A

All the questions are compulsory. Write the option code and the corresponding answer. 20 x 1 = 20

1. If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$ then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is.....a) $|\vec{a}||\vec{b}||\vec{c}|$ b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ c) 1 d) –1
2. If $\vec{u} = \vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b})$ then a) u is a unit vector b) $\vec{u} = \vec{a} + \vec{b} + \vec{c}$ c) $\vec{u} = \vec{o}$ d) $\vec{u} \neq \vec{o}$
3. The slope of the normal to the curve f(x) = 2cos4x at $x = \frac{\pi}{12}$ is a) $-4\sqrt{3}$ b) –4 c) $\frac{\sqrt{3}}{12}$ d) $4\sqrt{3}$
4. The minimum value of the function |3 – x| + 9 is a) 0 b) 3 c) 6 d) 9
5. An asymptote to the curve $y^2(a + 2x) = x^2(3a - x)$ is a) x = 3a b) $x = -\frac{a}{2}$ c) $x = \frac{a}{2}$ d) x = 0
6. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$ then λ is a) 17 b) 14 c) 19 d) 21
7. If $A = \begin{bmatrix} k & 3 \\ 3 & k \end{bmatrix}$ and $|A^3| = 343$ then the value of k is a) ±1 b) ±2 c) ±3 d) ±4
8. If z is a non-zero complex number such that $2iz^2 = \bar{z}$ then |z| is a) $\frac{1}{2}$ b) 1 c) 2 d) 3
9. The principal arguement of $\frac{3}{-1+i}$ is a) $\frac{-5\pi}{6}$ b) $\frac{-2\pi}{3}$ c) $\frac{-3\pi}{4}$ d) $\frac{-\pi}{2}$
10. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is a) xy = k b) y = klogx c) y = kx d) logy = k
11. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is a) 2 b) 4 c) 1 d) ∞
12. The value of $\int_0^{2/3} \frac{dx}{\sqrt{4-9x^2}}$ is a) $\frac{\pi}{6}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) π
13. The area between $y^2 = 4x$ and its latus rectum is a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{8}{3}$ d) $\frac{5}{3}$
14. The value of $\int_0^1 x^4(1-x)^5\,dx$ a) $\frac{25}{126}$ b) $\frac{15}{126}$ c) $\frac{1}{1260}$ d) $\frac{25}{625}$
15. If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$ the value of $\tan^{-1} x$ is a) $-\frac{\pi}{10}$ b) $\frac{\pi}{5}$ c) $\frac{\pi}{10}$ d) $\frac{-\pi}{5}$
16. $\sin(\tan^{-1} x)$, |x| < 1 is equal to a) $\frac{x}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{x}{\sqrt{1+x^2}}$
17. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?
    a) $\frac{1}{31}$ b) $\frac{1}{5}$ c) 5 d) 31
18. If $f(x, y) = e^{xy}$ then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to a) xye^{xy} b) $(1 + xy)e^{xy}$ c) $(1 + y)e^{xy}$ d) $(1 + x)e^{xy}$
19. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is a) 1 b) $\sqrt{10}$ c) 3 d) $\sqrt{11}$
20. If x + y = k is a normal to the parabola $y^2 = 12x$ then the value of k is a) 3 b) –1 c) 1 d) 9

PART - II

Answer any seven questions. Q.No.30 is compulsory. 7 x 2 = 14

21. Find the rank of the matrix by minor method. $A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$
22. Find the nature of roots of the equation $2x^2 - 6x + 7 = 0$
23. Find the principal value of $\sin^{-1}(2)$ of it exists
24. Find the length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
25. Find the angle between the planes $\vec{r}.(\hat{i} + \hat{j} - 2\hat{k}) = 3$ and $2x - 2y + x = 2$

26. Find the integrating factor of the differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$

27. Find the volume of the region bounded by y = x, y = 1 and x = 0 is rotated about y-axis.

28. Find the value of $\lim_{x \to \infty} \frac{x}{\log x}$

29. Use the linear approximatiion to find approximate value of $\sqrt[3]{26}$

30. Simplify $\sum_{n=1}^{12} i^n$

PART - III

7 x 3 = 21

Answer any 7 questions. Q.No.40 is compulsory.

31. Determine whether the 3 vectors $2\hat{i} + 3\hat{j} + \hat{k}$, $\hat{i} - 2\hat{j} + 2\hat{k}$ and $3\hat{i} + \hat{j} + 3\hat{k}$ are coplanar.

32. A circle of area 9π square units has two of its diameters along the lines x + y = 5 and x – y = 1. Find the equation of the circle.

33. Test the consistency and if possible solve the system of equations.
2x + 2y + z = 5 x –,y + z = 1 3x + y + 2z = 4

34. If |z| = 3 show that $7 \leq |z + 6 - 8i| \leq 13$

35. Find the condition that the roots of $ax^3 + bx^2 + cx + d = 0$ are in geometric progression. Assume a, b, c, d ≠ 0

36. Find the value of $\sin^{-1}\left[\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right]$

37. Evaluate $\int_{-\pi/2}^{\pi/2} x \cos x dx$

38. If $w(x, y, z) = x^2y + y^2z + z^2x$, $x, y, z \in R$. Find the differential dw.

39. Find the differential equation corresponding to the family of curves represented by the equation $y = Ae^{5x} + Be^{-5x}$, where A and B are arbitrary constants.

40. Compute the value of C satisfied by the Rolle's theorem for the function $f(x) = x^2 (1 - x)^2$ $x \in [0, 1]$

PART - IV

7 x 5 = 35

Answer all the questions.

41. a) Show that the curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants cuts orthogonally. **(OR)**
b) Find the area of the region in the first quadrant bounded by the parabola $y^2 = 4x$ the line x + y = 3 and y - axis.

42. a) Investigate the values of λ and μ the system of linear equation 2x + 3y + 5z = 9, 7x + 3y – 5z = 8, 2x + 3y + λz = μ have i) no solution ii) unique solution iii) infinite number of solutions. **(OR)**
b) Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

43. a) The rate of increase in the number of bacteria in a certain bacteria culture if proportional to the number present. Given that the number triples in 5 hours. Find how many bacteria will be present after 10 hours. **(OR)**
b) Evaluate $\int_{\pi/8}^{3\pi/8} \frac{1}{1 + \sqrt{\tan x}} dx$

44. If z = x + iy and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$ show that $x^2 + y^2 = 1$ **(OR)**
Solve : (2x – 1) (x + 3) (x – 2) (2x + 3) + 20 = 0

45. a) Find the vertex, focus, equation of the directrix and length of the latus rectum of the parabola $x^2 - 2x + 8y + 17 = 0$ **(OR)**
b) Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

46. a) Find the parametric form of vector equation and cartesian equations of the plane passing through the points (2, 2, 1) (9, 3, 6) and perpendicualr to the plane 2x + 6y + 6z = 9 **(OR)**
b) Find the dimensions of the largest rectangle that can be inscribed in a circle of radius 'r'.

47. a) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x} + \sqrt{y}}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$ **(OR)**
b) On lighting a rocket craker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.