HYM
12 - Std

HALF YEARLY EXAMINATION - 2024

MATHEMATICS

Time : 3.00 hrs. Marks : 90

Part - I

Note : i) All questions are compulsory $20 \times 1 = 20$

ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer

1. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

   (1) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (3) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (4) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

2. A is order of $n, \lambda \neq 0$ then $adj\ (\lambda A) =$

   (1) $\lambda^{n-1} adj\ (A)$ (2) $\lambda^{n-2} adj\ (A)$ (3) $\frac{1}{\lambda} adj\ (A)$ (4) $\lambda^{n} adj\ (A)$

3. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

   (1) z (2) $\bar{z}$ (3) $\frac{1}{z}$ (4) 1

4. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals

   (1) (1,0) (2) (−1,1) (3) (0,1) (4) (1,1)

5. A polynomial equation in x of degree n always has

   (1) n distinct roots (2) n real roots
   (3) n imaginary roots (4) at most one root

6. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

   (1) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (2) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (3) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (4) $\tan^{-1}\left(\frac{1}{2}\right)$

7. $\sin(\tan^{-1} x), |x| < 1$ is equal to

   (1) $\frac{x}{\sqrt{1-x^2}}$ (2) $\frac{1}{\sqrt{1-x^2}}$ (3) $\frac{1}{\sqrt{1+x^2}}$ (4) $\frac{x}{\sqrt{1+x^2}}$

8. The radius of the circle passing through the point (6,2) two of whose diameters are $x + y = 6$ and $x + 2y = 4$ is

   (1) 10 (2) $2\sqrt{5}$ (3) 6 (4) 4

9. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

   (1) $2ab$ (2) ab (3) $\sqrt{ab}$ (4) $\frac{a}{b}$

10. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to

   (1) 2 (2) −1 (3) 1 (4) 0

11. If the distance of the point $(1,1,1)$ from the origin is half of its distance from the plane $x+y+z+k=0$, then the values of k are

(1) ± 3 (2) ± 6 (3) $-3, 9$ (4) $3, -9$

12. The minimum value of the function $|3-x|+9$ is

(1) 0 (2) 3 (3) 6 (4) 9

13. If we measure the side of a cube to be 4cm with an error of 0.1cm, then the error in our calculation of the volume is

(1) 0.4cu.cm (2) 0.45cu.cm (3) 2 cu.cm (4) 4.8cu.cm

14. If W is a function of x and y; and x and y are functions of t, then which of the following is undefined ?

(1) $\frac{\partial w}{\partial x}$ (2) $\frac{\partial w}{\partial y}$ (3) $\frac{\partial x}{\partial t}$ (4) $\frac{dy}{dt}$

15. The value of $\int_0^1 x(1-x)^{99}dx$ is

(1) $\frac{1}{11000}$ (2) $\frac{1}{10100}$ (3) $\frac{1}{10010}$ (4) $\frac{1}{10001}$

16. The differential equation representing the family of curves $y = A\cos(x+B)$, where A and B are parameters, is

(1) $\frac{d^2y}{dx^2} - y = 0$ (2) $\frac{d^2y}{dx^2} + y = 0$ (3) $\frac{d^2y}{dx^2} = 0$ (4) $\frac{d^2x}{dy^2} = 0$

17. The integrating factor of $\frac{dx}{dy} + px = Q$ is

(1) $e^{\int p dy}$ (2) $e^{\int p dx}$ (3) $e^{\int Q dy}$ (4) $e^{\int Q dx}$

18. Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result. The value of $E[X]$ is

(1) 0.11 (2) 1.1 (3) 11 (4) 1

19. Which one of the following is not true?

(1) Negation of a negation of a statement is the statement itself.

(2) If the last column of the truth table contains only T then it is a tautology.

(3) If the last column of its truth table contains only F then it is a contradiction.

(4) If p and q are any two statements then $p \leftrightarrow q$ is a tautology.

20. The identity element under addition exists in?

(1) N (2) $C \setminus \{0\}$ (3) $(0, \infty)$ (4) $-3 \le x \le 3$

Part – II

Note : i) Answer any **Seven** questions. 7 × 2 = 14

ii) Question number **30** is compulsory

21. Find $adj(adj(A))$ if $adjA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

22. If $z_1 = 3-2i$ and $z_2 = 6+4i$ find $\frac{z_1}{z_2}$ in the rectangular form.

23 . Find the principal value of $cos^{-1}\left(\frac{1}{2}\right)$.

24 . Find the equation of the circle with centre $(2, -1)$ and passing through the point $(3,6)$ in standard form.

25 . Find the volume of the parallelepiped whose coterminus edges (adjacent edges) are given by the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}, \hat{i} + 2\hat{j} - \hat{k}$ and $3\hat{i} - \hat{j} + 2\hat{k}$.

26 . Evaluate : $\lim_{x \to 1}\left(\frac{x^2-3x+2}{x^2-4x+3}\right)$.

27 . Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2mm to 2.1mm, how much is cross-sectional area increased approximately?

28 . Evaluate : $\int_0^{\frac{\pi}{2}} sin^{10} x\, dx$

29 . A pair of fair dice is rolled once. Find the probability mass function to get the number of fours.

30 . Show that the differential equation corresponding to $y = A\, sinx$, where A is an arbitrary constant is $y = y'\ tanx$.

Part – III

Note : i) Answer any **Seven** questions. $7 \times 3 = 21$

ii) Question number **40** is compulsory.

31. Solve the following system of linear equations, using matrix inversion method :
$5x + 2y = 3, 3x + 2y = 5$.

32 . Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

33 . Find the equation of the parabola in each of the cases given below :

(i) focus(4,0) and directrix $x = -4$.

34 . Find the angle between the straight line $\frac{x+3}{2} = \frac{y-1}{2} = \frac{-z}{1}$ with coordinate axes.

35. The price of a product is related to the number of units' available (supply) by the equation $Px + 3P - 16x = 234$, where P is the price of the product per unit in Rupees and x is the number of units. Find the rate at which the price is changing with respect to time when 90 units are available and the supply is increasing at a rate of 15 units/week.

36 . Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator.

37 . Evaluate : $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\, dx$.

38 . The mean and variance of a binomial variate X are respectively 2 and 1.5. Find
(i) $P(X = 0)$ (ii) $P(X = 1)$ (iii) $P(X \geq 1)$

39 . Let A be $Q - \{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is $*$ a binary on A. If so, examine the closure, commutative, associative properties.

40 . If α and β are roots of $x^2 + 5x + 6 = 0$ then show that $\alpha^2 + \beta^2 = 13$.

Part – IV

Note : i) Answer all the questions $7 \times 5 = 35$

41 . a) Prove by vector method that $sin\,(\alpha+\beta) = sin\,\alpha\, cos\,\beta + cos\,\alpha\, sin\,\beta$. **(OR)**

b) Find the value of $sin^{-1}(-1) + cos^{-1}\left(\frac{1}{2}\right) + cot^{-1}(2)$.

42 . a) Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table. **(OR)**

b) Solve the equation : $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

43 . a) A six sided die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find

(i) the probability mass function (ii) the cumulative distribution function

(iii) $P(4 \le X \le 10)$ (ii) $P(X \ge 6)$ **(OR)**

b) Find the acute angle between $y = x^2$ and $y = (x-3)^2$.

44 . a) Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$,$2x + 3y + \lambda z = \mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(OR)**

b) Find the vector equation and Cartesian equation of the plane passing through the point $(1,-2,4)$and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$.

45 . a) If $z = x + iy$ and arg $\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$. **(OR)**

b) Find the area of the region bounded between the parabolas $y^2 = 4x$ and $x^2 = 4y$.

46 . a) Find two positive numbers whose product is 20 and their sum is minimum. **(OR)**

b) The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

47. a) If $u = sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}tan\,u$. **(OR)**

b) At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75m from the point of origin.