HSS

12 - Std

Time : 3.00 hrs.

HALF YEARLY EXAMINATION - 2024

MATHEMATICS

Marks : 90

PART – A

CHOOSE THE CORRET ANSWER :- **20 X 1 = 20**

1) If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is

(a) 15 (b) 12 (c) 14 (d) 11

2) If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0$, $(\cos\theta)\,x - y + z = 0$, $(\sin\,\theta)x + y - z = 0$ has a non-trivial solution then θ is

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

3) If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

(a) -2 (b) -1 (c) 1 (d) 2

4) A zero of $x^3 + 64$ is

(a) 0 (b) 4 (c) $4i$ (d) -4

5) If $\sin^{-1} x = 2\sin^{-1} \alpha$ has a solution, then

(a) $|\alpha| \le \frac{1}{\sqrt{2}}$ (b) $|\alpha| \ge \frac{1}{\sqrt{2}}$ (c) $|\alpha| < \frac{1}{\sqrt{2}}$ (d) $|\alpha| > \frac{1}{\sqrt{2}}$

6) The equation $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

(a) no solution (b) unique solution

(c) two solutions (d) infinite number of solutions

7) Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the straight line $2x - y = 1$. One of the points of contact of tangents on the hyperbola is

(a) $\left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (b) $\left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (c) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (d) $(3\sqrt{3}, -2\sqrt{2})$

8) If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is

(a) $2x + 1 = 0$ (b) $x = -1$ (c) $2x - 1 = 0$ (d) $x = 1$

9) If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$, where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{b} \cdot \vec{c} \ne 0$ and $\vec{a} \cdot \vec{b} \ne 0$, then $\vec{a}$ and $\vec{c}$ are

(a) perpendicular (b) parallel

(c) inclined at an angle $\frac{\pi}{3}$ (d) inclined at an angle $\frac{\pi}{6}$

10) If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1, \lambda > 0$ is $\frac{1}{5}$, then the value of λ is

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) 0 (d) 1

11) A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

(a) 2 (b) 2.5 (c) 3 (d) 3.5

12) The number given by the Rolle's theorem for the function $x^3 - 3x^2, x \in [0,3]$ is

(a) 1 (b) $\sqrt{2}$ (c) $\frac{3}{2}$ (d) 2

13) If $v(x,y) = \log(e^x + e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to

(a) $e^x + e^y$ (b) $\frac{1}{e^x+e^y}$ (c) 2 (d) 1

14) The value of $\int_{-4}^{4} \left[\tan^{-1}\left(\frac{x^2}{x^4+1}\right) + \tan^{-1}\left(\frac{x^4+1}{x^2}\right)\right] dx$ is

(a) π (b) 2π (c) 3π (d) 4π

15) The differential equation of the family of curves $y = Ae^x + Be^{-x}$, where A and B are arbitrary constants is

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$ (c) $\frac{dy}{dx} + y = 0$ (d) $\frac{dy}{dx} - y = 0$

16) The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is

(a) $y + \sin^{-1} x = c$ (b) $x + \sin^{-1} y = 0$

(c) $y^2 + 2\sin^{-1} x = C$ (d) $x^2 + 2\sin^{-1} y = 0$

17) If $z = \frac{-2}{1+i\sqrt{3}}$., then the principal argument of z is

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

18) The integrating factor of $x\frac{dy}{dx} + y\log x = e^x$ is

(a) $e^{\frac{\log x}{2}}$ (b) $e^{\frac{x^2}{2}}$ (c) $x^{\log\sqrt{x}}$ (d) $x^{\log x}$

19) The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin|x| + \cos|x|)\, dx$ is

(a) 1 (b)2 (c) 3 (d) 4

20) If the Rolle's theorem holds for the function $f(x) = x^3 - 6x^2 + ax + 5$, $x \in [1,3]$ with $c = 2 + \frac{1}{\sqrt{3}}$ then the value of a is

(a) 11 (b)-6 (c) 5 (d) 0

PART – B

ANSWER ANY SEVEN QUESTIONS (Q.NO : 30 IS COMPULSORY) :-

7 X 2 = 14

21. If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.
22. Prove that : z is real if and only if $z = \bar{z}$
23. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.
24. Find the equation of the parabola whose vertex is $(5, -2)$ and focus $(2, -2)$.
25. Find the angle between the straight line $\vec{r} = (2\hat{i} + 3\hat{j} + \hat{k}) + t(\hat{i} - \hat{j} + \hat{k})$ and the plane $2x - y + z = 5$.
26. Evaluate : $\lim\limits_{x \to \infty} \left(\frac{x^2+17x+29}{x^4}\right)$.
27. Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2mm to 2.1mm, how much is cross-sectional area increased approximately?
28. Prove that $\int_0^\infty e^{-x} x^n dx = n!$, where n is a positive integer.
29. Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant.
30. Find the domain for $cos^{-1}\left(\frac{2x}{1+x}\right)$.

PART – C

ANSWER ANY SEVEN QUESTIONS (Q.NO : 40 IS COMPULSORY) : 7 X 3 = 21

31. Solve the following system of linear equations, using matrix inversion method:

    $5x + 2y = 3, 3x + 2y = 5$.
32. If $z = x + iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$, show that the locus of z is real axis.
33. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.
34. Find the value of $tan^{-1}(-1) + cos^{-1}\left(\frac{1}{2}\right) + sin^{-1}\left(-\frac{1}{2}\right)$.
35. If the equation $3x^2 + (3-p)xy + qy^2 - 2px - 8pq = 0$ represents a circle, find p and q. Also determine the centre and radius of the circle.
36. Show that the straight lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ are skew and hence find the shortest distance between them.
37. Determine the intervals of concavity of the curve $y = 3 + sin\,x$.
38. Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator.

39. Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded by the curves $y = \log x$, $y = 0$, $x = 0$ and $y = 2$.

40. Solve : $x\frac{dy}{dx} + y = y^2$.

PART – D

ANSWER ALL THE QUESTIONS :- **7 X 5 = 35**

41. a. Solve the systems of linear equations by Cramer's rule:
$3x + 3y - z = 11, 2x - y + 2z = 9, 4x + 3y + 2z = 25$ ***(OR)***
b. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

42. a. Find the centre, foci and eccentricity of the hyperbola
$11x^2 - 25y^2 - 44x + 50y - 256 = 0$.
(OR) b. Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10 percentage of the original number of radioactive nuclei have undergone disintegration in a period of 100 years.What radioactive nuclei will remain after 1000 years? (Take the initial amount as A_0)

43. a. Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$. ***(OR)***
b. Father of a family wishes to divide his square field bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ along the curve $y^2 = 4x$ and $x^2 = 4y$ into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them.

44. a. Find the number of solutions of the equation
$tan^{-1}(x-1) + tan^{-1}x + tan^{-1}(x+1) = tan^{-1}3x$. ***(OR)***
b. If $f(x,y) = tan^{-1}\left(\frac{x}{y}\right)$, then prove that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

45. a. Prove by vector method that the perpendiculars (altitudes) from the vertices to the opposite sides of a triangle are concurrent. ***(OR)***
b. Solve : $(1 - x^2)\frac{dy}{dx} - xy = 1$

46. a. Find the acute angle between $y = x^2$ and $y = (x-3)^2$. ***(OR)***
b. Find the parametric form of vector and Cartesian equations of the plane containing the line
$\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + t(2\hat{i} - \hat{j} + 4\hat{k})$ and perpendicular to plane $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = 8$.

47. a. The top and bottom margins of a poster are each 6 cms and side margins are each 4 cms. If the area of the printed material on the poster is fixed at 384 cm^2, find the dimensions of the poster with the smallest area. ***(OR)***
b. Evaluate : $\int_0^1 \frac{\log(1+y)}{1+y^2} dy$.