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Thoothukudi District

T COMMON HALF YEARLY EXAMINATION - 2024

Standard - XII Reg.No. 2419

Time: 3.00 hrs. **MATHEMATICS** *Marks: 90*

Part - A **20×1=20**

Choose the correct answer:

1) If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I_2 - A =$

a) A^{-1} b) $\frac{A^{-1}}{2}$ c) $3A^{-1}$ d) $2A^{-1}$

2) The principal argument of $(\sin 40^\circ + i \cos 40^\circ)^5$ is

a) -110° b) -70° c) 70° d) 110°

3) The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

a) 2 b) 4 c) 1 d) ∞

4) If $\cot^{-1}\left(\sqrt{\sin\alpha}\right) + \tan^{-1}\left(\sqrt{\sin\alpha}\right) = u$, then cos2u is equal to

a) $\tan^2\alpha$ b) 0 c) -1 d) $\tan 2\alpha$

5) The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

a) $4(a^2+b^2)$ b) $2(a^2+b^2)$ c) a^2+b^2 d) $\frac{1}{2}(a^2+b^2)$

6) The locus of a point whose distance from $(-2, 0)$ is $\frac{2}{3}$ times is distance from the line $x = \frac{-9}{2}$ is

a) a parbola b) a hyperbola c) an ellipse d) a circle

7) The coordinates of the point where the line $\bar{r} = (6\hat{i} - \hat{j} - 3\hat{k}) + t(-\hat{i} + 4\hat{j})$ meets the plane $\bar{r}.(\hat{i} + \hat{j} - \hat{k}) = 3$ are

a) (2, 1, 0) b) (7, −1, −7) c) (1, 2, −6) d) (5, −1, 1)

8) The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0, 3]$ is

a) 1 b) $\sqrt{2}$ c) $\frac{3}{2}$ d) 2

9) If $v(x, y) = \log(e^x + e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to

a) $e^x + e^y$ b) $\frac{1}{e^x + e^y}$ c) 2 d) 1

10) The value of $\int_0^1 x(1-x)^{99}\,dx$ is

a) $\frac{1}{11000}$ b) $\frac{1}{10100}$ c) $\frac{1}{10010}$ d) $\frac{1}{10001}$

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11) The differential equation of the family of curves $y = Ae^x + Be^{-x}$, where A and B are arbitary constants is

a) $\frac{d^2y}{dx^2} + y = 0$ b) $\frac{d^2y}{dx^2} - y = 0$ c) $\frac{dy}{dx} + y = 0$ d) $\frac{dy}{dx} - y = 0$

12) The number of arbitrary constants in the particular solution of a differential equation of third order is

a) 3 b) 2 c) 1 d) 0

13) Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result. The value of E(X) is

a) 0.11 b) 1.1 c) 11 d) 1

14) If in 6 trials, X is a binomial variable which follows the relation $9P(X=4)=P(X=2)$, then the probability of success is

a) 0.125 b) 0.25 c) 0.375 d) 0.75

15) Subtraction is not a binary operation is

a) R b) Z c) N d) Q

16) In the set Q define $a \odot b = a+b+ab$. For what value of y, $3 \odot (y \odot 5) = 7$?

a) $y = \frac{2}{3}$ b) $y = \frac{-2}{3}$ c) $y = \frac{-3}{2}$ d) $y = 4$

17) If A is a matrix of order $m \times n$, then $\rho(A)$ is _____.

a) m b) n c) $\leq \min(m, n)$ d) $\geq \min(m, n)$

18) The amplitude of $\frac{1}{i}$ is equal to _____.

a) 0 b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) π

19) The slope of the tangent to the curve $x = 3t^2+1$, $y = t^3-1$ at $t = 1$ is _____.

a) 1/2 b) 0 c) -2 d) ∞

20) The area bounded by the parabola $y^2 = x$ and its latus rectum is _____.

a) $\frac{4}{3}$ b) $\frac{1}{6}$ c) $\frac{2}{3}$ d) $\frac{8}{3}$

Part - B

Answer any SEVEN questions: Q.No. 30 is compulsory. **7×2=14**

21) If $adj(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A.

22) Find the value of $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$.

23) Prove that $[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}] = 0$.

24) Let $g(x) = x^2+\sin x$. Calculate the differential dg.

25) Evaluate the following: $\int_0^{\pi/2} \sin^{10} x \, dx$

26) Determine the order and degree (if exists) of the following differential equations: $\left(\frac{d^4y}{dx^4}\right)^3 + 4\left(\frac{dy}{dx}\right)^7 + 6y = 5\cos 3x$

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27) Find a polynomial equation of minimum degree with rational coefficients, having $2-\sqrt{3}$ as a root.

28) Prove that the function $f(x) = x^2+2$ is strictly increasing in the interval (2, 7) and strictly decreasing in the interval (−2, 0).

29) Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

30) If z_1 and z_2 are $1-i$, $-2+4i$ then find $\text{Im}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$.

Part - C

Answer any SEVEN questions: Q.No. 40 is compulsory. **7×3=21**

31) If α and β are the roots of the quadratic equation $17x^2+43x-73 = 0$, costruct a quadratic equation whose roots are $\alpha+2$ and $\beta+2$.

32) Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x$, $|x| > 1$.

33) Find the equations of the tangent and normal to hyperbola $12x^2-9y^2 = 108$ at $\theta = \frac{\pi}{3}$. (Hint : use parametric form)

34) Evaluate: $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)$.

35) Evaluate the following integrals using properties of integration. $\int_0^1 |5x-3|\,dx$

36) Prove that $q \to p \equiv \neg p \to \neg q$.

37) Find the rank of the matrix $\begin{bmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{bmatrix}$.

38) If the straight lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{m^2}$ and $\frac{x-3}{1} = \frac{y-2}{m^2} = \frac{z-1}{2}$ are coplanar, find the distinct real values of m.

39) Prove that $g(x, y) = x\log\left(\frac{y}{x}\right)$ is homogeneous, what is the degree?

40) For the random variable X with the given probability mass function as below, find the mean and variance. $f(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$

Part - D

Answer all the questions: **7×5=35**

41) a) Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

(OR)

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b) Salt is poured a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

42) a) Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

(OR)

b) Show that $\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$ is real.

43) a) Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that 1/3 is a solution.

(OR)

b) Let $w(x,y,z)=\frac{1}{\sqrt{x^2+y^2+z^2}}$, $(x, y, z)\neq(0, 0, 0)$ show that

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}=0.$$

44) a) Prove that $\tan^{-1}x+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$.

(OR)

b) Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points (3, 6, −2), (−1, −2, 6) and (6, 4, −2).

45) a) A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m, and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be?

(OR)

b) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?

46) a) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

(OR)

b) Find the area of the region bounded by the line $y=2x+5$ and the parabola $y=x^2-2x$.

47) a) A random variable X has the following probability mass function.

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find (i) $P(2<X<6)$ (ii) $P(2\leq X<5)$ (iii) $P(X\leq 4)$ (iv) $P(3<X)$

(OR)

b) Verify:
- i) closure property
- ii) commutative property
- iii) associative property
- iv) existence of identity and
- v) existence of inverse for the operation $+_5$ and Z_5 using table corresponding to addition modulo 5.

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