Class : 12

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COMMON HALF YEARLY EXAMINATION-2024-25

MATHEMATICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

PART - I

Answer all the questions: 20x1=20

1. If $A^T A^{-1}$ is symmetric, then $A^2 =$

   (a) A^{-1} (b) $(A^T)^2$ (c) A^T (d) $(A^{-1})^2$

2. If $(1+i)(1+2i)(1+3i)\ldots\ldots(1+ni) = x+iy$, then $2.5.10\ldots\ldots(1+n^2)$ is

   (a) 1 (b) i (c) $x^2 + y^2$ (d) $1+n^2$

3. The equation $\tan^{-1} x - \cot^{-1} x = \tan^{-1}(1/\sqrt{3})$ has

   (a) No solution (b) Unique Solution
   (c) Two solutions (d) Infinite number of solutions

4. The number of positive roots of the polynomial $\sum_{r=0}^{n} nC_r (-1)^r x^r$ is

   (a) 0 (b) n (c) <n (d) r

5. The radius of the circle $3x^2 + 6y^2 + 4bx - 6by + b^2 = 0$ is

   (a) 1 (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$

6. If Z is a non - zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is

   (a) ½ (b) 1 (c) 2 (d) 3

7. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

   (a) 0 (b) 1 (c) 2 (d) 3

8. If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then n is

   (a) 10 (b) 5 (c) 8 (d) 9

9. A stone is thrown up vertically. The height it reaches at time 't' seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time 't' seconds is given by

   (a) 2 (b) 2.5 (c) 3 (d) 3.5

10. The integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is x, then P(x) is

    (a) x (b) $\frac{x^2}{2}$ (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$

11. If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, Then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is

    (a) $|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ (b) $\frac{1}{3}|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ (c) 1 (d) -1

12. The population P in any year 't' is such that the rate of increase in the population is proportional to the population. Then

    (a) $P = Ce^{kt}$ (b) $P = Ce^{-kt}$ (c) $P = Ckt$ (d) $P = C$

13. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

(a) $\frac{1}{31}$ (b) $\frac{1}{5}$ (c) 5 (d) 31

14. If P (x = 0) = 1 - P(x=1). If E(X) = 3 var(x), then P(x = 0) is

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3}$

15. The maximum slope of the tangent to the curve $y = e^x \sin x, x \in [0, 2\pi]$ is at ------------

(a) $x = \frac{\pi}{4}$ (b) $x = \frac{\pi}{2}$ (c) $x = \pi$ (d) $x = \frac{3\pi}{2}$

16. Which one is the inverse of the statament $(p \vee q) \rightarrow (p \wedge q)$?

(a) $(p \wedge q) \rightarrow (p \vee q)$ (b) $\neg(p \vee q) \rightarrow (p \wedge q)$

(c) $(\neg p \vee \neg q) \rightarrow (\neg p \wedge \neg q)$ (d) $(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$

17. If A is a matrix of order m x n then P(A) is

(a) m (b) n (c) $\leq$ min {m,n} (d) $\geq$ min {m,n}

18. The number of tangents to the circle from inside the circle is

(a) 2 real (b) 0 (c) 2 imaginary (d) cant'be determined

19. Which one is meaningful?

(a) $(\vec{a} \times \vec{b}) \times (\vec{b} \cdot \vec{c})$ (b) $\vec{a} \times (\vec{5} + \vec{b})$ (c) $(\vec{a}.\vec{b}) \times (\vec{c}.\vec{d})$ (d) $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d})$

20. Which of the following is not a binary operation on R?

(a) + (b) - (c) ÷ (d) x

PART - II

Answer any 7 Questions. Question Nunmber 30 is compulsory. **7x2=14**

21. Show that th vectors $\hat{i} + 2\hat{j} - 3\hat{k}$; $2\hat{i} - \hat{j} + 2\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ are coplanar.

22. If $x^2 + 2(k+2)x + 9k = 0$ has equal roots, Find K.

23. Find the square root of -6 + 8i.

24. Find the principal value of $\cos^{-1}(1/2)$

25. Examine the position of the point (2,3) with respect to the circle $x^2 + y^2 - 6x - 8y + 12 = 0$

26. A person learnt 100 words for an English test. The number of words the person remembers in 't' days after learning is given by $w(t) = 100 \times (1 - 0.1t)^2$ $0 \leq t \leq 10$. What is the rate at which the person forgets the words 2 days after learning?

27. The probability density function of x is given by

$$f(x) = \begin{cases} Ke^{-x/3} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

Find (i) The value of K

28. **Find the partial derivatives of the functions.**

$g(x,y) = 3x^2 + y^2 + 5x + 2$ at the point (1,-2)

29. The following physical statement. Write in the form of differential equation.
A saving amount pays 8% interest per year, compounded continuously. In addition, the income from another investment is credited to the amount continuously at the rate of ₹.400 per year.

30. Solve, by Cramer's rule, the system of equations. $5x - 2y + 16 = 0, x + 3y - 7 = 0$

PART - III

III. **Answer any 7 Questions. Question Number 40 is compulsory.** **7x3=21**

31. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$ Verify that $(AB)^{-1} = B^{-1}A^{-1}$

32. Prove that $\tan(\sin^{-1}x) = \dfrac{x}{\sqrt{1-x^2}}$ for $|x| < 1$

33. Obtain the Cartesian form of the locus of $z = x+iy$ in $|z+i| = |z-1|$

34. Show that $\Gamma(n) = 2\int_0^\infty e^{-x^2} x^{2n-1}\,dx$.

35. If $X \sim B(n,p)$ such that $4P(x=4) = P(x=2)$ and $n = 6$ Find the distribution, mean and standard deviation of X.

36. If $u(x,y,z) = xy^2z^3$, $x = \sin t$; $y = \cos t$; $z = 1 + e^{2t}$ Find $\dfrac{du}{dt}$

37. Evaluate : $\lim\limits_{x\to 0^+} x \log x$

38. Solve : $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$

39. Using the truth table for the following statement $(p \vee q) \vee \neg q$.

40. Find the angle between the planes $\vec{r}\cdot(\hat{i} + \hat{j} - 2\hat{k}) = 3$ and $2x - 2y + z = 2$.

PART - IV

Answer all the questions **7x5=35**

41. a) If $ax^2 + bx + c$ is divided by $x + 3$, $x - 5$ and $x - 1$, The remainders are 21, 61 and 9 respectively. Find a, b, and c. [Use Gaussian Elimination method]

(OR)

b) Find the vector parametric, vector non - parametric and cartesian form of the equation of the plane passing through the points (-1, 2, 0), (2, 2, -1) and parallel to the straight line. $\dfrac{x-1}{1} = \dfrac{2y+1}{2} = \dfrac{z+1}{-1}$

42. a) Solve the equation $z^3 + 27 = 0$ **(OR)**

b) A random variable x has the following probability mass function.

X	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find i) $P(2 < x < 6)$ ii) $P(2 \le x < 5)$ iii) $P(x \le 4)$ iv) $P(3 < x)$

43. a) Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root.

(OR)

b) Find the angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection (0,0) and (1,1)

44. a) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and $0 < x, y, z < 1$ Show that $x^2 + y^2 + z^2 + 2xyz = 1$

(OR)

b) Verify i) Closure property ii) Commutative property iii) Associative property iv) Existence of identity and v) existence of inverse for the operation X_{11} on a subset A = { 1, 3, 4, 5, 9} of the set of remainders {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

45. a) Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolis path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How for beyond this vertical line will the water strike the ground?

(OR)

b) $U = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$ then prove that $x\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} = \sin 2u$

46. a) Show that the lines $\frac{x-3}{3} = \frac{y-3}{-1}$, $z - 1 = 0$ and $\frac{x-6}{2} = \frac{z-1}{3}$, $y - 2 = 0$ intersect. Also find the point of inflection.

(OR)

b) Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.

47. a) Find the volume of a right - circular cone of base radius 'r' and height 'h'.

(OR)

b) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cyclinder that can be hidden underneath is $4/9$ times volume of the cone.