CHENGALPATTU - DISTRICT

# COMMON HALF YEARLY EXAMINATION - 2024

**Standard XII** Reg.No.

## MATHEMATICS

*Time : 3.00 hrs* **Part - A** *Marks : 90*

**I. Choose the correct answer:** **20 x 1 = 20**

1. If A is a non–singular matrix of odd order, then adj A is
   a) negative b) zero c) |adj A| d) positive

2. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then
   a) $c = \pm 3$ b) $c = \pm\sqrt{3}$ c) $c > 0$ d) $0 < c < 1$

3. Which of the following is the principle value branch of $\cos^{-1}x$ ?
   a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ b) $(0, \pi)$ c) $[0, \pi]$ d) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$

4. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents
   a) straight lines b) circles c) parabola d) ellipse

5. Centre (–3,–4)and radius 3 units then find the general equation of the circle is
   a) $x^2 + y^2 + 6x + 8y + 16 = 0$ b) $x^2 + y^2 + 6x + 8y = 16$
   c) $x^2 + y^2 + 6x + 8y - 16 = 0$ d) $x^2 + y^2 - 6x - 8y = 16$

6. If $\vec{a}, \vec{b}, \vec{c}$ are non–coplanar, non–zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$, then $\{[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]\}^2$ is equal to
   a) 81 b) 9 c) 27 d) 18

7. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies
   a) $|k| \le 6$ b) $k = 0$ c) $|k| > 6$ d) $|k| \ge 6$

8. If $P(X = 0) = 1 - P(X = 1)$. If $E(X) = 3\,Var(X)$, then $P(X = 0)$ is
   a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

9. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by
   a) 2 b) 2.5 c) 3 d) 3.5

10. In the set $\mathbb{R}$ of real numbers * is defined as follows. Which one of the following is not a binary operation on $\mathbb{R}$ ?
    a) a* b = min (a. b) b) a* b= max (a, b) c) a* b = a d) $a * b = a^b$

11. The sum of the focal distances of any point on the ellipse is equal to
    a) length of the minor axis b) length of the major axis
    b) length of the letus rectum d) eccentricity

12. The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0, 3]$ is
    a) 1 b) $\sqrt{2}$ c) $\frac{3}{2}$ d) 2

13. If $\rho(A) = \rho([A/B])$, then the system $AX = B$ of linear equations is
a) consistent and has a unique solution b) consistent
c) consistent and has infinitely many solution d) inconsistent

14. If $z + z^{-1} = 1$, then $z^{100} + z^{-100} =$
a) $i$ b) $-i$ c) $+1$ d) $-1$

15. The principle argument of $(\sin 40^\circ + i \cos 40^\circ)^2$ is
a) $-110^\circ$ b) $-70^\circ$ c) $70^\circ$ d) $110^\circ$

16. The point of inflection of the curve $y = (x-1)^3$ is
a) (0,0) b) (0,1) c) (1,0) d) (1,1)

17. Find the value of $\int_0^{\pi/2} \sin^4 x \cos^6 x \, dx$
a) $\frac{3\pi}{512}$ b) $\frac{3}{512}$ c) $\frac{3\pi}{256}$ d) $\frac{3}{256}$

18. A polynomial equation in x of degree n always has
a) n distinct roots b) n real roots c) n complex roots d) atmost one root.

19. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ then $\cos^{-1}x + \cos^{-1}y =$
a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$ c) $\frac{\pi}{6}$ d) $\pi$

20. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is
a) 3 b) $-1$ c) 1 d) 9

## Part - B

**II. Answer any 7 questions. (Q.No.30 is compulsory)** **7 x 2 = 14**

21. If $\text{adj}\, A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find $A^{-1}$

22. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form.

23. If $\alpha, \beta, \gamma$ and $\delta$ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$, find a quadratic equation with integer coefficients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$

24. Find the equations of the tangent and normal to the circle $x^2 + y^2 = 25$ at $P(-3, 4)$

25. Find the acute angle between the straight lines $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$ state whether they are parallel or perpendicular.

26. Suppose f(x) is differentiable function for all x with $f'(x) \le 29$ and $f(2) = 17$. What is the maximum value of f(7) ?

27. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 3$, and $dx = 0.02$.

28. Evaluate : $\int_{-\pi/2}^{\pi/2} x \cos x \, dx$

29. Solve the differential equations $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

30. Prove that in an algebraic structure the identity element (if exists) must be unique.

**Part - C**

**III. Answer any 7 questions. (Q.No.40 is compulsory)** **7 x 3 = 21**

31. Find the rank matrix by minor method $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$

32. Find the square root of $-5 - 12i$

33. Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.

34. With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

35. Evaluate : $\lim_{x\to\infty} \frac{2x^2-3}{x^2-5x+3}$

36. If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$

37. The maximum and minimum distances of the Earth from the Sun respectively are $152 \times 10^6$ km and $94.5 \times 10^6$ km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

38. Find the differential equation of the family of circles passing through the origin and having their centres on the x –axis.

39. A pair of fair dies is rolled once. Fine the probability mass function to get the number of four.

40. Evaluate : $\int_1^4 \left(\frac{8}{\sqrt{x}} - 12\sqrt{x^3}\right)dx$

**Part - D**

**IV. Answer all the questions.** **7 x 5 = 35**

41. a) Investigate the values of $\lambda$ and $\mu$ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$ (i) no solution (ii) a unique solution (iii) an infinite number of solutions

*(OR)*

b) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, show that $x + y + z = xyz$

42. a) Find all cube roots of $\sqrt{3} + i$ *(OR)*

b) Prove that $\int_0^{\pi/4} \log(1+\tan x)dx = \frac{\pi}{8}\log 2$

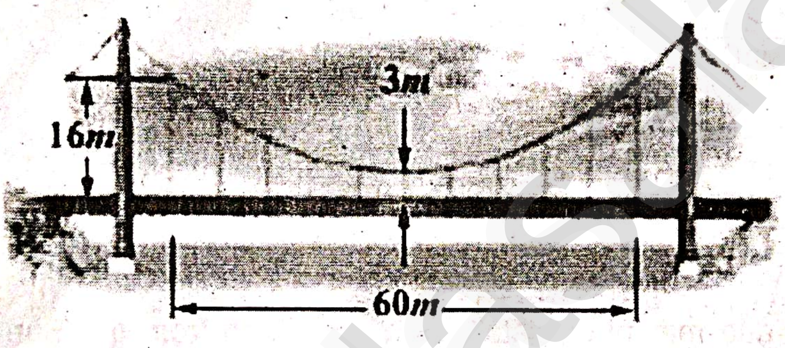
43\. a) Prove that among all the rectangle of the given perimeter, the square has the maximum areas. *(OR)*

b) Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

44\. a) Solve the following equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ *(OR)*

b) A radioactive isotope has an initial mass 200mg , After two years it is decreased by 50mg . Find the expression for the amount of the isotope remaining at any time. What is its half–life? (half–life means the time taken for the radioactivity of a specified isotope to fall to half its original value).

45\. a) Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.




*(OR)*

b) Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function

$f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & \text{Otherwise} \end{cases}$, Find (i) the value of k (ii) the distribution function (iii) the probability that daily sales will fall between 300 litres and 500 litres?

46 a) Find the non–parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$ *(OR)*

b) Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$

47\. a) Find the equations of tangent and normal to the curve given by $x = 7\cos t$ and $y = 2\sin t$, $t \in R$ at any point on the curve *(OR)*

b) Using the equivalence property, Show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

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16.12.24

ST. JOSEPH'S HR. SEC. SCHOOL, CHENGALPATTU

COMMON HALF YEARLY EXAMINATION - 2024

STD - 11 MATHEMATICS - KEY MARKS: 90

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I

| | | |
|---|---|---|
| 1 | | MA |
| 2 | b | $c = \pm\sqrt{3}$ |
| 3. | c | $[0, \pi]$ |
| 4. | c | parabola |
| 5. | a | $x^2+y^2+6x+8y+16=0$ |
| 6. | a | 81 |
| 7. | d | $\lvert k \rvert \geq 6$ |
| 8. | d | $1/3$ |
| 9. | b | 2.5 |
| 10. | d | $a*b = a^b$ |
| 11 | b | length of the major axis |
| 12 | d | 2 |
| 13 | b | consistent |
| 14 | d | $-1$ |
| 15 | a | $-110°$ |
| 16 | c | $(1,0)$ |
| 17 | a | $3\pi/512$ |
| 18 | c | n complex roots |
| 19 | a | $\pi/3$ |
| 20. | d | 9 |

II

21. $\lvert adj A \rvert = 9$, $A^{-1} = \pm \frac{1}{\sqrt{\lvert adj A \rvert}} adj A$

$A^{-1} = \pm \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

22. $\left(\frac{1+i}{1-i}\right) = i$, $\left(\frac{1-i}{1+i}\right) = -i$

$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = i^3 - (-i)^3 = -i - i = -2i$

23. $\alpha+\beta+\gamma+\delta = -5/2$, $\alpha\beta\gamma\delta = 8/2 = 4$

$(x+\frac{5}{2})(x-4) = 0 \Rightarrow 2x^2-3x-20=0$

24. $xx_1 + yy_1 = a^2 \Rightarrow x(-3)+y(4) = 25$

eqn. of Tgt : $-3x+4y = 25$

eqn. of normal : $4x+3y = 0$

25. $\vec{a} = 2\hat{i}+\hat{j}-2\hat{k}$, $\vec{b} = 4\hat{i}-4\hat{j}+2\hat{k}$

$\hat{a}\cdot\hat{b} = 8-4-4 = 0 \Rightarrow \perp r$, $\theta = \pi/2$

26. $\frac{f(7)-f(2)}{7-2} \leq 29 \Rightarrow f(7) \leq 162$

27. $dy = (2x+3)\,dx$

$= 9 \times 0.02$

$= 0.18$

28. $f(x) = x\cos x$, $f(-x) = -x\cos x = -f(x)$

$f(x)$ is an odd fn.

$\int_{-\pi/2}^{\pi/2} x\cos x\,dx = 0$

29. $\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}} \Rightarrow \sin^{-1}y = \sin^{-1}x + C$

30. $a*a_1 = a_1*a = e \Rightarrow a*a_2 = a_2*a = e$

$a_1 = a_1*e = a_1*(a*a_2) = (a_1*a)*a_2 = e*a_2 = a_2$

$a_1 = a_2$ inverse of a is unique

III

31. $\lvert A \rvert = 1(-4+6) + 2(-2+30) + 3(2-20)$

$= 1(2)+2(28)+3(-10) = 2+56-30 \neq 0$

$\rho(A) = 3$

32. $\sqrt{-5-12i} = \pm\left(\sqrt{\frac{13-5}{2}} - i\sqrt{\frac{13+5}{2}}\right) = \pm(\sqrt{4} - i\sqrt{9})$

$= \pm(2-3i)$

33. Let $\cos^{-1}(-x) = y \Rightarrow \cos y = -x$

$x = -\cos y = +\cos(\pi-y)$

$\cos^{-1}x = \pi - y \Rightarrow \cos^{-1}x = \pi - \cos^{-1}(-x)$

$\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

34. $\lvert\vec{a}\times\vec{b}\rvert = ab\sin C$

$\lvert\vec{a}\times\vec{b}\rvert = \lvert\vec{b}\times\vec{c}\rvert = \lvert\vec{c}\times\vec{a}\rvert$

$ab\sin C = bc\sin A = ac\sin B$

$\div abc$ $\quad \frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b}$

35. $\lim_{x\to\infty} \frac{2-3/x^2}{1-5/x+3/x^2} = \frac{2}{1} = 2$

36. $u(tx, ty) = t^{3/2}u(x,y)$

By Euler's Theorem.

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$

37. $a+c = 152\times10^6$

$a-c = 94.5\times10^6$

$2c = 57.5\times10^6$

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38. centre (h, 0)

$(x-h)^2+(y-0)^2=h^2$ $\quad x-h=-yy'$, $h=x+yy'$

$2(x-h)+2yy'=0$

$y^2=x^2+2xy\frac{dy}{dx}$

39.

| value of Random variable x | 0 | 1 | 2 | Total |
|---|---|---|---|---|
| No. of elements in inverse image | 25 | 10 | 1 | 36 |

Probability mass function

| x | 0 | 1 | 2 |
|---|---|---|---|
| f(x) | 25/36 | 10/36 | 1/36 |

40. $=\left[8\frac{x^{1/2}}{1/2}-12\cdot\frac{x^{5/2}}{5/2}\right]_1^4=$

$=-\frac{664}{5}$

IV

41 a. $(A, B)=\begin{bmatrix}2 & 3 & 5 & 9\\ 0 & -5 & -45 & -47\\ 0 & 0 & \lambda-5 & \mu-9\end{bmatrix}$

i. $\lambda=5$, $\mu\neq 9 \Rightarrow$ no soln

ii. $\lambda\neq 5 \Rightarrow$ unique

iii. $\lambda=5, \mu=9 \Rightarrow$ infinite

b. $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

$\tan\pi=0$

$x+y+z=xyz$

42. a. $\sqrt{3}+i=2(\cos\pi/6+i\sin\pi/6)$

$(\sqrt{3}+i)^{1/3}=2^{1/3}\left[\cos\left(\frac{\pi+12k\pi}{18}\right)+i\sin\left(\frac{\pi+12k\pi}{18}\right)\right]$ $\quad k=0,1,2$

$k=0$ $\quad z=2^{1/3}\,\mathrm{cis}\,\pi/18$

$k=1$ $\quad z=2^{1/3}\,\mathrm{cis}\,13\pi/18$

$k=2$ $\quad z=2^{1/3}\,\mathrm{cis}\,\frac{25\pi}{18}=2^{1/3}\left(-\mathrm{cis}\,\frac{7\pi}{18}\right)$

b. $I=\int_0^{\pi/4}\log(1+\tan x)\,dx$

$I=\int_0^{\pi/4}\log[1+\tan(\pi/4-x)]\,dx$

$=\int_0^{\pi/4}\log\left[1+\frac{1-\tan x}{1+\tan x}\right]dx$

$=\int_0^{\pi/4}[\log 2-\log(1+\tan x)]\,dx$

$=\log 2\int_0^{\pi/4}dx-\int_0^{\pi/4}\log(1+\tan x)\,dx$

$=\pi/4\log 2-I$

$2I=\pi/4\log 2 \Rightarrow I=\pi/8\log 2$

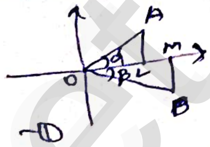
43 a. $p=2(x+y)$ $\quad A=xy$ $\quad \frac{dA}{dx}=\frac{p}{2}-2x$

$\frac{dA}{dx}=0 \Rightarrow x=p/4$, $\frac{d^2A}{dx^2}=-ve$ (max)

$x=p/4$, $y=p/4$ $\quad \therefore$ square

b. $\vec{OA}=\cos\alpha\hat{i}+\sin\alpha\hat{j}=\vec{a}$

$\vec{OB}=\cos\alpha\hat{i}-\sin\alpha\hat{j}=\vec{b}$




By value $\hat{b}\times\hat{a}=\sin(\alpha+\beta)\hat{k}$ — ①

By def $\hat{b}\times\hat{a}=\begin{vmatrix}i & j & k\\ \cos\beta & -\sin\beta & 0\\ \cos\alpha & \sin\alpha & 0\end{vmatrix}=(\sin\alpha\cos\beta+\cos\alpha\sin\beta)\hat{k}$ — ②

① & ② $\quad \sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

44 a. $\div x^2$ $\quad x^2-10x+26-\frac{10}{x}+\frac{1}{x^2}=0$

$(x^2+1/x^2)-10(x+1/x)+26=0$

$x+1/x=t \Rightarrow t^2-2-10t+26=0$

$t=6, 4$

$x+1/x=6 \Rightarrow x=3\pm 2\sqrt{2}$

$x+1/x=4 \Rightarrow x=2\pm\sqrt{3}$

b. $\frac{dA}{dt}\propto A \Rightarrow A=ce^{-kt}$

(i) $c=200 \Rightarrow A=200e^{-kt}$

(ii) $t=2$ $\quad A=150 \Rightarrow A=\frac{1}{2}\log(4/3)$

(iii) $A=100$ $\quad t=?$ $\Rightarrow t=\frac{2\log(1/2)}{\log(3/4)}$

45. a. $x^2=4ay$ — ①

(i) (30, 13) is a pt $\quad 4a=\frac{900}{13}$

① $\Rightarrow x^2=\frac{900}{13}y$

B(6, y) is a pt $\Rightarrow y=0.52$

cable from the road $=3+0.52=3.52$ m

(ii) B(12, y) $\Rightarrow 12^2=\frac{30^2}{13}y \Rightarrow y=2.08$

cable from the road $=3+2.08=5.08$

b (i) $\int_{200}^{600}k\,dx=1 \Rightarrow k=\frac{1}{400}$

(ii) $F(x)=\begin{cases}0 & x<200\\ \frac{x}{400}-\frac{1}{2} & 200\le x\le 600\\ 1 & x>600\end{cases}$

(iii) $P(300\le x\le 500)=F(500)-F(300)=\frac{1}{2}$

46 a. $\vec{a}=2\hat{i}+3\hat{j}+6\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$, $\vec{v}=2\hat{i}-5\hat{j}-3\hat{k}$

$\vec{u}\times\vec{v}=-4\hat{i}+8\hat{j}-16\hat{k}$

$(\vec{r}-\vec{a})\cdot(\vec{u}\times\vec{v})=0$

$[\vec{r}-(2\hat{i}+3\hat{j}+6\hat{k})]\cdot[-4\hat{i}+8\hat{j}-16\hat{k}]=0$

$\vec{r}\cdot(\hat{i}-2\hat{j}+4\hat{k})=20$

$x-2y+4z=20$

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46 b. $A = 2\left[\int_0^2 y_1\,dx + \int_2^4 y_2\,dx\right]$

$= 2\left[\int_0^2 \sqrt{6}\sqrt{x}\,dx + \int_2^4 \sqrt{16-x^2}\,dx\right]$

$= \frac{4}{3}(4\pi + \sqrt{3})$

47. a. $\frac{dy}{dx} = m = \frac{dy/dt}{dx/dt} = \frac{2\cos t}{-7\sin t}$

eqn. of tangent: $y - y_1 = m(x - x_1)$

$y - 2\sin t = -\frac{2\cos t}{7\sin t}(x - 7\cos t)$

$\boxed{2x\cos t + 7y\sin t = 14}$

eqn of normal: $y - y_1 = -\frac{1}{m}(x - x_1)$

$7x\sin t - 2y\cos t = 45\sin t\cos t$

b. $p \Leftrightarrow q \equiv (p\wedge q) \vee (\neg p \wedge \neg q)$

| $p$ | $q$ | $p\Leftrightarrow q$ | $p\wedge q$ | $\neg p$ | $\neg q$ | $\neg p\wedge\neg q$ | $(p\wedge q)\vee(\neg p\wedge\neg q)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | F | F | F | T |
| T | F | F | F | F | T | F | F |
| F | T | F | F | T | F | F | F |
| F | F | T | F | T | T | T | T |

$p\Leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$

$\equiv (\neg p \vee q) \wedge (p \vee \neg q)$

$\equiv (\neg p \wedge (p \vee \neg q)) \vee (q \vee (p \vee \neg q))$

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee (q \wedge \neg q)$

$\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$p\Leftrightarrow q \equiv (p\wedge q) \vee (\neg p \wedge \neg q)$