12

Register No. ______

Half Yearly Examination - 2024

Time : 3.00 Hrs. **MATHEMATICS** Marks : 90

PART - A

Choose the correct or the most suitable answer. **20 x 1 = 20**

1. If A $\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix} = \begin{bmatrix}6 & 0\\0 & 6\end{bmatrix}$ then A = a) $\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}$ b) $\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}$ c) $\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}$ d) $\begin{bmatrix}4 & -1\\2 & 1\end{bmatrix}$

2. If A = $\begin{bmatrix}2 & 3\\5 & -2\end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is a) 17 b) 14 c) 19 d) 21

3. The area of the triangle formed by the complex number z, iz, and z + iz in the Argand's diagram is
a) $\frac{1}{2}|z|^2$ b) $|z|^2$ c) $\frac{3}{2}|z|^2$ c) $2|z|^2$

4. If $|z - 2 + i| \le 2$, then the greatest value of $|z|$ is a) $\sqrt{3}-2$ b) $\sqrt{3}+2$ c) $\sqrt{5}-2$ d) $\sqrt{5}+2$

5. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only it, k satisfies a) $|k| \le 6$ b) k = 0 c) $|k| > 0$ d) $|k| \ge 6$

6. If $\sin^{-1}\frac{x}{5} + \operatorname{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$ - then the value of x is a) 4 b) 5 c) 2 d) 3

7. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

8. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

9. If x + y = k is a normal to the parabola $y^2 = 12x$, then the value of k is a) 3 b) –1 c) 1 d) 9

10. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a} \times \vec{b}] = \frac{1}{4}$ then the angle between $\vec{a}$ and $\vec{b}$ is a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

11. The angle between the line $\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + t(2\hat{i} + \hat{j} - 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} + \hat{j}) + 4 = 0$ is a) 0^0 b) 30^0 c) 45^0 d) 90^0

12. The position of a particle moving along a horizontal line of any time r is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is a) t = 0 b) t = $\frac{1}{3}$ c) t = 1 d) t = 3

13. The number given by the Mean value theorem for the function $\frac{1}{x}$, $x \in [1, 9]$ is a) 2 b) 2.5 c) 3 d) 3.5

14. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31? a) $\frac{1}{31}$ b) $\frac{1}{5}$ c) 5 d) 31

15. If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to a) xye^{xy} b) $(1 + xy)e^{xy}$ c) $(1 + y)e^{xy}$ d) $(1 + x)e^{xy}$

16. The value of $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$ is a) $\frac{\pi}{6}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) π

17. The value of $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x \, dx$ is a) $\frac{3}{2}$ b) $\frac{1}{2}$ c) 0 d) $\frac{2}{3}$

18. The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is
a) $y + \sin^{-1} x = c$ b) $x + \sin^{-1} y = 0$ c) $y^2 + 2\sin^{-1} x = c$ d) $x^2 + 2\sin^{-1} y = 0$

19. The integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is x, then P(x) a) x b) $\frac{x^2}{2}$ c) $\frac{1}{x}$ d) $\frac{1}{x^2}$

20. The polynomial $x^3 + 2x + 3$ has
a) one negative and two imaginary zeros b) one positive and two imaginary zeros c) three real zeros d) no zeros

PART - B

i) Answer any seven questions. ii) Question No.30 is compulsory. **7 x 2 = 14**

21. If adj(A) = $\begin{bmatrix}0 & -2 & 0\\6 & 2 & -6\\-3 & 0 & 6\end{bmatrix}$, find A^{-1}

22. If $z_1 = 3 - 2i$ and $z_2 = 6 + 4i$, find $\frac{z_1}{z_2}$ in the rectangular form.

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23. Find a polynomial equation of minimum degree with rational coefficients, having 2i + 3 as a root.

24. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

25. If y = 4x + c is a tangent to the circle $x^2 + y^2 = 9$, find c.

26. Determine whether the three vectors $2\hat{i}+3\hat{j}+\hat{k}$, $\hat{i}-2\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}+3\hat{k}$ are coplanar.

27. A particle moves along a straight line in such a way that after t seconds its distance from the origin is $s = 2t^2 + 3t$ metres. Find the average velocity between t = 3 and t = 6 seconds.

28. If U (x, y, z) = $\log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$.

29. Evaluate $\int_0^1 x^3(1-x)^4\,dx$

30. Determine the order and degree (if exists) of the differential equation $\left(\frac{d^4y}{dx^4}\right)^3 + 4\left(\frac{dy}{dx}\right)^7 + 6y = 5\cos 3x$.

PART - C

i) Answer any seven quetsions. (ii) Question No.40 is compulsory. **7 x 3 = 21**

31. Solve the following system of linear equations, using matrix inversion method. 5x + 2y = 3, 3x + 2y = 5.

32. Show that $(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$ is purely imaginary.

33. Find the exact number of real zeros and imaginary of the polynomial $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$

34. Find the value of $\cos\left(\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right)$

35. Find the equation of the ellipse with foci (0, $\pm$4) and end points of major axis are (0, $\pm$5)

36. If the two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-m}{2} = z$ intersect at a point, find the value of m.

37. Write the Maclaurin series expansion of e^x.

38. Use the linear approximation to find approximate values of $\sqrt[3]{26}$.

39. Evaluate : $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$

40. Show that $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

PART - D

Answer all the questions. **7 x 5 = 35**

41. a) Investigate for what values of λ and μ the system of linear equations x + 2y + z = 7, x + y + λz = μ, x + 3y – 5z = 5 has i) no solution ii) a unique solution iii) an infinite number of solutions. **(OR)**

b) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and 0 < x, y, z < 1, show that $x^2 + y^2 + z^2 + 2xyz = 1$

42. a) A random variable X has the following probability mass function

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find i) P (2 < x < 6) ii) P(2 ≤ x < 5) iii) P(x ≤ 4) iv) P(3 < x) **(OR)**

b) Determine the intervals of concavity of the curve $f(x) = (x-1)^3 \cdot (x-5)$, $x \in R$ and points of inflection if any

43. a) Find the foci, vertices and length of major and minor axis of the conic $4x^2 + 36y^2 + 40x - 288y + 532 = 0$. **(OR)**

b) Find the parametric form of vector equation and Cartesian equations of the plane containing the line $\vec{r} = (\hat{i}-\hat{j}+3\hat{k}) + t(2\hat{i}-\hat{j}+4\hat{k})$ and perpendicular to plane $\vec{r}\cdot(\hat{i}+2\hat{j}+\hat{k}) = 8$.

44. a) Prove by vector method that $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ **(OR)**

b) Find the area of the region bounded by the line y = 2x + 5 and the parabola $y = x^2 - 2x$.

45. a) If $u = \sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$. **(OR)**

b) Solve the equation $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

46. a) In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperatue of the person before death was 98.6°F at what time did the murder occur? [log (2.43) = 0.88789 ; log (0.5) = –0.69315] **(OR)**

b) If z = x + iy and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$ show that $x^2 + y^2 + 3x - 3y + 2 = 0$

47. a) At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5 m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75m from the point of origin. **(OR)**

b) Prove that $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

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Part – A

1. c) $\begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$
2. c) 19
3. a) $\frac{1}{2}|z|^2$
4. d) $\sqrt{5}+2$
5. d) $|k| \geq 6$
6. d) 3
7. d) $\tan^{-1}(1/2)$
8. c) $\sqrt{10}$
9. d) 9
10. a) $\pi/6$
11. c) 45°
12. b) $t = 1/3$
13. ~~b) 1/5~~ c) 3
14. b) $1/5$
15. b) $(1+xy)\,e^{xy}$
16. ~~b) (1+xy) e^xy~~ a) $\pi/6$
17. d) $2/3$
18. a) $y + \sin^{-1}x = c$
19. c) $1/x$
20. a) 1st & 2nd IR

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Part - B

21) Gn.: adj A = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

$A^{-1} = \pm \frac{1}{\sqrt{|adj A|}} (adj A)$

$= \pm \frac{1}{6} \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

22) $z_1 = 3-2i, z_2 = 6+4i$

$\frac{z_1}{z_2} = \frac{3-2i}{6+4i} \times \frac{6-4i}{6-4i}$

$= \frac{10-24i}{52} = \frac{\cancel{2}(5-12i)}{\cancel{52}\,26}$

$= \frac{5-12i}{26}$

23) Gn.: One root : $3+2i$

Other root : $3-2i$

SOR : 6 & POR : 13

Req. eqn : $x^2 - 6x + 13 = 0$.

24) P.V. $\cos^{-1}(\sqrt{3}/2)$

$= \frac{\pi}{6}$

25) $m = 4, c = c, a^2 = 9$

$c^2 = a^2(1+m^2)$

$= 9 \times 17$

$\boxed{c = \pm 3\sqrt{17}}$

26) $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 2 \\ 3 & 1 & 3 \end{vmatrix}$

$= 2(-8) - 3(-3) + 1(7)$

$= -16 + 9 + 7$

$= 0.$

∴ The gn Vrs are co-planar.

27) $s(t) = 2t^2 + 3t$.

$s(3) = 27$ & $s(6) = 90$

Vel. $= \frac{90-27}{3} = \frac{63}{3} = 21$ m/s.

28) $u = \log(x^3+y^3+z^3)$

$\frac{\partial u}{\partial x} = \frac{1}{x^3+y^3+z^3} \times 3x^2$.

$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3(x^2+y^2+z^2)}{x^3+y^3+z^3}$

29) $I = \int_0^1 x^3(1-x)^4 dx$

$= \frac{3! \times 4!}{(3+4+1)!} = \frac{(\cancel{3}\times\cancel{2})\times\cancel{4!}}{8\times7\times\cancel{6}\times5\times\cancel{4!}}$

$= \frac{1}{280}$

30) Order : 4

Degree : 3

Part - C

31) $A = \begin{pmatrix} 5 & 2 \\ 3 & 2 \end{pmatrix}; B = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

$|A| = 4$

$X = \frac{1}{4}\begin{pmatrix} 2 & -2 \\ -3 & 5 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix}$

$X = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

32) Let $z = (2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$

$\bar{z} = \overline{(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}}$

$= (2-i\sqrt{3})^{10} - (2+i\sqrt{3})^{10}$

$\bar{z} = -z$

∴ z is purely imaginary.

33) $P(x) = x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$

$P(-x) = -x^9 - 9x^7 - 7x^5 - 5x^3 - 3x$

$x = 0$ is a root

∴ No of real zeros : 1

No of imaginary : 8

(34) $\cos\left[\cos^{-1}(4/5)+\sin^{-1}(4/5)\right]$

$= \cos \pi/2$

$= 0$

(35) Gn.: $ae = 4$ & $a = 5$

$5 \times e = 4$ | $\boxed{a^2 = 25}$

$\boxed{e = 4/5}$

& $b^2 = a^2(1-e^2)$

$= 25\left(1-\frac{16}{25}\right)$

$\boxed{b^2 = 9}$

Eqn of the ellipse

T-II $\boxed{\frac{x^2}{9} + \frac{y^2}{25} = 1}$

(36) Condition:

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0.$$

$$\Rightarrow \begin{vmatrix} 2 & m+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 2(-5)-(m+1)(-2) \\ -1(1) \} = 0$

$\Rightarrow -10 + 2m + 2 - 1 = 0$

$\boxed{m = 9/2}$

(37) $f(x) = e^x$ | $f(0) = 1$

$f'(x) = e^x$ | $f'(0) = 1$

$f''(x) = e^x$ | $f''(0) = 1$

$f'''(x) = e^x$ | $f'''(0) = 1$

M.S

$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{x^2}{2!}f''(0) + \cdots$

$\boxed{e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots}$

(38) Let $f(x) = x^{1/3}$ | $x_0 = 27$

$f'(x) = \frac{1}{3}x^{-2/3}$ | $\Delta x = -1$

$= \frac{1}{3x^{2/3}}$ | $x_0 + \Delta x = 26$

$\boxed{L(x) = f(x_0) + f'(x_0)(x - x_0)}$

$= 3 + \frac{1}{27}(-1)$

$= 3 - 0.037$

$\boxed{L(x) = 2.963}$

(39) $I = \int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$ — (1)

$x \to 5-x$

$I = \int_2^3 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx$ — (2)

(1) + (2)

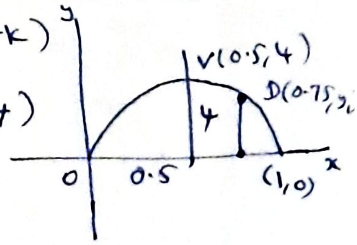
$2I = (x)_2^3 \Rightarrow \boxed{I = 1/2}$

(40)

p	q	$p \wedge q$	$\sim(p\wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

47) a) $(x-h)^2 = -4a(y-k)$

$(x-0.5)^2 = -4a(y-4)$



$(0,0)$: $4a = \frac{0.25}{4} = \frac{1}{16}$

$D(0.75, y_1)$: $\frac{1}{16} = -\frac{1}{16}(y_1 - 4) \Rightarrow \boxed{y_1 = 3m}$

47) b)

p	q	$p \leftrightarrow q$	$p \to q$ (r)	$q \to p$ (s)	$r \wedge s$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

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Part - D

41) a)

$$[A, B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 1 & \lambda & \mu \\ 1 & 3 & -5 & 5 \end{array}\right]$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & -1 & \lambda-1 & \mu-7 \\ 0 & 1 & -6 & -2 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & -1 & \lambda-1 & \mu-7 \\ 0 & 0 & \lambda-7 & \mu-9 \end{bmatrix}$$

(i) No soln. : $\lambda = 7, \mu \neq 9$

(ii) Unique : $\lambda \neq 7, \mu \in \mathbb{R}$

(iii) ∞'ly many : $\lambda = 7, \mu = 9$

(b) Gn. : $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ — ①

Let $\alpha = \cos^{-1}x$, $\beta = \cos^{-1}y$

$\Rightarrow x = \cos\alpha$, $y = \cos\beta$

① $\Rightarrow$ $\boxed{\alpha + \beta = \pi - \cos^{-1}z}$

WKT $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\pi - \cos^{-1}z) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$-\cos(\cos^{-1}z) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$-xy - z = -\sqrt{1-x^2}\sqrt{1-y^2}$

$\Rightarrow \boxed{x^2 + y^2 + z^2 + 2xyz = 1}$

42) a) WKT $\sum f(x) = 1$

$30k = 1$

$\boxed{k = 1/30}$

(i) $P(2 < x < 6) = P(x=3) + P(x=4) + P(x=5)$

$= \cancel{\tfrac{17}{30}}\ \tfrac{17}{30}$

ii) $P(2 \le x < 5) = P(x=2) + P(x=3) + P(x=4)$

$= \frac{13}{30}$

iii) $P(x \le 4) = 1 - P(x > 4) = 1 - \{P(x=5) + P(x=6)\}$

$= \frac{14}{30}$

iv) $P(3 < x) = P(x > 3) = P(x=4) + P(x=5) + P(x=6)$

$= 21k = \frac{21}{30}$

42) b) $f(x) = (x-1)^3(x-5)$

$f'(x) = (x-1)^3 + 3(x-1)^2 \cdot (x-5)$

$(x-1) + 3x - 15 = 4x - 16 = 4(x-4)$

$f'(x) = (x-1)^2 \cdot 4(x-4)$

$f''(x) = 4(x-4) \cdot 2(x-1) + 4(x-1)^2$

$8(x-4) + 4(x-1) = \boxed{12x - 36}$

$= 12(x-1)(x-3)$

$f''(x) = 0$

$\Rightarrow \boxed{x = 1, x = 3}$

$-\infty \quad 1 \quad 3 \quad \infty$

$(-\infty, 1) \cup (3, \infty)$ concave $\uparrow$

$(1, 3)$ " $\downarrow$

Pts of Inflection : $(1, 0)$ & $(3, -16)$

43) a) $\dfrac{(x+5)^2}{36} + \dfrac{(y-4)^2}{4} = 1$

$A(1, 4)$

$A'(-11, 4)$

$F(-5 \pm 4\sqrt{2}, 4)$

$\boxed{a^2 = 36}$ $\boxed{b^2 = 4}$

$c^2 = a^2 - b^2$

$\boxed{c^2 = 32}$

$\boxed{c = 4\sqrt{2}}$

L. Major : $2a = 12$ units

L. Minor : $2b = 4$ units

43) b) V.E

$\vec{r} = \bar{a} + s\bar{b} + t\bar{c}$

$\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + s(2\hat{i} - \hat{j} + 4\hat{k}) + t(\hat{i} + 2\hat{j} + \hat{k})$

C.E

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y+1 & z-3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \left.\begin{array}{l} -9(x-1) - (y+1)(-2) \\ +(z-3)(5) \end{array}\right\} = 0$

$\Rightarrow \boxed{9x - 2y - 5z + 4 = 0}$

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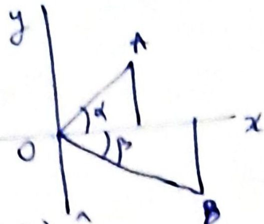
44) a) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$



$\hat{a} = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$

$\hat{b} = \cos\beta\,\hat{i} - \sin\beta\,\hat{j}$

$\hat{b}\times\hat{a} = \sin(\alpha+\beta)\,\hat{k}$

$$\hat{b}\times\hat{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\beta & -\sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix}$$

$\hat{b}\times\hat{a} = \hat{i}(0) - \hat{j}(0) + \hat{k}(\sin\alpha\cos\beta + \cos\alpha\sin\beta)$

$\Rightarrow \boxed{\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}$

45) b) $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

$6\left(x^2 + \frac{1}{x^2}\right) - 35\left(x + \frac{1}{x}\right) + 62 = 0$

$y = x + 1/x \;;\quad y^2 - 2 = x^2 + 1/x^2$

$6y^2 - 35y + 50 = 0$

$(3y-10)(2y-5) = 0$

$y = 10/3$ (or) $y = 5/2$

$x + \frac{1}{x} = \frac{10}{3}$

$3x^2 - 10x + 3 = 0$

$\boxed{x = 3 \text{ (or) } x = 1/3}$

$x + \frac{1}{x} = \frac{5}{2}$

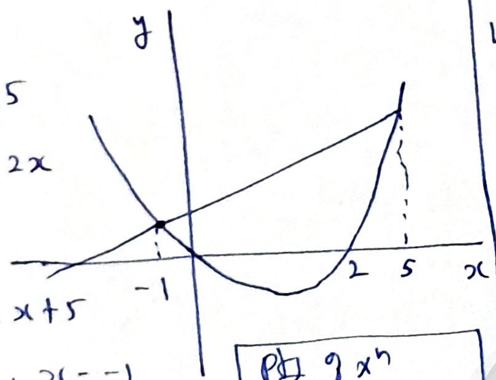
$2x^2 - 5x + 2 = 0$

$\boxed{x = 2 \text{ (or) } x = 1/2}$

44) b)

$y = 2x + 5$

$y = x^2 - 2x$



$x^2 - 2x = 2x + 5$

$\Rightarrow x = 5,\ x = -1$

$y = 15,\ y = 3$

P.O.I of x^n
$(-1,3), (5,15)$

$$A = \int_{-1}^{5} [(2x+5) - (x^2 - 2x)]\,dx$$

$$= \int_{-1}^{5} (5 + 4x - x^2)\,dx$$

$$= \left(5x + 2x^2 - x^3/3\right)_{-1}^{5}$$

$A = 36$ sq-units.

46) a) $\frac{dT}{dt} = k(T - 50)$

t	0	2	?
T	70	60	98.6

$\Rightarrow \boxed{50 - T = ce^{kt}}$ ①

$\left.\begin{array}{l} t = 0 \\ T = 70 \end{array}\right\}$ $\boxed{c = -20}$

$\left.\begin{array}{l} t = 2 \\ T = 60 \end{array}\right\}$ $\boxed{e^{2k} = \frac{1}{2}}$

$2k = \log 1/2$

$\boxed{k = \frac{1}{2}\log(1/2)}$

① ⇒

$50 - T = -20\, e^{\frac{1}{2}t\log(\frac{1}{2})}$

$50 - T = -20 \times \left(\frac{1}{2}\right)^{t/2}$

$\boxed{T = 50 + 20\left(\frac{1}{2}\right)^{t/2}}$

$\left.\begin{array}{l} T = 98.6 \\ t = ? \end{array}\right\}$ $t = 2\left[\dfrac{\log\left(\frac{48.6}{20}\right)}{\log(1/2)}\right] = -2.56$

murder time $\simeq$ 5.30 pm

45). a) Given: $u = \sec^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$

$\sec u = \frac{x^3 - y^3}{x + y} = f(x, y)$

$f(\lambda x, \lambda y) = \frac{\lambda^3(x^3 - y^3)}{\lambda(x + y)} = \lambda^2 f(x, y)$

E.T. $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2f$

$\sec u \tan u\left(\frac{\partial u}{\partial x}\cdot x + y\frac{\partial u}{\partial y}\right) = 2\sec u$

$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u}$

46) b) $\arg\left(\frac{z - i}{z + 2}\right) = \frac{\pi}{4}$

$\arg[x + i(y-1)] - \arg[(x+2) + iy] = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{y-1}{x}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left[\frac{(x+2)(y-1) - yx}{x(x+2) + y(y-1)}\right] = \frac{\pi}{4}$

$\Rightarrow (x+2)(y-1) - xy = x(x+2) + y(y-1)$

$\Rightarrow \boxed{x^2 + y^2 + 3x - 3y + 2 = 0}$

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