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Register No. [ ]

Half Yearly Examination - 2024

MATHEMATICS

Time : 3.00 Hrs. Marks : 90

PART - I

Note : i) All questions are compulsory. **20 x 1 = 20**

ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1. If A^TA^{-1} is symmetric, then A^2 = ...a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$
2. If A is a matrix of order 3, then det (KA) a) K^3 det (A) b) K^2 det(A) c) K det(A) d) det(A)
3. The solution of the equation $|z| - z = 1 + 2i$ is a) $\frac{3}{2} - 2i$ b) $-\frac{3}{2} + 2i$ c) $2 - \frac{3}{2}i$ d) $2 + \frac{3}{2}i$
4. The modulus of the complex number $2 + i\sqrt{3}$ is a) $\sqrt{3}$ b) $\sqrt{13}$ c) $\sqrt{7}$ d) 7
5. A polynomial equation in x of degree n always has
   a) n distinct roots b) n real roots c) n complex roots d) at most one root
6. The value of $\sin^{-1}(\cos x)$, $0 \le x \le \pi$ is a) $\pi - x$ b) $x - \pi/2$ c) $\pi/2 - x$ d) $x - \pi$
7. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is a) [1, 2] b) [–1, 1] c) [0, 1] d) [–1, 0]
8. If $x + y = k$ is normal to the parabola $y^2 = 12x$, then the value of k is a) 3 b) –1 c) 1 d) 9
9. The focus of the parabola $x^2 = 16y$ is a) (4, 0) b) (0, 4) c) (–4, 0) d) (0, –4)
10. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then a) $c = \pm 3$ b) $c = \pm\sqrt{3}$ c) $c > 0$ d) $0 < c < 1$
11. The value of $[\vec{i}+\vec{j},\ \vec{j}+\vec{k},\ \vec{k}+\vec{i}]$ is equal to a) 0 b) 1 c) 2 d) 4
12. The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0, 3]$ is a) 1 b) $\sqrt{2}$ c) $\frac{3}{2}$ d) 2
13. The point of inflection of the curve $y = (x-1)^3$ is a) (0, 0) b) (0, 1) c) (1, 0) d) (1, 1)
14. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is
    a) $0.3\,xdx\ m^3$ b) $0.03\ xm^3$ c) $0.03\ x^2\ m^3$ d) $0.03\ x^3 m^3$
15. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is a) $x + \frac{\pi}{2}$ b) $-x + \frac{\pi}{2}$ c) $x - \frac{\pi}{2}$ d) $-x - \frac{\pi}{2}$
16. The value of $\int_{-1}^{2} |x|\,dx$ is a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$
17. The value of $\int_{0}^{\pi} \sin^4 x\,dx$ is a) $\frac{3\pi}{10}$ b) $\frac{3\pi}{8}$ c) $\frac{3\pi}{4}$ d) $\frac{3\pi}{2}$
18. The solution of $\frac{dy}{dx} + P(x)\,y = 0$ is a) $y = ce^{\int pdx}$ b) $y = ce^{-\int pdx}$ c) $x = ce^{-\int pdy}$ d) $x = ce^{\int pdy}$
19. The number of arbitrary constants in the particular solution of a differential equation of third order is
    a) 3 b) 2 c) 1 d) 0
20. A zero of $x^3 + 64$ is a) 0 b) 4 c) 4i d) –4

PART - II

Note : i) Answer any seven questions. 2) Question number 30 is compulsory. **7 x 2 = 14**

21. Find the square root of $-8 - 6i$.
22. If $\text{adj}(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ find A^{-1}.
23. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$
24. Find the principal value of $\text{cosec}^{-1}(-\sqrt{2})$
25. Find centre and radius of the following circle $x^2 + y^2 + 6x - 4y + 4 = 0$
26. A particle moves so that the distance moved is according to the law $s(t) = \frac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero.
27. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.
28. Evaluate : $\int_{0}^{3} (3x^2 - 4x + 5)\,dx$

29. Determine the order and degree of the following differential equation : $\left(\frac{d^3y}{dx^3}\right)^{2/3} - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4 = 0$

30. Show that the vectors $\vec{i} + 2\vec{j} - 3\vec{k}, 2\vec{i} - \vec{j} + 2\vec{k}$ and $3\vec{i} + \vec{j} - \vec{k}$ are coplanar.

PART - III

Note : 1) Answer any seven questions. 2) Question number 40 is compulsory. **7 x 3 = 21**

31. Solve the following system of linear equations, by matrix inversion method. $2x - y = 7, \; 3x - 2y = 11$
32. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.
33. Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1}x, \; |x| > 1$.
34. Find the equation of the hyperbola with vertices $(0, \pm 4)$ and foci $(0, \pm 6)$
35. If the two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{2} = \frac{y-m}{2} = z$ intersect at a point, find the value of m.
36. Evaluate : $\lim_{x \to \infty} \frac{2x^2-3}{x^2-5x+3}$
37. If $v(x, y, z) = x^3 + y^3 + z^3 + 3xyz$, show that $\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial z \partial y}$
38. Evaluate : $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} dx$
39. Solve the following linear differential equation $(1 - x^2)\frac{dy}{dx} - xy = 1$
40. Write in polar form of the following complex number $3 - i\sqrt{3}$

PART - IV

Note : Answer all the questions. **7 x 5 = 35**

41. a) Solve the following systems of linear equations by Cramer's rule :
$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0, \; \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0, \; \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$ **(OR)**
b) Solve the equation $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

42. a) If $z = x + iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$ **(OR)**
b) Find the angle between $y = x^2$ and $y = (x - 3)^2$.

43. a) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$ **(OR)**
b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

44. a) For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2. **(OR)**
b) Find the area of the region bounded between the parabola $y^2 = 4ax$ and its latus rectum.

45. a) If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}, \; \vec{b} = 3\vec{i} + 5\vec{j} + 2\vec{k}, \; \vec{c} = -\vec{i} - 2\vec{j} + 3\vec{k}$ verify that $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$ **(OR)**
b) Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.

46. a) Sketch the curve $y = f(x) = x^2 - x - 6$. **(OR)**
b) Find the value of k for which the equations $kx - 2y + z = 1, \; x - 2ky + z = -2, \; x - 2y + kz = 1$ have
i) no solution ii) unique solution iii) infinitely many solution

47. a) Find the non-parametric form of vector equation, and Cartesian equations of the plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$. **(OR)**
b) Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

HALF YEARLY - ANSWER KEY - XII - DEC - 2024 (CH - 1 to 10)

1 MARKS

1) b) $(A^T)^2$
2) a) $K^3 \det(A)$
3) a) $\frac{3}{2} - 2i$
4) c) $\sqrt{7}$
5) c) n complex roots
6) c) $\frac{\pi}{2} - x$
7) a) $[1, 2]$
8) d) 9
9) b) $(0, 4)$
10) b) $c = \pm\sqrt{3}$
11) c) 2
12) d) 2
13) c) $(1, 0)$
14) d) $0.03\,x^3 m^3$
15) b) $-x + \frac{\pi}{2}$
16) c) $\frac{5}{2}$
17) b) $3\pi/8$
18) b) $y = ce^{-\int P dx}$
19) d) 0
20) d) -4

2 MARKS

21) $z = -8 - 6i$

$|z| = \sqrt{(-8)^2 + (-6)^2} \Rightarrow \sqrt{64+36}$

$|z| = 10$

$\sqrt{-8-6i} = \pm\left(\sqrt{\frac{10-8}{2}} - i\sqrt{\frac{10+8}{2}}\right)$

$= \pm\left(\sqrt{\frac{2}{2}} - i\sqrt{\frac{18}{2}}\right)$

$= \pm(1 - i\sqrt{9})$

$= \boxed{\pm(1-3i)}$

22) $\text{adj } A = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

$A^{-1} = \pm \frac{1}{\sqrt{|\text{adj } A|}} \text{adj}(A)$

$|\text{adj } A| = 36$

$\sqrt{|\text{adj } A|} = \pm 6$

$A^{-1} = \pm \frac{1}{6}\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

23) $2x^4 - 8x^3 + 6x^2 - 3 = 0$

$\Sigma_1 = 4, \quad \Sigma_2 = 3$

$\alpha+\beta+\gamma+\delta = 4,$

$\alpha\beta+\beta\gamma+\gamma\delta+\alpha\gamma+\alpha\delta+\beta\delta = 3$

$(\alpha+\beta+\gamma+\delta)^2 = \alpha^2+\beta^2+\gamma^2+\delta^2 + 2(\alpha\beta+\beta\gamma+\gamma\delta+\alpha\gamma+\alpha\delta+\beta\delta)$

$(4)^2 = \alpha^2+\beta^2+\gamma^2+\delta^2 + 2(3)$

$\alpha^2+\beta^2+\gamma^2+\delta^2 = 10$

24) $\text{cosec}^{-1}(-\sqrt{2})$

let $\text{cosec}^{-1}(-\sqrt{2}) = y$

$\text{cosec } y = -\sqrt{2}$

$\Rightarrow \sin y = -\frac{1}{\sqrt{2}}$

principal value $y = -\frac{\pi}{4}$

25) $x^2+y^2+6x-4y+4=0$

$g = 3, \quad f = -2, \quad c = 4,$

$r = \sqrt{g^2+f^2-c}$

$\boxed{r = 3}$ $\boxed{\text{centre } (-3, 2)}$

26) $s(t) = \frac{t^3}{3} - t^2 + 3$

$v(t) = t^2 - 2t$

$a(t) = 2t - 2$

$t^2 - 2t = 0 \Rightarrow \boxed{t = 0, 2}$

velocity is zero at $\boxed{t = 0, 2}$

$2t - 2 = 0 \Rightarrow \boxed{t = 1}$

acceleration is zero at $\boxed{t = 1}$

27) Let $y = x^{1/n}$

$\log y = \frac{1}{n}\log x$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{n}\frac{1}{x}$

$\frac{dy}{y} = \frac{1}{n}\frac{dx}{x}$

$\frac{dy}{y} \times 100 = \frac{1}{n}\frac{dx}{x} \times 100$

$\simeq \frac{1}{n}$ (Percent. error in number)

28) $\int_0^3 (3x^2 - 4x + 5)\,dx$

$\cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_0^3 - \cancel{4}^{2}\left[\frac{x^2}{\cancel{2}}\right]_0^3 + 5[x]_0^3$

$= 27 - 18 + 15 = \boxed{24}$

29) $\left(\frac{d^3y}{dx^3}\right)^{2/3} - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4 = 0$

order $= 3$,

Degree $= 2$

30) $\vec{i} + 2\vec{j} - 3\vec{k},$

$2\vec{i} - \vec{j} + 2\vec{k},$

$3\vec{i} + \vec{j} - \vec{k}$

$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix} = \boxed{0}$

Vectors are coplanar.

3 MARKS

31) $2x - y = 7,$

$3x - 2y = 11$

$A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$

$X = A^{-1}B, \quad A^{-1} = \frac{1}{|A|}\text{adj } A$

$|A| = -4 + 3 = \boxed{-1}$

$\text{adj } A = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{-1}\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

$X = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 7 \\ 11 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$x = 3, \quad y = -1$

32) $3x^3 - 16x^2 + 23x - 6 = 0$

Roots are $\alpha, \frac{1}{\alpha}, \beta,$

$\Sigma_1 = \alpha + \frac{1}{\alpha} + \beta = \frac{16}{3}$

$\Sigma_3 = \cancel{\alpha} \cdot \frac{1}{\cancel{\alpha}} \cdot \beta = 2 \Rightarrow \boxed{\beta = 2}$

Sub in $\Sigma_1 \Rightarrow 3\alpha^2 - 10\alpha + 3 = 0$

$\alpha = 3, \ \alpha = \frac{1}{3}$

Roots are $3, \frac{1}{3}, 2$ (or)

$\frac{1}{3}, 3, 2$

33) $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x$

$|x| > 1$

let $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \alpha$

$\cot\alpha = \frac{1}{\sqrt{x^2-1}}$

$\Rightarrow \sec\alpha = x$

$\alpha = \sec^{-1} x$

$\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x, \quad |x|>1$

34) vertices $(0, \pm 4)$, foci $(0, \pm 6)$

centre $\Rightarrow (0,0)$,

Transverse axis along y-axis.

$AA' = 2a = 8 \Rightarrow \boxed{a=4}$

$SS' = 2c = 12 \Rightarrow \boxed{c=6}$

$b^2 = c^2 - a^2 \Rightarrow 36-16 = \boxed{20}$

Eq. of hyperbola is

$\frac{y^2}{16} - \frac{x^2}{20} = 1$

35) $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ &

$\frac{x-3}{2} = \frac{y-m}{2} = z$

intersect at a point

$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = s$

$\Rightarrow \boxed{(2s+1, 3s-1, 4s+1)}$

$\frac{x-3}{2} = \frac{y-m}{2} = z = t$

$\Rightarrow \boxed{(t+3, 2t+m, t)}$

$4s+1 = t$

$2s+1 = t+3$, sub t value

$\boxed{s = -\frac{3}{2}}$, sub s in any value, $\boxed{t=-5}$

sub, s & t $\Rightarrow \boxed{m = \frac{9}{2}}$

36) $\lim_{x\to\infty} \frac{2x^2-3}{x^2-5x+3}$

$\frac{\infty}{\infty}$ (IDF), using l'Hopital rule,

$\lim_{x\to\infty} \frac{4x}{2x-5} = \frac{\infty}{\infty}$ (IDF)

$\lim_{x\to\infty} \frac{4}{2} \Rightarrow \boxed{2}$

37) $v(x,y,z) = x^3+y^3+z^3+3xyz$

$\frac{\partial^2 v}{\partial y \partial z} \Rightarrow \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial z}\right)$

$\frac{\partial}{\partial y}(3z^2+3xy) \Rightarrow \boxed{3x}$

$\frac{\partial^2 v}{\partial z \partial y} = \frac{\partial}{\partial z}\left(\frac{\partial v}{\partial y}\right)$

$= \frac{\partial}{\partial z}(3y^2+3xz)$

$= \boxed{3x}$

$\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial z \partial y}$

38) $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$

$I = \int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx \to ①$

$I = \int_2^3 \frac{\sqrt{2+3-x}}{\sqrt{5-(2+3-x)}+\sqrt{2+3-x}}\,dx$

$I = \int_2^3 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx \to ②$

① + ② $\Rightarrow$

$2I = \int_2^3 \frac{\sqrt{x}+\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx$

$I = \frac{1}{2}[x]_2^3 \Rightarrow \boxed{I = \frac{1}{2}}$

39) $(1-x^2)\frac{dy}{dx} - xy = 1$

$\div$ by $(1-x^2)$

$\frac{dy}{dx} - \frac{xy}{1-x^2} = \frac{1}{1-x^2}$

$P = \frac{-x}{1-x^2}, \quad Q = \frac{1}{1-x^2}$

$e^{\int P dx} = e^{\log\sqrt{1-x^2}} \Rightarrow \boxed{\sqrt{1-x^2}}$

$y e^{\int P dx} = \int Q e^{\int P dx}\,dx + c$

$y\sqrt{1-x^2} = \int \frac{1}{1-x^2}\sqrt{1-x^2}\,dx + c$

$y\sqrt{1-x^2} = \int \frac{1}{\sqrt{1-x^2}}\,dx + c$

$y\sqrt{1-x^2} = \sin^{-1} x + c$

40) $3 - i\sqrt{3}$

$r = \sqrt{12} \Rightarrow 2\sqrt{3}$

$\alpha = \pi/6 \Rightarrow \theta = -\pi/6$

$3 - i\sqrt{3} = 2\sqrt{3}\left(\cos(2k\pi - \pi/6) + i\sin(2k\pi - \pi/6)\right)$

5 MARKS

41) a) $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0$

$\frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0$

$\frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$

$3a - 4b - 2c = 1$

$a + 2b + c = 2$

$2a - 5b - 4c = -1$

$\Delta = -15, \quad \Delta_a = -15,$

$\Delta_b = -5, \quad \Delta_c = -5$

$a = 1, \; b = \frac{1}{3}, \; c = \frac{1}{3}$

$\boxed{x = 1, \; y = 3, \; z = 3}$

b) $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

$6\left(x^2 + \frac{1}{x^2}\right) - 35\left(x + \frac{1}{x}\right) + 62 = 0$

$x + \frac{1}{x} = y,$

$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$6(y^2-2) - 35y + 62 = 0$

$6y^2 - 35y + 50 = 0$

$y = \frac{10}{3}, \frac{5}{2}$

if $y = \frac{10}{3} \Rightarrow \boxed{x = \frac{1}{3}, 3}$

if $y = \frac{5}{2} \Rightarrow \boxed{x = \frac{1}{2}, 2}$

42) a) $z = x + iy$

$Im\left(\frac{2z+1}{iz+1}\right) = 0$

$\frac{2z+1}{iz+1} \Rightarrow \frac{2(x+iy)+1}{i(x+iy)+1}$

$\frac{(2x+1)+i2y}{(1-y)+ix} \times \frac{(1-y)-ix}{(1-y)-ix}$

$Im\left(\frac{2z+1}{iz+1}\right) =$

$\frac{-(2x+1)x + (1-y)2y}{(1-y)^2 + x^2} = 0$

$= 2x^2 + 2y^2 + x - 2y = 0$

b) $y=x^2$, $y=(x-3)^2$

Eq. $x^2=(x-3)^2$

$\Rightarrow \boxed{x=\frac{3}{2}}$

pt. of intersection is

$\left(\frac{3}{2}, \frac{9}{4}\right)$

$y=x^2 \Rightarrow \frac{dy}{dx}=2x$

$m_1=\frac{dy}{dx}$ at $\left(\frac{3}{2},\frac{9}{4}\right)=\boxed{3}$

$y=(x-3)^2$, $\frac{dy}{dx}=2(x-3)$

$m_2=\frac{dy}{dx}$ at $\left(\frac{3}{2},\frac{9}{4}\right)$ is $=\boxed{-3}$

$\tan\theta=\left|\frac{3-(-3)}{1-9}\right|$

$\theta=\tan^{-1}\left(\frac{3}{4}\right)$

43) a)

$\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$,

W.K.T,

$\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=$

$\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]=\pi$

$\frac{x+y+z-xyz}{1-xy-yz-zx}=\tan\pi$

$\frac{x+y+z-xyz}{1-xy-yz-zx}=0$

$x+y+z=xyz$

b) $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

u is not homogeneous,

So $f(x,y)=\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

$f(\lambda x,\lambda y)=\lambda^{1/2}f(x,y)$

f is homogeneous with order 1/2, ($f=\sin u$)

$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=\frac{1}{2}f(x,y)$

$x\cos u\frac{\partial u}{\partial x}+y\cos u\frac{\partial u}{\partial y}=\frac{1}{2}\sin u$

÷ by $\cos u$,

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$

44) a) $4x^2+y^2+24x-2y+21=0$

$4(x^2+6x+9-9)+(y^2-2y+1-1)+21=0$

$4(x+3)^2-36+(y-1)^2-1+21=0$

$4(x+3)^2+(y-1)^2=16$

$\frac{(x+3)^2}{4}+\frac{(y-1)^2}{16}=1$

centre $\Rightarrow (-3,1)$

$a^2=16$, $b^2=4$

$a=4$, $b=2$

$c^2=16-4=12$

$\boxed{c=\pm 2\sqrt{3}}$

Foci: $(h, k\pm c)$

$=\boxed{(-3, 1\pm 2\sqrt{3})}$

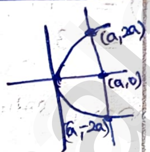
vertices: $(h, k\pm a)$

$=\boxed{(1,5) \text{ \& } (1,-3)}$

length of latus rectum $=\frac{2b^2}{a}$

$=\frac{2(4)}{4}\Rightarrow\boxed{2}$

b) $y^2=4ax$



$y=2\sqrt{ax}$

$A=2\int_0^a y\,dx$

$=2\int_0^a 2\sqrt{ax}\,dx$

$=4\sqrt{a}\left[\frac{2}{3}x^{3/2}\right]_0^a$

$=\boxed{\frac{8a^2}{3}}$

45) a) $\vec{a}=2\vec{i}+3\vec{j}-\vec{k}$

$\vec{b}=3\vec{i}+5\vec{j}+2\vec{k}$

$\vec{c}=-\vec{i}-2\vec{j}+3\vec{k}$

$(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$

$\vec{a}\times\vec{b}=11\hat{i}-7\hat{j}+\hat{k}$

$\boxed{(\vec{a}\times\vec{b})\times\vec{c}=-19\hat{i}-34\hat{j}-29\hat{k}}$

$\vec{a}\cdot\vec{c}=-2-6-3=\boxed{-11}$

$\boxed{(\vec{a}\cdot\vec{c})\vec{b}=-33\hat{i}-55\hat{j}-22\hat{k}}$

$\vec{b}\cdot\vec{c}=-3-10+6=\boxed{-7}$

$\boxed{(\vec{b}\cdot\vec{c})\vec{a}=-14\hat{i}-21\hat{j}+7\hat{k}}$

$(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}=$

$\boxed{-19\hat{i}-34\hat{j}-29\hat{k}}$

b) population at time $t=A$

$\frac{dA}{dt}\propto A\Rightarrow\frac{dA}{dt}=kA$

$\int\frac{dA}{A}=k\int dt$

$\log A=kt+\log c$

$\boxed{A=ce^{kt}}\rightarrow$ ①

at $t=0\Rightarrow A=3,00,000$

$\boxed{C=3,00,000}$

$\boxed{A=3,00,000\,e^{kt}}\rightarrow$ ②

at $t=40$, $A=4,00,000$

Sub in ②,

$4,00,000=3,00,000\,e^{k(40)}$

$e^{40k}=\frac{4}{3}$

$(e^{40k})^{1/40}=\left(\frac{4}{3}\right)^{1/40}$

$\boxed{e^k=\left(\frac{4}{3}\right)^{1/40}}$

Pop. at time t is

$\boxed{A=3,00,000\left(\frac{4}{3}\right)^{t/40}}$

46) a) $y=f(x)=x^2-x-6$

$f(x)=(x-3)(x+2)$

1) Domain − R

2) Range − $y\geq-\frac{25}{4}$

3) x-intercept − $(-2,0)$, $(3,0)$

4) y-intercept − $(0,-6)$

5) $f'(x)=2x-1$

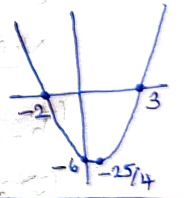
critical point $x=\frac{1}{2}$

6) Local minimum $=-\frac{25}{4}$

7) concave upwards

8) No point of inflection

9) No asymptotes



b) $kx-2y+z=1$

$x-2ky+z=-2$

$x-2y+kz=1$

$[A|B]=\left[\begin{array}{ccc|c} k & -2 & 1 & 1 \\ 1 & -2k & 1 & -2 \\ 1 & -2 & k & 1 \end{array}\right]$

$R_1\leftrightarrow R_3\Rightarrow\left[\begin{array}{ccc|c} 1 & -2 & k & 1 \\ 1 & -2k & 1 & -2 \\ k & -2 & 1 & 1 \end{array}\right]$

$R_2\rightarrow R_2-R_1$
$R_3\rightarrow R_3-kR_1$ $\left[\begin{array}{ccc|c} 1 & -2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & -2+2k & 1-k^2 & 1-k \end{array}\right]$

$R_3 \to R_3 + R_2$ $\left[\begin{array}{ccc|c} 1 & -2 & k & 1 \\ 0 & 2(1-k) & 1-k & -3 \\ 0 & 0 & 2-k-k^2 & k+2 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & k & 1 \\ 0 & 2(1-k) & (1-k) & -3 \\ 0 & 0 & (k+2)(1-k) & k+2 \end{array}\right]$

(i) No solution:

if $k=1 \Rightarrow \rho(A) \neq \rho(A|B)$

$2 \neq 3$

(ii) Unique solution:

if $k \neq 1$, $k \neq -2$

$\rho(A) = \rho(A|B) = n$

$3 = 3 = 3$

(iii) infinitely many solutions:

if $k = -2$,

$\rho(A) = \rho(A|B) < n$

$2 = 2 < 3$

47) a) points $(2,2,1)$ & $(9,3,6)$

$\perp^r$ to $2x+6y+6z=9$

$\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = 9\hat{i} + 3\hat{j} + 6\hat{k}$

$\vec{c} = 2\hat{i} + 6\hat{j} + 6\hat{k}$

N.P. form: $(\vec{r}-\vec{a}) \cdot ((\vec{b}-\vec{a}) \times \vec{c}) = 0$

$(\vec{b}-\vec{a}) \times \vec{c} = -24\hat{i} - 32\hat{j} + 40\hat{k}$

$(\vec{r}-\vec{a}) \cdot ((\vec{b}-\vec{a}) \times \vec{c}) \Rightarrow$

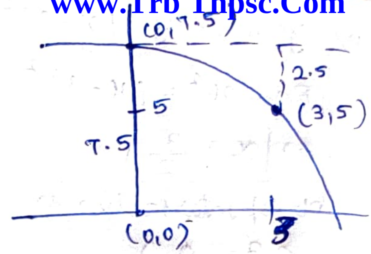
$\boxed{\vec{r} \cdot (3\hat{i} + 4\hat{j} - 5\hat{k}) = 9}$

cartesian form: $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$\begin{vmatrix} x-2 & y-2 & z-1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix} = 0$

$\Rightarrow \boxed{3x + 4y - 5z = 9}$

b)



$(x-h)^2 = -4a(y-k)$

sub $(0, 7.5)$

$\boxed{x^2 = -4a(y-7.5)}$

sub $(3,5) \Rightarrow 4a = \frac{9}{2.5}$

$x^2 = -\frac{9}{2.5}(y-7.5)$

sub $(x, 0) \Rightarrow \boxed{x = 3\sqrt{3}\ m}$

water strike ground at $3\sqrt{3}$ m