HALF YEARLY EXAMINATION - 2024

CLASS: XII **MATHEMATICS** Reg.No

Time : 3.00 Hours **MARKS : 90**

PART – I

Answer all the questions . **20 x 1 = 20**

1. If $A = \begin{pmatrix} 7 & 3 \\ 4 & 2 \end{pmatrix}$, then $9I_2 - A =$

   1) A^{-1} 2) $\frac{A^{-1}}{2}$ 3) $3A^{-1}$ 4) $2A^{-1}$

2. In the non-homogenous system of equations with 3 unknowns if $\rho(A) = \rho([A/B]) = 2$ then the system has

   1) unique solution 2) one parameter family of solutions
   3) two parameter family of solutions 4) no solution

3. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

   1) z 2) $\bar{z}$ 3) $\frac{1}{z}$ 4) 1

4. The solution of the equation $|z| - z = 1 + 2i$ is

   1) $\frac{3}{2} - 2i$ 2) $-\frac{3}{2} + 2i$ 3) $2 - \frac{3}{2}i$ 4) $2 + \frac{3}{2}i$

5. The number of real numbers in $[0,2\pi]$ satisfying $sin^4 x - 2\,sin^2 x + 1$ is

   1) 2 2) 4 3) 1 4) ∞

6. If $cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, then value of $tan^{-1} x$ is

   1) $\frac{-\pi}{10}$ 2) $\frac{\pi}{5}$ 3) $\frac{\pi}{10}$ 4) $\frac{-\pi}{5}$

7. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

   1) 1 2) 3 3) $\sqrt{10}$ 4) $\sqrt{11}$

8. If the parabola $y^2 = 4ax$ passes through the point (3,2) then the length of its latus rectum is

   1) $\frac{2}{3}$ 2) $\frac{4}{3}$ 3) $\frac{1}{3}$ 4) 1

9. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a} \times \vec{b}] = \frac{1}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

   1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

10. The coordinates of the point where the line $\vec{r} = (6\hat{\imath} - \hat{\jmath} - 3\hat{k}) + t(-\hat{\imath} + 4\hat{k})$ meets the plane $\vec{r}.(\vec{\imath} + \vec{\jmath} - \vec{k}) = 3$ are

    1) (2,1,0) 2) (7,-1,-7) 3) (1,2,-6) 4) (5,-1,1)

11. The value of the limit $\lim_{x \to 0}\left(cot\, x - \frac{1}{x}\right)$

    1) 0 2) 1 3) 2 4) -1

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12. The point of inflection of the curve $y = (x-1)^3$ is

1) (0,0) 2) (0,1) 3) (1,0) 4) (1,1)

13. If $f(x,y) = e^{xy}$,then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to

1) xye^{xy} 2) $(1+xy)e^{xy}$ 3) $(1+y)e^{xy}$ 4) $(1+x)e^{xy}$

14. The value of $\int_{-1}^{2} |x| dx$ is

1) $\frac{1}{2}$ 2) $\frac{3}{2}$ 3) $\frac{5}{2}$ 4) $\frac{7}{2}$

15. The value of $\int_0^1 x^2(1-x)^3\, dx$ is

1) $\frac{1}{30}$ 2) $\frac{1}{20}$ 3) $\frac{1}{60}$ 4) $\frac{1}{2}$

16. The order and degree of the differential equation $\sqrt{\sin x}\,(dx + dy) = \sqrt{\cos x}\,(dx - dy)$

1) 1,2 2) 2,2 3) 1,1 4) 2,1

17. Integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is

1) e^{-x} 2) x 3) $\frac{1}{x}$ 4) e^{-y}

18. Let X have a Bernoulli distribution with mean 0.4, then the variance of (2X-3) is

1) 0.24 2) 0.48 3) 0.6 4) 0.96

19. The value of $Var(3X-5)$ is

1) $3Var(X)$ 2) -5 3) $25Var(X)$ 4) $9Var(X)$

20. The operation * defined by $a * b = \frac{ab}{7}$ is not a binary operation on

1) $\mathbb{Q}^+$ 2) $\mathbb{Z}$ 3) $\mathbb{R}$ 4) $\mathbb{C}$

PART – II

Answer any seven questions. Question No.30 is compulsory. 7 x 2 = 14

21. If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ find A^{-1}.

22. Find the square root of $6 - 8i$.

23. If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.

24. Find centre and radius of the circle $x^2 + y^2 + 6x - 4y + 4 = 0$.

25. Explain why Lagrange's mean value theorem is not applicable to the function in the respective intervals $f(x) = |3x+1|,\ x \in [-1,3]$.

26. Let $g(x) = x^2 + \sin x$ .Calculate the differential dg.

27. Evaluate $\int_0^\infty x^5 e^{-3x} dx$.

28. Show that $y = e^{-x} + mx + n$ is a solution of the differential equation $e^{x}\left(\frac{d^2y}{dx^2}\right) - 1 = 0$.

29. The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive.

30. Find the angle between the following lines. $x = y + 2 = z$ and $x + 2 = 2y = 2z$.

PART – III

Answer any Seven questions. Question No.40 is compulsory. 7 x 3 = 21

31. Solve the systems of linear equations by Cramer's rule: $5x - 2y + 16 = 0, x + 3y - 7 = 0$.

32. If $z = x + iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$ show that the locus of z is real axis.

33. Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary roots.

34. Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$.

35. Let, $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, show that $[\vec{a}, \vec{b}, \vec{c}]^2 = \frac{1}{4}|\vec{a}|^2|\vec{b}|^2$.

36. Find the equations of tangent and normal to the curve $y = x^2 + 3x - 2$ at the point $(1,2)$.

37. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

38. Evaluate: $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$

39. Establish the equivalence property: $p \rightarrow q \equiv \neg p \vee q$

40. Solve : $\frac{dy}{dx} = xy - 1 + x - y ; y(0) = 0$.

PART – IV

Answer all the questions. 7 x 5 = 35

41. **(a)** Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9, 7x + 3y - 5z = 8, 2x + 3y + \lambda z = \mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(or)

(b) Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

42. **(a)** If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are $-1, 1 - 2\omega, 1 - 2\omega^2$.

(or)

(b) Prove that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$.

43. (a) A bridge has a parabolic arch that is $10m$ high in the centre and $30m$ wide at the bottom. Find the height of the arch $6m$ from the centre, on either sides.

(or)

(b) Prove that $f(x,y) = x^3 - 2x^2y + 3xy^2 + y^3$ is homogeneous; what is the degree? Verify Euler's Theorem for f.

44. (a) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

(or)

(b) A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find (i) $P(2 < X < 6)$ (ii) $P(2 \le X < 5)$ (iii) $P(X \le 4)$ (iv) $P(3 > X)$.

45. (a) Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + s(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and $\vec{r} = (\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) + t(\hat{\imath} + \hat{\jmath} - \hat{k})$.

(or)

(b) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall,
(i) how fast is the top of the ladder moving down the wall?
(ii) at what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?

46. (a) A rectangular page is to contain 24 cm^2 of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.

(or)

(b) Find the area of the region enclosed by the parabolas $y = x^2 - 5x$ and $y = 7x - x^2$.

47. (a) The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

(or)

(b) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on Z_5 using table corresponding to addition modulo 5.