COMMON HALF YEARLY EXAMINATION – 2024

S **Standard XII** Reg.No.: | | | | | | |

MATHEMATICS

Time: 3.00 hrs. **Part - I** **Marks: 90**

I. Answer all the questions. **20 x 1 = 20**

1. If $A = \begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is

   a) $-4/5$ b) $-3/5$ c) $3/5$ d) $4/5$

2. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

   a) 19 b) 17 c) 21 d) 14

3. Identify the incorrect statement.

   a) $|z|^2 = 1 \Rightarrow \frac{1}{z} = \bar{z}$ b) $Re(z) \le |z|$ c) $||z_1| - |z_2|| \ge |z_1 + z_2|$ d) $|z^n| = |z|^n$

4. If $z = x + iy$ is a complex number such that $|z+2| = |z-2|$ then the locus of z is

   a) real axis b) imaginary axis c) elipse d) circle

5. A zero of $x^3 + 64$ is

   a) 0 b) 4 c) 4i d) –4

6. If $\cot^{-1} x \frac{2\pi}{5}$ for some $x \in R$, then the value of $\tan^{-1} x$ is

   a) $\frac{-\pi}{10}$ b) $\frac{\pi}{5}$ c) $\frac{\pi}{10}$ d) $\frac{-\pi}{5}$

7. The properties closure, associative, identity, inverse and commutative under addition such the set

   a) R b) N c) {1, –1, 0} d) Q \ {0}

8. The operation * defined by $a * b = \frac{ab}{7}$ is not a binary operation on

   a) R b) Q^+ c) C d) Z

9. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

   a) $\sqrt{ab}$ b) 2ab c) a/b d) ab

10. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ having centre at (0,3) is

    a) $x^2 + y^2 - 6y - 7 = 0$ b) $x^2 + y^2 - 6y + 7 = 0$
    c) $x^2 + y^2 - 6y - 5 = 0$ d) $x^2 + y^2 - 6y + 5 = 0$

11. The volume of the parallelepiped with its edges represented by the vectors $\hat{i} + \hat{j}$, $\hat{i} + 2\hat{j}$, $\hat{i} + \hat{j} + \pi\hat{k}$ is

    a) π b) $\pi/2$ c) $\pi/4$ d) $\pi/3$

12. The co-ordinates of the point where the line $\vec{r} = (6\hat{i} - \hat{j} - 3\hat{k}) + t(-\hat{i} + 4\hat{k})$ meet the plane $\vec{r}.(\hat{i} + \hat{j} - \hat{k}) = 3$ are

a) (2,1,0) b) (7,–1,–7) c) (1,2,–6) d) (5,–1,1)

13. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

a) $t = 3$ b) $t = 0$ c) $t = \frac{1}{3}$ d) $t = 1$

14. The maximum value of the function $x^2 e^{-2x}$, $x > 0$ is

a) $\frac{1}{e^2}$ b) $\frac{1}{e}$ c) $\frac{4}{e^4}$ d) $\frac{1}{2e}$

15. If $w(x,y) = x^y$, $x > 0$ then $\frac{\partial w}{\partial x}$ is equal to

a) $x^y \log x$ b) $y \log x$ c) $y x^{y-1}$ d) $x \log y$

16. The value of $\int_{-1}^{2} |x|\, dx$ is

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$

17. The value of $\int_{0}^{1} x(1-x)^{99}\, dx$ is

a) $\frac{1}{10010}$ b) $\frac{1}{11000}$ c) $\frac{1}{10001}$ d) $\frac{1}{10100}$

18. The solution of differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is

a) $x\phi\left(\frac{y}{x}\right) = k$ b) $\phi\left(\frac{y}{x}\right) = kx$ c) $y\phi\left(\frac{y}{x}\right) = k$ d) $\phi\left(\frac{y}{x}\right) = ky$

19. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

a) $y = kx$ b) $xy = k$ dc) $\log y = kx$ d) $y = k \log x$

20. A random variable X has a binomial distribution with n = 25 and p = 0.8, then the standard deviation of X is

a) 2 b) 6 c) 4 d) 3

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. Fnd the least positive integer n such that $\left(\frac{1+i}{1-i}\right)^n = 1$

22. Find the principal value of $\tan^{-1}\left(\sqrt{3}\right)$

23. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$

24. If $Y = 4x + c$ is tangent to the circle $x^2 + y^2 = 9$, find c.

25. Find the acute angle between the straight lines $\frac{x-4}{2}=\frac{y}{1}=\frac{z+1}{-2}$ and $\frac{x-1}{4}=\frac{y+1}{-4}=\frac{z-2}{2}$.

26. Find the value in the interval $\left(\frac{1}{2}, 2\right)$ satisfied by the Rolle's theorem for the function $f(x)=x+\frac{1}{x}, x\in\left[\frac{1}{2}, 2\right]$

27. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$

28. Evaluate : $\int_0^\infty x^5 e^{-3x}\,dx$

29. How many rows are needed for following statement formulae?
    i) $p \vee \neg t \wedge (p \vee \neg s)$ (ii) $(p \wedge q) \vee (\neg r \vee \neg s) \wedge (\neg t \wedge v)$

30. If $\text{adj}\,A=\begin{bmatrix}-1 & 2 & 2\\ 1 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, find A^{-1}

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. Find the rank of the matrix $\begin{bmatrix}1 & -2 & 3\\ 2 & 4 & -6\\ 5 & 1 & -1\end{bmatrix}$

32. Find z^{-1}, if $z = (3 + 2i)(1 - i)$

33. Prove that the point of intersection of the tangents at t_1 and t_2 on the parabola $y^2 = 4ax$ is $[a\,t_1t_2, a(t_1 + t_2)]$

34. Find two positive numbers whose sum is 12 and their product is maximum.

35. If $z = y e^{x^2}$, $x = 2t$ and $y = 1 - t$ then find $\frac{dz}{dt}$

36. Prove that $\int_0^{\pi/3} \frac{\sec x \tan x}{1+\sec^2 x}\,dx = \tan^{-1}(2)=\frac{-\pi}{4}$

37. Let X be a continuous random variable and f(x) is defined as $f(x)=\begin{cases}kx(1-x)^{10} & 0<x<1\\ 0 & \text{otherwise}\end{cases}$
    Find the value of k

38. Show that $p \to q$ and $q \to p$ are not equivalent.

39. A particle acted on by constant forces $8\hat{i}+2\hat{j}-6\hat{k}$ and $6\hat{i}+2\hat{j}-2\hat{k}$ is displaced from the point (1,2,3) to the point (5,4,1). Find the total work done by the forces.

40. If $a + b + c = 0$ and a, b, c are rational numbers then, prove that the roots of the equation $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ are rational numbers.

Part - IV

IV. Answer all the questions. 7 x 5 = 35

41. a) Solve the equation $z^3 + 8i = 0$ where $z \in C$

(OR)

b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}$

42. a) Using vector method, prove that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

(OR)

b) Suppose the amount of milk sold daily at a milk booth is distributed with a minimum 200 litres and a maximum of 600 litres with probability density function

$f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & \text{otherwise} \end{cases}$. Find (i) the value of k (ii) the distribution function

(iii) the probability that daily sales will full between 300 litres and 500 litres.

43. a) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 m when it is 6 m away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection.

(OR)

b) Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

44. a) Solve by cramer's rule, the system of equations
$3x + 3y - z = 11$, $2x - y + 2z = 9$, $4x + 3y + 2z = 25$

(OR)

b) Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius r cm.

45. a) Find the parametric form of vector equation and cartesian equation of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$

(OR)

b) Solve the equation : $2x^3 + 11x^2 - 9x - 18 = 0$

46. a) Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

(OR)

b) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, Find how many bacteria will be present after 10 hours?

47. a) Prove that the ellipse $x^2 + 4y^2 = 8$ and hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

(OR)

b) Evaluate : $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$

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