Half Yearly – 2024 (MATHS)

12 – Std

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Part – I

1) ④ $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

2) ① $\lambda^{n-1} \text{adj}(A)$

3) ① z

4) ④ $(1,1)$

5) ③ n imaginary roots (or) Complex roots

6) ④ $\tan^{-1}\left(\frac{1}{2}\right)$

7) ④ $\frac{x}{\sqrt{1+x^2}}$

8) ② $2\sqrt{5}$

9) ① $2ab$

10) ④ 0

11) ④ $3, -9$

12) ④ 9

13) ④ 4.8 cu.cm

14) ③ $\frac{\partial x}{\partial t}$

15) ② $\frac{1}{10100}$

16) ② $\frac{d^2y}{dx^2} + y = 0$

17) ① $e^{\int P dy}$

18) ② 1.1

19) ④ If p and q are any two statements then $p \leftrightarrow q$ is a tautology

20) ④ $-3 \le x \le 3$

Part – II

21) $\text{adj}(\text{adj} A) = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix}^T$

$\therefore \text{adj}(\text{adj} A) = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$

22) $\frac{z_1}{z_2} = \frac{3-2i}{6+4i} = \frac{3-2i}{6+4i} \times \frac{6-4i}{6-4i} \Rightarrow \frac{10-24i}{52}$

$\frac{z_1}{z_2} = \frac{5}{26} - \frac{6}{13} i$

23) Principal Value of $\cos^{-1}\left(\frac{1}{2}\right)$

$= \cos^{-1}(\cos(60^\circ)) \Rightarrow \frac{\pi}{3} \in [0, \pi]$

24) Centre $(h,k) = (2,-1)$

Radius $CP = \sqrt{(3-1)^2 + (6+1)^2} = \sqrt{1+49}$

$\therefore r = \sqrt{50}$

Equation of Circle $(x-h)^2 + (y-k)^2 = r^2$

$(x-2)^2 + (y+1)^2 = (\sqrt{50})^2$

$x^2 + y^2 + (-4x) + 2y - 45 = 0$

25) $|[a\ b\ c]| = \left| \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} \right|$

$= |2(4-1) + 3(2+3) + 4(-1-6)|$

$= |6 + 15 - 28| \Rightarrow |-7|$

$= 7$ cubic units

26) $\lim_{x \to 1} \left(\frac{x^2 - 3x + 2}{x^2 - 4x + 3} \right) = \frac{0}{0}$ (I.d.f)

By using L-Hôpital Rule

$= \lim_{x \to 1} \left(\frac{2x-3}{2x-4} \right) = \frac{1}{2}$

27) $A = \pi r^2$; $r = 2$

$\frac{dA}{dr} = 2\pi r$; $dr = 2.1 - 2$

$dr = 0.1$

$dA = 2\pi(2)(0.1) \Rightarrow 0.4\pi$

Area increased $\simeq 0.4\pi\ mm^2$.

28) $n = 10$ (even)

$$\int_0^{\frac{\pi}{2}} \sin^{10} x\,dx = \frac{n-1}{n} \times \frac{n-3}{n-2} \times \frac{n-5}{n-4} \times \cdots \times \frac{1}{2} \times \frac{\pi}{2}$$

$$= \frac{9}{10} \times \frac{7}{8} \times \frac{5}{6} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2}$$

$$= \frac{63\pi}{512}$$ //

29) Let 'X' be a random Variable

$X = 0, 1, 2$

Values of R.V	0	1	2	Total
No. of elements in inverse images	25	10	1	36

$f(0) = P(X=0) = \frac{25}{36}$

$f(1) = P(X=1) = \frac{10}{36}$

$f(2) = P(X=2) = \frac{1}{36}$

$\leq f(x) = \frac{25}{36} + \frac{10}{36} + \frac{1}{36} = \frac{36}{36} = 1$

Prob. mass function

x	0	1	2
$f(x)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

30) $y = A \sin x$ —— ①

① $\Rightarrow$ $y' = A\cos x$

$A = \frac{y'}{\cos x}$

① $\Rightarrow$ $y = \left(\frac{y'}{\cos x}\right)\sin x$

$\therefore y = y' \tan x$; Hence Proved

Part - III

31) $\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$|A| = 4$, $A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

$X = A^{-1}B$

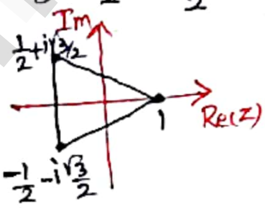
$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 6-10 \\ -9+25 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix}$

$x = -1$, $y = 4$

32) Let $z_1 = 1$, $z_2 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, $z_3 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

$|z_1 - z_2| = \sqrt{\frac{9}{4} + \frac{3}{4}} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$|z_2 - z_3| = \sqrt{3}$, $|z_3 - z_1| = \sqrt{3}$



33) From the given data the Equation of form of the parabola $y^2 = 4ax$

Focus $(a, 0) = (4, 0)$, & DR $\Rightarrow x = -a$, $x = -4$

$\therefore a = 4$

$\therefore$ The Equ. of Parabola $y^2 = 4(4)x$

$y^2 = 16x$

34) If $\hat{b}$ is a unit Vector parallel to the given line $\hat{b} = \frac{2\hat{i}+2\hat{j}-\hat{k}}{|2\hat{i}+2\hat{j}-\hat{k}|} = \frac{2\hat{i}+2\hat{j}-\hat{k}}{3}$

$\cos\alpha = \frac{2}{3}$, $\cos\beta = \frac{2}{3}$, $\cos\gamma = -\frac{1}{3}$

angles made by $\hat{b}$ with the Co-ordinate axes, we have

$\alpha = \cos^{-1}\left(\frac{2}{3}\right)$, $\beta = \cos^{-1}\left(\frac{2}{3}\right)$, $\gamma = \cos^{-1}\left(-\frac{1}{3}\right)$

respectively.

35) $P = \frac{234+16x}{x+3}$; $\frac{dP}{dt} = \frac{-186}{(x+3)^2} \times \frac{dx}{dt}$

Substituting $x = 90$, $\frac{dx}{dt} = 15$

; $\frac{dP}{dt} = -\frac{186}{93^2} \times 15 = \frac{-10}{31} \simeq -0.32$ rupee/week

That is the price is changing, in fact decreasing at ₹0.32 per week

36) $f(x)=\sqrt{x}$, $x_0=9$, $x=9.2$

$\Delta x = dx = 0.2$

$L(x)=f(x_0)+f'(x_0)(x-x_0)$

$\sqrt{9.2} \simeq f(9)+f'(9)(0.2)$

$\simeq 3+\frac{0.2}{6} = 3.0333$

37) $I=\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$ —— ①

$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

① ⇒ $I=\int_2^3 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx$ —— ②

①+② ⇒ $2I = \int_2^3 \frac{\cancel{\sqrt{x}+\sqrt{5-x}}}{\cancel{\sqrt{x}+\sqrt{5-x}}}\,dx$

$2I=\int_2^3 [1]\,dx$

$2I=[x]_2^3$

$\boxed{I=\frac{1}{2}}$

38) Mean $=np=2$, Variance $=npq=1.5$

$\frac{npq}{np}=\frac{1.5}{2}=\frac{3}{4}$; $q=\frac{3}{4}$, $q=\frac{1}{4}$

$\therefore n=\frac{2}{p}=8$

$P(X=x)=f(x)={}^8C_x\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{8-x}$

$x=0,1,2\ldots8$

i) $P(X=0)=\left(\frac{3}{4}\right)^8$

ii) $P(X=1)=2\left(\frac{3}{4}\right)^7$

iii) $P(X\geq 1)=1-P(X<1)=1-P(X=0)$

$=1-\left(\frac{3}{4}\right)^8$

39) $A=Q-\{1\}$, $x,y\in A$

$x\neq 1$, $y\neq 1$

$*$ is defined by $x*y=x+y-xy$

i) Closure Property

Let $x,y\in A$, $x\neq 1$, $y\neq 1$

$x-1\neq 0$, $y-1\neq 0$

$(x-1)(y-1)\neq 0$

$xy-x-y+1\neq 0$

$1\neq x+y-xy$

$x*y\neq 1 \Rightarrow x*y\in A$

$x,y\in A \Rightarrow x*y\in A$, $*$ closed on A

ii) Commutative property

$x*y=y*x=x+y-xy$ is true

iii) Associative property

$x*(y*z)=(x*y)*z=x+y+z-xy-yz-zx+xyz$

40) If α and β are roots of $x^2+5x+6=0$ then $\alpha=3$, $\beta=2$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(3+2)^2-2(3)(2)$

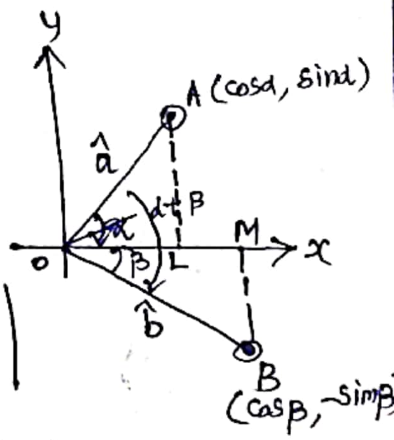
$=5^2-12$

$=25-12$

$\alpha^2+\beta^2=13$

Part - IV

41) a)



$\vec{a}=\cos\alpha\,\hat{i}+\sin\alpha\,\hat{j}$

$\vec{b}=\cos\beta\,\hat{i}-\sin\beta\,\hat{j}$

By Defn.

$\vec{b}\times\vec{a}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \cos\beta & -\sin\beta & 0\\ \cos\alpha & \sin\alpha & 0\end{vmatrix}$

$\vec{b}\times\vec{a}=\hat{k}(\cos\alpha\sin\beta+\sin\alpha\cos\beta)$ —— ①

By Value:

$\vec{b}\times\vec{a}=|\vec{b}||\vec{a}|\sin(\alpha+\beta)\hat{k}$ —— ②

From ① & ②

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

Hence Proved.

41) b) $\sin^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\cot^{-1}(2)$

$=\sin^{-1}\left(-\sin\frac{\pi}{2}\right)+\cos^{-1}\left(\cos\frac{\pi}{3}\right)+\cot^{-1}(2)$

$=-\frac{\pi}{2}+\frac{\pi}{3}+\cot^{-1}(2)$

$=\frac{-3\pi+2\pi}{6}+\cot^{-1}(2)$

$=\frac{-\pi}{6}+\cot^{-1}(2)$

$=\cot^{-1}(2)-\frac{\pi}{6}$

42) a)

p	q	r	$\neg p$	$\neg q$	$(\neg q\vee r)$	$p\to(\neg q\vee r)$	$\neg p\vee(\neg q\vee r)$
T	T	T	F	F	T	T	T
T	T	F	F	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	F	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T
1	2	3	4	5	6	7	8

$7\equiv 8$

$p\to(\neg q\vee r)\equiv\neg p\vee(\neg q\vee r)$ Hence proved.

42) b) Type 1 even degree

$\div$ by x^2 $\quad 6x^2-35x+62-\frac{35}{x}+\frac{6}{x^2}=0$

$\Rightarrow 6\left(x^2+\frac{1}{x^2}\right)-35\left(x+\frac{1}{x}\right)+62=0$

Put $x^2+\frac{1}{x^2}=y^2-2$; $x+\frac{1}{x}=y$

$6(y^2-2)-35y+62=0$

$y=\frac{10}{3}$; $y=\frac{5}{2}$

$x+\frac{1}{x}=\frac{10}{3}$; $x+\frac{1}{x}=\frac{5}{2}$

$x=\frac{1}{3},3$ | $x=\frac{1}{2},2$

$\therefore$ The roots $3,\frac{1}{3},2,\frac{1}{2}$.

43) a) P.m.f

i) Pro

x	2	4	6	8	10
$f(x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

ii) The cumulative distribution function

x	2	4	6	8	10
$F(x)$	$\frac{1}{36}$	$\frac{5}{36}$	$\frac{15}{36}$	$\frac{27}{36}$	1

iii) $P(4\le X<10)=P(X=4)+P(X=6)+P(X=8)$

$=\frac{4}{36}+\frac{10}{36}+\frac{12}{36}$

$=\frac{26}{36}\Rightarrow\frac{13}{18}$

iv) $P(X\ge 6)=P(X=6)+P(X=8)+P(X=10)$

$=\frac{10}{36}+\frac{12}{36}+\frac{9}{36}$

$=\frac{31}{36}$

43) b) $y=x^2$, $y=(x-3)^2\Rightarrow x^2=(x-3)^2$

$\therefore$ Point of intersection $\left(\frac{3}{2},\frac{9}{4}\right)$

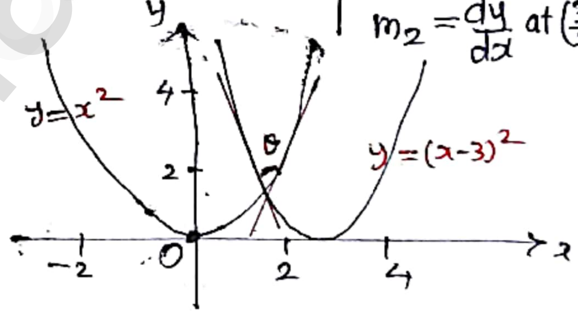
$\frac{dy}{dx}=2x$

$m_1=\frac{dy}{dx}$ at $\left(\frac{3}{2},\frac{9}{4}\right)=3$

$y=(x-3)^2$

$\frac{dy}{dx}=2(x-3)$

$m_2=\frac{dy}{dx}$ at $\left(\frac{3}{2},\frac{9}{4}\right)=-3$



$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\frac{3}{4}$

$\theta=\tan^{-1}\left(\frac{3}{4}\right)$

44) a) $[A/B]=\left[\begin{array}{ccc|c}2&3&5&9\\7&3&-5&8\\2&3&\lambda&\mu\end{array}\right]$

$\sim\left[\begin{array}{ccc|c}2&3&5&9\\0&-15&-45&-47\\0&0&\lambda-5&\mu-9\end{array}\right]$ $\quad R_2\to 2R_2-7R_1$, $R_3\to R_3-R_1$

i) If $\lambda=5$ & $\mu\ne 9$ then $\rho(A)=2$, $\rho(A/B)=3$ no solution.

ii) $\lambda\ne 5$ $\rho(A)=\rho[A/B]=3$ Unique solution

iii) $\lambda=5$, $\mu=9$ Infinitely many solutions

44) b) Non parametric Form :

$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$

$(\vec{b}\times\vec{c}) = -\hat{i}-10\hat{j}-7\hat{k} \Rightarrow -1(\hat{i}+10\hat{j}+7\hat{k})$

$[\vec{r}-(\hat{i}-2\hat{j}+4\hat{k})]\cdot(-1)(\hat{i}+10\hat{j}+7\hat{k})=0$

$\vec{r}\cdot(\hat{i}+10\hat{j}+7\hat{k})=9$

Cartesian Form :

Put $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+10\hat{j}+7\hat{k})=9$

$x+10y+7z=9$

45) a) $\dfrac{z-1}{z+1} = \dfrac{(x^2+y^2-1)+i(2y)}{(x+1)^2+y^2}$

$\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2} \Rightarrow \tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right)=\frac{\pi}{2}$

$\dfrac{2y}{x^2+y^2-1}=\tan\frac{\pi}{2} \Rightarrow x^2+y^2-1=0$

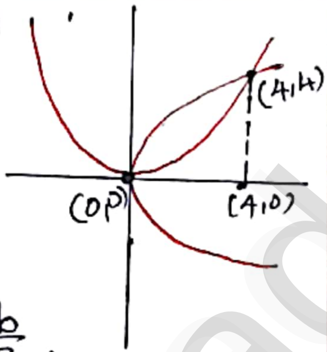
$\Rightarrow x^2+y^2=1$

45) b) $y^2=4x$ & $x^2=4y$

Eliminating y. $x^4=64x$

$\therefore x=0$ & $y=0$

$x=4$ & $y=4$



$A=\int_0^4 (y_u-y_v)\,dx$

$A=\int_0^4\left(2\sqrt{x}-\frac{x^2}{4}\right)dx=\frac{16}{3}$

46) a Let x & y be the two positive numbers. $xy=20$

$y=\frac{20}{x}$ —①

$S=x+y$

$S=x+\frac{20}{x}$

$\frac{dS}{dx}=1-\frac{20}{x^2}$

$\frac{dS}{dx}=0$

$1-\frac{20}{x^2}=0$

$1=\frac{20}{x^2}$

$x^2=20$

$x=\pm\sqrt{20}$

Since x is a +ve number

$x=\sqrt{4\times5}=2\sqrt{5}$

At $x=2\sqrt{5}$

$\frac{d^2S}{dx^2}=\frac{40}{(2\sqrt{5})^3}=+ve$

S is minimum when ~~x=2√5~~

$x=2\sqrt{5}$

① $\Rightarrow y=\frac{20}{2\sqrt{5}}=\frac{10}{\sqrt{5}}$

$y=\frac{2\times\sqrt{5}\times\sqrt{5}}{\sqrt{5}}=2\sqrt{5}$

$\therefore$ The two +ve numbers are $2\sqrt{5}$ & $2\sqrt{5}$

46) b) Let $x(t)$ be the population at time t. then $\frac{dx}{dt}=kx$

By Separating the variables, we obtain

$\frac{dx}{x}=k\,dt.$

$\int\frac{dx}{x}=k\int dt \Rightarrow \log|x|=kt+\log|c|$

(or) $x=Ce^{kt}$

Let x_0 be the population when $t=0$ and $c=x_0$

$x=x_0e^{kt}$

$x=2x_0$, when $t=50$ & $k=\frac{1}{50}\log 2.$

Hence $x=x_0 2^{\frac{t}{50}}$ is the population at t.

$x=3x_0$, when $t=t_1$

$t_1=50\left(\frac{\log 3}{\log 2}\right).$

$\therefore$ the population is tripled in $50\left(\frac{\log 3}{\log 2}\right)$ years.

47) a) $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

$f(x,y)=\sin u=\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

$f(tx,ty)=\sin u=t\cdot t^{-1/2}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

Degree $=\frac{1}{2}$ $(\therefore n=\frac{1}{2})$

By Euler's theorem; $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf$

$x\frac{\partial(\sin u)}{\partial x}+y\frac{\partial(\sin u)}{\partial y}=\frac{1}{2}\sin u$

$\cos u\left[x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right]=\frac{1}{2}\sin u$

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u.$

47 b) $V\left(\frac{1}{2},4\right)$

Equation of the parabola

$(x-h)^2=-4a(y-k)$

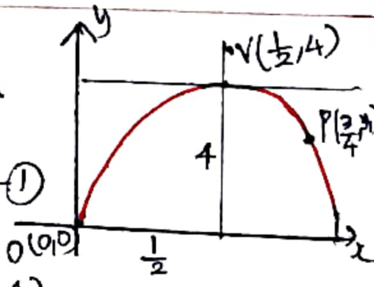
$(x-\frac{1}{2})^2=-4a(y-4)$ —①

at $(0,0)$

① $\Rightarrow (0-\frac{1}{2})^2=-4a(0-4)$

$4a=\frac{1}{16}$

② $\Rightarrow (x-\frac{1}{2})^2=-\frac{1}{16}(y_1-4)$ —②



$(\frac{3}{4}-\frac{1}{2})^2=-\frac{1}{16}(y_1-4)$

$-\frac{16}{16}=(y_1-4)$

$4-1=y_1$

$y_1=3$

Height $=3m$