XII STD

VELLORE EDUCATIONAL DISTRICT
HALF YEARLY EXAMINATION, DEC-2024

SUB: MATHEMATICS

(ANSWER KEY-EM)

Max.Marks:90

PART - I (20 x 1= 20)

Q.No	Correct Choice	Correct Answer
1	d)	Positive **(MA)**
2	b)	$c = \pm\sqrt{3}$
3	c)	$[0, \pi]$
4	c)	Parabola
5	a)	$x^2 + y^2 + 6x + 8y + 16 = 0$
6	a)	81
7	d)	$\|k\| \geq 6$
8	d)	$\frac{1}{3}$
9	b)	2.5
10	d)	$a * b = a^b$
11	b)	Length of the major axis
12	d)	2
13	b)	Consistent
14	d)	-1
15	**(MA)**	100°
16	c)	$(1,0)$
17	a)	$\frac{3\pi}{512}$
18	c)	n complex roots
19	a)	$\frac{\pi}{3}$
20	d)	9

PART - II (7 x 2 = 14)

Answer any 7 questions. Q.No. 30 is compulsory.

21	$A^{-1} = \pm\frac{1}{\sqrt{\|adjA\|}}(adjA)$ → (1) $A^{-1} = \pm\frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ → (1)
22	$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = i^3 - (-i)^3$ →(1) $= -i - i = -2i$ →(1)
23	$\sum 1 = \frac{-5}{2}$ & $\sum 4 = 4$ →(1) $2x^2 - 3x - 20 = 0$ →(1)
24	Tangent : $3x - 4y = -25$ →(1) Normal : $4x + 3y = 0$ →(1)
25	$\vec{u} = 2\vec{\imath} + \vec{j} - 2\vec{k}$ & $\vec{v} = 4\vec{\imath} - 4\vec{j} + 2\vec{k}$ →(1) $\vec{u} \cdot \vec{v} = 0 \Rightarrow$ Perpendicular →(1)
26	By mean value theorem $\frac{f(7)-f(2)}{7-2} \leq 29$ →(1) $f(7) \leq 162$ →(1)
27	$df = f^1(x)\,dx = (2x+3)dx$ → (1) $df = 0.18$ → (1)
28	$f(-x) = -f(x)$; **f(x) is odd function** →(1) $I = 0$ →(1)
29	$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$ →(1) $\sin^{-1} y = \sin^{-1} x + c$ →(1)
30	$e_2 * e_1 = e_1 * e_2 = e_2$ $e_1 * e_2 = e_2 * e_1 = e_1$ } →(1) $e_1 = e_2$ →(1)

PART -III (7x3=21)

Answer any 7 questions. Q.No. 40 is compulsory.

Q.NO.	ANSWER KEY
31	$\|A\| = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{vmatrix} = 4 \neq 0$ →(2) Rank = 3 →(1)
32	$\sqrt{a-ib} = \pm\left[\sqrt{\frac{\|z\|+a}{2}} - i\sqrt{\frac{\|z\|-a}{2}}\right]$ →(1) $\|-5-12i\| = \sqrt{25+144} = 13$ →(1) $\sqrt{-5-12i} = \pm(2-3i)$ →(1)
33	Yes it is true →(1) $cos\alpha = -x$ $x = -cos\alpha = cos(\pi-\alpha)$ →(1) $\alpha = \pi - cos^{-1}x$ →(1) $cos^{-1}(-x) = \pi - cos^{-1}x$
34	By the property of area of triangle $\|\vec{a}\times\vec{b}\| = \|\vec{b}\times\vec{c}\| = \|\vec{c}\times\vec{a}\|$ →(1) $absin(\pi-C) = bcsin(\pi-A) = casin(\pi-C)$ →(1) $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$ →(1)
35	$\lim_{x\to\infty} \frac{2x^2-3}{x^2-5x+3} = \frac{\infty}{\infty}\ form$ (1) $= \lim_{x\to\infty} \frac{4x}{2x-5}$ (1) $= \lim_{x\to\infty} \frac{4}{2} = 2$ (1)
36	**f is a homogeneous with degree** $n = \frac{3}{2}$ →(1) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = n\,f$ →(1) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$ →(1)
37	$a+c = 152\times10^6\ km$ (1) $a-c = 94.5\times10^6\ km$ (1) Distance of the sun from other focus $SS' = 2c = 57.5\times10^6$ $= 575\times10^5\ km$ (1)
38	Circle $(x-a)^2+y^2 = a^2$ →(1) $x^2+2xy\frac{dy}{dx}-y^2 = 0$ →(2)
39	X: 0, 1, 2, Total Image: 25, 10, 1, 36 →(1) X: 0, 1, 2 P(x): $^{25}/_{36}$, $^{10}/_{36}$, $^{1}/_{36}$ →(2)
40	$\left[8\frac{x^{1/2}}{1/2} - 12\frac{x^{5/2}}{5/2}\right]_1^4$ →(1) $= \frac{-664}{5}$ →(2)

PART - IV (7 x 5 = 35)

Answer All the questions:

41(a)	$[A/B] = \begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & 1 & -6 & -2 \\ 0 & 0 & \lambda-7 & \mu-9 \end{bmatrix}$ →(2) i) No solution $\lambda = 7, \mu \neq 9 \Rightarrow \rho(A) \neq \rho(A/B)$ →(1) ii) Unique solution $\lambda \neq 7, \mu \neq 9 \Rightarrow \rho(A) = \rho(A/B) = 3$ →(1) iii) Many solutions $\lambda = 7, \mu = 9 \Rightarrow \rho(A) = \rho(A/B) = 2 < 3$ →(1)
41(b)	$tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right) = \pi$ →(2) $\frac{x+y+z-xyz}{1-xy-yz-zx} = tan\pi = 0$ →(1) $x+y+z-xyz = 0$ →(1) $x+y+z = xyz$ →(1)
42(a)	$\sqrt{3}+i = 2\left(CIS\frac{\pi}{6}\right)$ →(1) $(\sqrt{3}+i)^{1/3} = \sqrt[3]{2}\,CIS\left(\frac{12k\pi+\pi}{18}\right)$ →(2) **Roots are** $\sqrt[3]{2}CIS\frac{\pi}{18}$, $\sqrt[3]{2}CIS\frac{13\pi}{18}$, $\sqrt[3]{2}CIS\frac{25\pi}{18}$ →(2)

42(b)	$I = \int_0^{\frac{\pi}{4}} log\left[1 + tan\left(\frac{\pi}{4} - x\right)\right] dx$ →(1) $I = \int_0^{\frac{\pi}{4}} log\, 2\, dx - \int_0^{\frac{\pi}{4}} log(1 + tan\, x)\, dx$ →(2) $2I = \frac{\pi}{4} log\, 2$ →(1) $I = \frac{\pi}{8} log\, 2$ →(1)
43(a)	Rough diagram →(1) $y = \frac{k}{2} - x$ Area $A = x\left(\frac{k}{2} - x\right) = \frac{k}{2}x - x^2$ →(1) $A'(x) = 0 \Rightarrow x = \frac{k}{4}$ →(1) $A''(x) = -2 \Rightarrow maximum$ →(1) $x = y = \frac{k}{4} \Rightarrow It\ is\ a\ square$ →(1)
43(b)	Rough diagram →(1) $\vec{a} = cos\,\alpha\,\vec{i} + sin\,\alpha\,\vec{j}$ $\vec{b} = cos\,\beta\,\vec{i} - sin\,\beta\,\vec{j}$ →(1) $\vec{b} \times \vec{a} = \vec{k}(sin\,\alpha\, cos\,\beta + cos\,\alpha\, sin\,\beta)$ →(1) $\vec{b} \times \vec{a} = \vec{k}\, sin(\alpha + \beta)$ →(1) $sin(\alpha + \beta) = sin\,\alpha\, cos\,\beta + cos\,\alpha\, sin\,\beta$ →(1)
44(a)	$y^2 - 10y + 24 = 0$ →(2) $y = 6, 4$ →(1) $x + \frac{1}{x} = 6 \Rightarrow x = 3 \pm 2\sqrt{2}$ →(1) $x + \frac{1}{x} = 4 \Rightarrow x = 2 \pm \sqrt{3}$ →(1)
44(b)	$\frac{dA}{dt} = kA$ $A = Ce^{kt}$ →(1) $t = 0 \Rightarrow C = 200$ →(1) $t = 2 \Rightarrow k = \frac{-1}{2} log\left(\frac{4}{3}\right)$ →(1) $A(t) = 200e^{\frac{-t}{2} log\left(\frac{4}{3}\right)}$ →(1) $t = \frac{2\, log\left(\frac{1}{2}\right)}{log\left(\frac{4}{3}\right)}$ →(1)

45(a)	Rough Diagram →(1) **Parabola** $x^2 = 4ay$ →(1) **Points are** $(6,\ y_1), (12,\ y_2)$ →(1) $4a = \frac{900}{13}$ →(1) **Height = 5.08 m** →(1)
45(b)	$\int_{200}^{600} f(x)dx = 1 \Rightarrow k = \frac{1}{400}$ → (2) $F(x) = \begin{cases} \frac{x}{400} - \frac{1}{2} & ; 200 \le x \le 600 \\ 1 & ; x > 600 \end{cases}$ →(2) $P(300 \le x \le 500) = \frac{1}{2}$ →(1)
46(a)	$\vec{a} = 2\vec{i} + 3\vec{j} + 6\vec{k}$ $\vec{u} = 2\vec{i} + 3\vec{j} + 1\vec{k}$ $\vec{v} = 2\vec{i} - 5\vec{j} - 3\vec{k}$ →(2) **Non-parametric equation** $(\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{c}) = 0$ $\vec{r} \cdot (\vec{i} - 2\vec{j} + 4\vec{k}) = 20$ →(1) $x - 2y + 4z = 20$ →(2)
46(b)	**Intersecting points** $(2, 2\sqrt{3}), (2, -2\sqrt{3})$ →(2) **Area** $\mathbf{A} = \int_c^d [\,\mathbf{x_R} - \mathbf{x_L}\,]\,\mathbf{dy}$ →(1) $= \int_{-2\sqrt{3}}^{2\sqrt{3}} \left[\sqrt{16 - y^2} - \frac{y^2}{6}\right]\,\mathbf{dy}$ →(1) $= \frac{4}{3}(4\pi + \sqrt{3})$ →(1)
47(a)	Slope $m = \frac{dy}{dx} = \frac{-2cost}{7sint}$ →(2) Tangent $y - y_1 = m(x - x_1)$ →(1) $2cost\ x + 7sint\ y = 14$ →(1) Normal : $7sint\ x - 2cost\ y = 45sint\ cost$ →(1)
47(b)	$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$ →(1) $\equiv (\neg p \vee q) \wedge (p \vee \neg q)$ →(1) $\equiv (\neg p \wedge (p \vee \neg q)) \vee (q \wedge (p \vee \neg q))$ →(1) $\equiv F \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee F$ →(1) $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$ →(1)