Tsi12M

16-12-24

Tenkasi District

Common Half Yearly Examination - 2024

Time: 3.00 Hours

Standard 12

MATHEMATICS

Marks: 90

PART - I

I. Answer all the questions. 20x1=20

1) If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adj\,(AB)| =$

a) −40 b) −80 c) −60 d) −20

2) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that, $\lambda A^{-1} = A$ then λ is

a) 17 b) 14 c) 19 d) 21

3) If $(1+i)(1+2i)(1+3i)\ldots\ldots(1+ni) = x+iy$ then $2.5.10\ldots(1+n^2)$ is

a) 1 b) i c) $x^2 + y^2$ d) $1 + n^2$

4) Which of the following is not a cube root of unity

a) $\frac{-1+i\sqrt{3}}{2}$ b) −1 c) $\frac{-1-i\sqrt{3}}{2}$ d) 1

5) The number of positive zeros of the polynomial $\sum_{r=0}^{n} {}^nC_r (-1)^r x^r$ is

a) 0 b) n c) <n d) r

6) The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

a) 2 b) 4 c) 1 d) ∞

7) Angle between $y^2 = x$ and $x^2 = y$ at the origin is

a) $\tan^{-1}\frac{3}{4}$ b) $\tan^{-1}\frac{4}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

8) The order and degree of the differential equation, $dy + (xy - \cos x)dx = 0$ are respectively

a) 2, 1 b) 1, 1 c) 1, 3 d) 1, 2

9) If $u(x, y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

a) $e^{x^2+y^2}$ b) 2xu c) x^2u d) y^2u

10) If $\sin^{-1}\frac{x}{5} + \text{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$, then the value of x is

a) 4 b) 5 c) 2 d) 3

11) Area of the greatest reactangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

a) 2ab b) ab c) $\sqrt{ab}$ d) $\frac{a}{b}$

12) If $\bar{a}.\bar{b} = \bar{b}.\bar{c} = \bar{c}.\bar{a} = 0$, then value of $[\bar{a}\bar{b}\bar{c}]$ is

a) $|\bar{a}||\bar{b}||\bar{c}|$ b) $\frac{1}{3}|\bar{a}||\bar{b}||\bar{c}|$ c) 1 d) −1

13) The angle between the line $\bar{r} = (\bar{i} + 2\bar{j} - 3\bar{k}) + t(2\bar{i} + \bar{j} - 2\bar{k})$ and the plane $\bar{r}.(\bar{i} + \bar{j}) + 4 = 0$ is

a) 0° b) 30° c) 45° d) 90°

14) In the set Q define $a \odot b = a + b + ab$. For what value of y, $3 \odot (y \odot 5) = 7$?

a) $y = \frac{2}{3}$ b) $y = \frac{-2}{3}$ c) $y = \frac{-3}{2}$ d) y=4

15) The number of rows in the truth table of $(P \vee \neg t) \wedge (P \vee \neg S)$ is

a) 6 b) 8 c) 9 d) 3

16) If P(X=0)=1−P(x=1). If E(x) = 3 Var(x), then P(x=0) is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

17) The solution of the differential equation $\frac{dy}{dx} = 2xy$ is

a) $y = ce^{x^2}$ b) $y=2x^2 + c$ c) $y = ce^{-x^2} + c$ d) $y = x^2 + c$

18) The volume of solid of revolution of the region bounded by $y^2 = x(a - x)$ about x-axis is

a) πa^3 b) $\frac{\pi a^3}{4}$ c) $\frac{\pi a^3}{5}$ d) $\frac{\pi a^3}{6}$

19) The type of conic section for $x^2 - 2y = x + 3$ is

a) hyperbola b) ellipse c) circle d) parabola

20) If $|adj\ (adj\ A)| = |A|^9$, then the order of the square matrix A is

a) 3 b) 4 c) 2 d) 5

PART - II

II. Answer any 7 questions. Q.No. 30 is compulsory **7x2=14**

21) Solve the system of linear equations by Crame's Rule. $5x - 2y + 16 = 0$, $x+3y-7=0$

22) Find a Polynomial equation of minimum degree with rational coefficients, having $2-\sqrt{3}$ is a root

23) Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$

24) Find two positive number whose sum is 12 and their product is maximum

25) Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $2\vec{i} - 3\vec{j} + 4\vec{k}$, $\vec{i} + 2\vec{j} - \vec{k}$ and $3\vec{i} - \vec{j} + 2\vec{k}$

26) A random variable x has the following probability mass function

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find the value of k?

27) Let $v(x, y, z) = xy+yz + zx$, $x, y, z \in R$, Find the differential dv

28) Find centre and radius of the circle $2x^2+2y^2-6x+4y+2=0$.

29) Find the differential equation corresponding to the family of curves represented by the equation $y = Ae^{7x} + Be^{-7x}$, where A and B are arbitrary constants

30) Evaluate: $\int_{-\log 2}^{\log 2} e^{-|x|}dx$

PART - III

III. Answer any 7 questions. Q.No. 40 is compulsory **7x3=21**

31) Find the rank of the matrices by row reduction method $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$

32) If $z = \cos\theta + i\sin\theta$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i\sin n\theta$

33) Solve the equation $9x^3-36x^2+44x-16=0$. If the roots form an arithmetic progression.

34) Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$

35) If the normal at the point 't_1' on the parabola $y^2=4ax$ meets the parabola again at the point 't_2', then prove that $t_2=-\left(t_1+\frac{2}{t_1}\right)$

36) Prove, using mean value theorem that $|\sin\alpha-\sin\beta|\le|\alpha-\beta|$ $\alpha,\beta\in R$

37) Find the volume of a right - circular cone of base radius r and height h.

38) Solve $\frac{dy}{dx}+\frac{y}{x}=\sin x$

39) Define an operation * on Q as follows:

$a*b=\left(\frac{a+b}{2}\right)$:a, b $\in$ Q. Examine the existence of identity and the existence of inverse for the operation * on Q

40) Prove that $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]=2[\vec{a}\ \vec{b}\ \vec{c}]$

PART - IV

IV. Answer all the questions. **7x5=35**

41) a) Investigate the value of λ and μ then system of linear equation $2x+3y+5z=9$; $7x+3y-5z=8$, $2x+3y+\lambda z=\mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(OR)

b) For the function $f(x)=4x^3+3x^2-6x+1$. Find the intervals of monotonicity, local extrema, intervals of cancavity and points of inflection

42) a) If $g(x,y)=\frac{x^3y}{x^6+y^2}$ when $(x,y)\neq(0,0)$ and $g(0,0)=0$ then prove that It is not continuous at (0,0)

(OR)

b) If $z=x+iy$ is a complex number such that $I_m\left(\frac{2z+1}{iz+1}\right)=0$, show that the locus of z is $2x^2+2y^2+x-2y=0$

43) a) If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and $0<x, y, z<1$, show that $x^2+y^2+z^2+2xyz=1$.

(OR)

b) Find the population of a city at any time t, given that the rate of increase of population is proportional to the populaiton at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000

44) a) For the ellipse $4x^2+y^2+24x-2y+21=0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2.

(OR)

b) Prove that $P\to(\neg q\vee r)\equiv\neg P\vee(\neg q\vee r)$ using truth table

45) a) Find the non-parametric form of vector equation and cartesian equation of the plane passing through the point (1, −2, 4) and perpendicular to the plane $x + 2y - 3z = 11$ and Parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$

(OR)

b) Suppose a discrete random variable can only take the values 0, 1 and 2. The probability mass function is defined by

$$f(x) = \begin{cases} \frac{x^2+1}{k}, & \text{for } x = 0,1,2 \\ 0 & \text{otherwise} \end{cases}$$

find (i) the value of k (ii) cumulative distribution function (iii) $P(x \geq 1)$

46) a) Find the area of the region common to the circle $x^2+y^2=16$ and the parabola $y^2=6x$

(OR)

b) If $\vec{a} = 2\vec{i}+3\vec{j}-\vec{k}$, $\vec{b} = 3\vec{i}+5\vec{j}+2\vec{k}$, $\vec{c} = -\vec{i}-2\vec{j}+3\vec{k}$, verify that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

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47) a) If $2 + i$ and $3 - \sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$

(OR)

b) Onlighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.
