VNR12M

Virudhunagar District
Common Half Yearly Exam, December - 2024

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Standard 12
MATHS

Time: 3.00 Hours | Marks: 90

PART - I

Answer all the quesitons. Choose the correct answer: **20×1=20**

1) If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}$ then K =
   a) 0 b) $\sin\theta$ c) $\cos\theta$ d) 1

2) If z = x+iy is a complex number such that $|z+2| = |z-2|$, then the locus of z is
   a) real axis b) imaginary axis c) ellipse d) circle

3) $\sum_{n=1}^{12} i^n$ is equal to
   a) 1 b) −1 c) i d) 0

4) Which of the following is/are correct?
   i) Adjoint of a symmetric matrix is also a symmetric matrix
   ii) Adjoint of a diagonal matrix is also a diagonal matrix
   iii) If A is a square matrix of order n and λ is a scalar, then $\text{adj}(\lambda A) = \lambda^n \text{adj}(A)$
   iv) $A(\text{adj } A) = (\text{adj } A)A = |A|I$
   a) only (i) b) (ii) and (iii) c) (iii) and (iv) d) (i) (ii) and (iv)

5) If α and β are the roots of $x^2+x+1 = 0$, then $\alpha^{2020}+\beta^{2020}$ is
   a) −2 b) −1 c) 1 d) 2

6) $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to
   a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

7) The domain of $\text{cosec}^{-1}x$ function is
   a) $R\backslash(-1, 1)$ b) $R\backslash[-1, 1]$ c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ d) $R\backslash\{0\}$

8) The radius of the circle $3x^2+by^2+4bx-6by+b^2 = 0$ is
   a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

9) If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is
   a) $2x+1 = 0$ b) $x = -1$ c) $2x-1 = 0$ d) $x = 1$

10) With usual notation which one is not equal to $\vec{a}.(\vec{b}\times\vec{c})$?
   a) $-\vec{a}.(\vec{c}\times\vec{b})$ b) $\vec{c}.(\vec{a}\times\vec{b})$ c) $-\vec{b}.(\vec{c}\times\vec{a})$ d) $(\vec{c}\times\vec{a}).\vec{b}$

11) If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a}\times\vec{b}] = \frac{1}{4}$ then the angle between $\vec{a}$ and $\vec{b}$ is
   a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

12) Find the point on the curve $6y = x^3+2$ at which y-coordinate changes 8 times as fast as x-coordinate is
   a) (4, 11) b) (4, −11) c) (−4, 11) d) (−4, −11)

13) What is the value of the limit $\lim_{x \to 0}\left(\cot x - \frac{1}{x}\right)$?

a) 0 b) 1 c) 2 d) ∞

14) If $v(x, y) = \log(e^x+e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to

a) e^x+e^y b) $\frac{1}{e^x + e^y}$ c) 2 d) 1

15) The solution of $\frac{dy}{dx} + P(x)\,y = 0$ is

a) $y = ce^{\int Pdx}$ b) $y = ce^{-\int Pdx}$ c) $x = ce^{-\int Pdy}$ d) $x = ce^{\int Pdy}$

16) The differential equation of the family of curves $y = Ae^x+Be^{-x}$, where A and B are arbitrary constants is

a) $\frac{d^2y}{dx^2} + y = 0$ b) $\frac{d^2y}{dx^2} - y = 0$ c) $\frac{dy}{dx} + y = 0$ d) $\frac{dy}{dx} - y = 0$

17) If $P(X = 0) = 1-P(X = 1)$. If $E(X) = 3Var(X)$, then $P(X = 0)$

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

18) A random variable X has binomial distribution with n = 25 and p = 0.8 then standard deviation of X is

a) 6 b) 4 c) 3 d) 2

19) In the set Q define $a*b = a+b+ab$. Then the solution of $3*(y*5) = 7$ is

a) $y = \frac{2}{3}$ b) $y = -\frac{2}{3}$ c) $y = \frac{-3}{2}$ d) $y = 4$

20) Which one of the following is incorrect?

a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$ b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

c) $\neg(p \vee q) \equiv \neg p \vee \neg q$ d) $\neg(\neg p) \equiv p$

PART - II

Answer any 7 questions. (Question number 30 is compulsory) 7×2=14

21) Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal

22) Find the square root of 4+3i

23) If α and β are the roots of the quadratic equation $17x^2+43x-73 = 0$, constuct a quadratic equation whose roots are $\alpha+2$, $\beta+2$.

24) Find the principal value of $\cos^{-1}\frac{1}{2}$

25) Identify the type of conic for the equation $4x^2-9y^2-16x+18y-29 = 0$.

26) If $\vec{a} = -3\vec{i} - \vec{j} + 5\vec{k}$, $\vec{b} = \vec{i} - 2\vec{j} + \vec{k}$, $\vec{c} = 4\vec{j} - 5\vec{k}$, find $\vec{a}.(\vec{b} \times \vec{c})$

27) Verify Rolle's theorem for the function $f(x) = \tan x$, $x\in[0, \pi]$

28) Form the differential equation by eliminating the arbitrary constants A and B from $y = A\cos x + B\sin x$.

29) Construct a truth table for $p \,\underline{\vee}\, q$

30) Show that $f(x, y) = \frac{6x^2y^3 - \pi y^5 + 9x^4y}{2020x^2 + 2019y^2}$ is a homogenous function of degree 3.

PART - III

Answer any 7 questions. (Question number 40 is compulsory) **7×3=21**

31) Find adj(adj A) if adj A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

32) Show that $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

33) Show that the equation $x^9-5x^5+4x^4+2x^2+1 = 0$ has atleast 6 imaginary solutions.

34) Find the domain of $\sin^{-1}(2-3x^2)$

35) Find the equation of the parabola with vertex (1, −2) and focus (4, −2)

36) Evaluate: $\lim_{x \to \infty} \frac{x^2+17x+29}{x^4}$

37) Solve: $(1+x^2)\frac{dy}{dx} = 1+y^2$

38) Suppose two coins are tossed once. If X denotes the number of tails (i) write down the sample space (ii) find the inverse image of 1 (iii) the values of the random variable and the number of elements in its inverse images.

39) Let A be Q\{1}. Define * on A by x*y = x+y − xy. is * a binary on A?

40) Find the distance between the planes $\vec{r}.(2\hat{i}-\hat{j}-2\hat{k}) = 6$ and $\vec{r}.(6\hat{i}-3\hat{j}-6\hat{k}) = 27$

PART - IV

Answer all the questions: **7×5=35**

41) a] Investigate the values of λ and μ the system of linear equations 2x+3y+5z = 9, 7x+3y−5z = 8, 2x+3y+λz = μ, have (i) no solution (ii) a unique solution (iii) an infinite number of solution.

(OR)

b] Sketch the curve $y = f(x) = x^2-x-6$

42) a] If z = x+iy and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2+y^2 = 1$.

(OR)

b] If $v(x, y, z) = x^3+y^3+z^3+3xyz$, show that $\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial z \partial y}$

43) a] Solve the equation: $x^4-10x^3+26x^2-10x+1 = 0$

(OR)

b] The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

44) a] If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z = \pi$, show that $x+y+z = xyz$

(OR)

b] Verify (i) closure property (ii) commutative property (iii) associative property (iv) existence of identity and (v) existence of inverse for the operation X_{11} on a subset A = {1, 3, 4, 5, 9} of the set of remainders {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

45) a] Find the equations of tangent and normal to the parabola $x^2+6x+4y+5 = 0$ at (1, −3)

(OR)

b] A random variable x has the following probability mass function

x	1	2	3	4	5	6
f(x)	K	2K	6K	5K	6K	10K

Find (i) $P(2 < x < 6)$ (ii) $P(2 \le X < 5)$ (iii) $P(X \le 4)$ (iv) $P(3 < X)$

46) a] By Vector method, prove that $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

(OR)

b] Prove that the ellipse $x^2+4y^2 = 8$ and the hyperbola $x^2-2y^2 = 4$ intersect orthogonally.

47) a] On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 m when it is 6 m away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection.

(OR)

b] Find the parametric form of vector equation and Cartesian equation of the plane passing through the points (2, 2, 1) (9, 3, 6) and perpendicular to the plane $2x+6y+6z = 9$

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