LONDON SCHOOL

ORATHANADU

+2

MATHS

VOLUME 1 & 2

2024 - 25

MINIMUM Q-BANK

AND BOOK BACK ONE WORDS WITH ANSWERS

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5 MARKS

CHAPTER 1 – Application of Martrices and Determents

1. If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$,verify that $A(adj\ A) = (adj\ A)A = |A|I_3$ **(Eg.1.1)**
2. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$. **(EX 1.1 -3)**
3. If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ find the products AB and BA and hence solve the system of equations $x - y + z = 4,\ \ x - 2y - 2z = 9, 2x + y + 3z = 1$. **(Eg.1.24)**
4. If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ find the products AB and BA and hence solve the system of equations $x - y + z = 4,\ \ x - 2y - 2z = 9, 2x + y + 3z = 1$. **(Eg.1.24)**
5. Solve, by Cramer's rule,system of equations $x_1 - x_2 = 3, 2x_1 + 3x_2 + 4x_3 = 17, x_2 + 2x_3 = 7$. **(Eg.1.25)** **June '23**
6. Solve the systems of linear equations by Cramer's rule: $3x + 3y - z = 11, 2x - y + 2z = 9, 4x + 3y + 2z = 25$ **(Ex. 1.4 1(iii)**
7. Solve $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0, \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0, \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$**(Ex. 1.4 1(iv)**
8. Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7, x + y + \lambda z = \mu, x + 3y - 5z = 5$has (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(EG 1.34)**
9. Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9, 7x + 3y - 5z = 8, 2x + 3y + \lambda z = \mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(EX 1.6 - 3)**
10. Determine the values of λ for which the following system of equations $x + y + 3z = 0, 4x + 3y + \lambda z = 0, 2x + y + 2z = 0$, has (i)a unique solution (ii) a non-trivial solution. **(EX 1.7 - 2)**
11. Find the value of k for which the equations $kx - 2y + z = 1, x - 2ky + z = -2, x - 2y + kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution **(EX 1.6 - 2)**
12. Solve the system of linear equations by matrix inversion method: $x + y + z - 2 = 0, 6x - 4y + 5z - 31 = 0, 5x + 2y + 2z = 13$ **(EX 1.3**

CHAPTER 2 – Complex Numbers

1. Show that (i) $(2 + i\sqrt{3})^{10} + (2 - i\sqrt{3})^{10}$is real and (ii) $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary. **(Eg.2.8)**
2. Show that $(i)\left(2 + i\sqrt{3}\right)^{10} + (2 - i\sqrt{3})^{10}$is real and (ii) $\left(\frac{19-9i}{9+i}\right)^{12} - \left(\frac{20-5i}{7-6i}\right)^{12}$ is purely imaginary. **(Ex.2.4 - 7)**
3. Let $z_1, z_2$and $z_3$be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$, P.T. $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right| = r$. **(Eg.2.15)**
4. If z_1, z_2 and $z_3$are three complex numbers such that $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, Prove that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$. **(Ex.2.5 - 7)**
5. If $z = x + iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$. Show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$. **(Ex.2.6 - 2)**
6. If $z = x + iy$and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, then show that $x^2 + y^2 = 1$. **(EG 2.27)**

7. If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, then show that $x^2 + y^2 + 3x - 3y + 2 = 0$. **(EX 2.7 - 6) June' 23**
8. Solve the equation $z^3 + 8i = 0$, where $z \in C$. **(EG 2.34) March '23**
9. Solve the equation $z^3 + 27 = 0$. **(EX 2.8 - 5)**
10. If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$are $-1, 1 - 2\omega, 1 - 2\omega^2$. **(EX 2.8 - 6)**
11. If $2\cos\alpha = x + \frac{1}{x}and2\cos\beta = y + \frac{1}{y}$show that
    $(i)\ \frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$ $(ii)\ xy - \frac{1}{xy} = 2i\sin(\alpha + \beta)$
    $(iii)\ \frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$ $(iv) x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$. **(EX 2.8 - 4)**
12. Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle. **(EG 2.14)**

CHAPTER 3 THEORY OF EQUATIONS

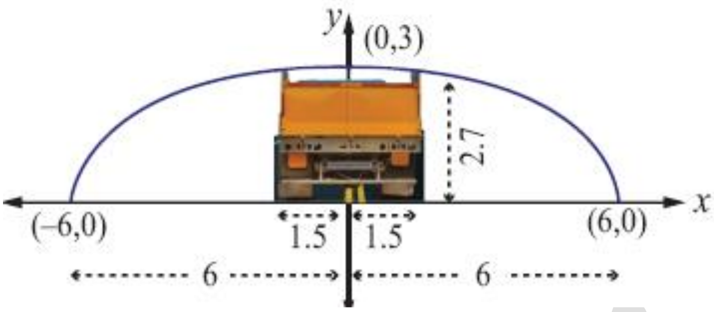
1. If $2 + i$and $3 - \sqrt{2}$are roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots. **(EG 3.15)**
2. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$and $\sqrt{3}$ are two of its zeros. **(EX 3.3 - 5)**
3. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$if it is known that $\frac{1}{3}$ is a solution. **(EX 3.5 - 7)**
4. Solve the equations $(i)\ 6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$, **(EX 3.5 - 5)**
5. Solve the equations $(ii) x^4 + 3x^3 - 3x - 1 = 0$. **(EX 3.5 - 5)**
6. Solve the equation $2x^3 + 11x^2 - 9x - 18 = 0$. **(EG 3.18) June '23**
7. Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$. **(EG 3.28)**

INVERSE TRIGONOMETRIC FUNCTIONS

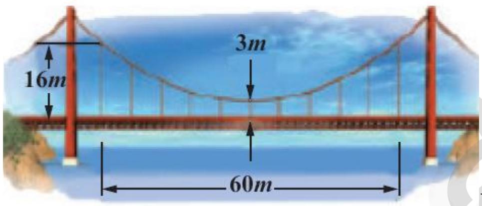
1. To draw $y - sinx$ and $y = sin^{-1}x$
2. To draw $y - \cos x$ and $y = cos^{-1}x$
3. To draw $y - \tan x$ and $y = tan^{-1}x$
4. Find the domain of $(i)\ f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + cos^{-1}\left(\frac{1-|x|}{4}\right)$ **EX. 4.2-6(i)**
5. Find the value of $(iii)\ cos\left(sin^{-1}\left(\frac{4}{5}\right) - tan^{-1}\left(\frac{3}{4}\right)\right)$ **(EX 4.3 - 4)**
6. Find the value of $(ii)\ sin\left(tan^{-1}\left(\frac{1}{2}\right) - cos^{-1}\left(\frac{4}{5}\right)\right)$.**(EX 4.3 - 4)**
7. Evaluate $\sin\left[sin^{-1}\left(\frac{3}{5}\right) + sec^{-1}\left(\frac{5}{4}\right)\right]$. **(EG 4.20)**
8. Prove that $(ii)\ sin^{-1}\frac{3}{5} + cos^{-1}\frac{12}{13} = sin^{-1}\frac{16}{65}$ **(EX 4.5 - 4)**
9. Prove that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$. **(EX 4.5 - 5**
10. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, show that $x + y + z = xyz$. **(EX 4.5 - 6)**
11. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$. **(EG 4.22) March '23**
12. If $a_1, a_2, a_3, \ldots, a_n$is an arithmetic progression with common difference d,
    P.T. $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + .. + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1a_n}$. **(EG 4.23)**
13. Solve $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$, if $6x^2 < 1$. **(EG 4.27)**
14. Find the number of solution of the equation $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$. **(EX 4.5 - 10)**

CHAPTER 5 - TWO DIMENSIONAL ANALYTICAL GEOMETRY-II

1. Find the equation of the circle passing through the points $(1,1), (2,-1)$ and $(3,2)$. **Eg.5.10**
2. A road bridge over an irrigation canal have two semi circular vents each with a span of $20m$ and the supporting pillars of width $2m$. to write the equations that model the arches. **Eg.5.13**
3. A semielliptical archway over a one-way road has a height of $3m$ and a width of $12m$. The truck has a width of $3m$ and a height of $27.m$. Will the truck clear the opening of the archway? $(Fig.5.6)$ **Eg.5.30**

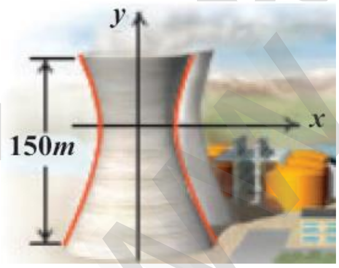


4. A bridge has a parabolic arch that is $10m$ high in the centre and $30m$ wide at the bottom. Find the height of the arch $6m$ from the centre, on either sides. **EX. 5.5-1**
5. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be $16m$, and the height at the edge of the road must be sufficient for a truck $4m$ high to clear if the highest point of the opening is to be $5m$ approximately . How wide must the opening be? **EX. 5.5-2**
6. Parabolic cable of a $60m$ portion of the roadbed of a suspension bridge are



positioned as shown below. Vertical Cables are to be spaced every $6m$ along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex. **EX. 5.5-5**

7. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$.



The tower is $150\ m$ tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower. **EX. 5.5-6**

8. A rod of length $1.2\ m$ moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is $0.3\ m$ from the end in contact with x –axis is an ellipse. Find the eccentricity. **EX. 5.5-7**
9. Assume that water issuing from the end of a horizontal pipe, $7.5\ m$above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position $2.5\ m$below

the line of the pipe, the flow of water has curved outward $3m$ beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground? **Eg.,6.5**

10. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4\ m$ when it is $6\ m$ away from the point of projection. Finally it reaches the ground $12\ m$ away from the starting point. Find the angle of projection. **EX. 5.5-9**
11. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact. **EX. 5.4-3**
12. Find the equations of the tangent and normal to hyperbola $12x^2 - 9y^2 = 108$ at $\theta = \frac{\pi}{3}$. (Hint: use parametric form) **(EX 5.4 - 6)**
13. Find the vertex, focus, directrix, and length of the latus rectum of the parabola $x^2 - 4x - 5y - 1 = 0$. **Eg.5.17**
14. For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2. **Eg.5.21**
15. Find the centre, foci, and eccentricity of the hyperbola $11x^2 - 25y^2 - 44x + 50y - 256 = 0$. **Eg.5.24**

CHAPTER 6- APPLICATIONS OF VECTOR ALGEBRA

1. By vector method, prove that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$. **Eg.,6.3**
2. Prove by vector method that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$. **Eg.,6.5**
3. Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$. **EX. 6. 1 – 9**
4. Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$. **EX. 6. 1 – 10**
5. Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent. **Eg.,6.7**
6. If $\vec{a} = 2\vec{i} - \vec{j}$, $\vec{b} = \vec{i} - \vec{j} - 4\vec{k}$, $\vec{c} = 3\vec{j} - \vec{k}$ and $\vec{d} = 2\vec{i} + 5\vec{j} + \vec{k}$, verify that (i) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$ (ii) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{c}, \vec{d}]\vec{b} - [\vec{b}, \vec{c}, \vec{d}]\vec{a}$ **Eg.,623**
7. If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = 3\vec{i} + 5\vec{j} + 2\vec{k}$, $\vec{c} = -\vec{i} - 2\vec{j} + 3\vec{k}$, verify that (i) $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$ (ii) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$. **EX. 6. 3 – 4**
8. Show that the lines $\frac{x-3}{3} = \frac{y-3}{-1}, z - 1 = 0$ and $\frac{x-6}{2} = \frac{z-1}{3}, y - 2 = 0$ intersect. Also find the point of intersection. **EX. 6. 5 – 4**
9. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r} = (\vec{i} + 2\vec{j} - 4\vec{k}) + s(2\vec{i} + 3\vec{j} + 6\vec{k})$ and $\vec{r} = (\vec{i} - 3\vec{j} + 5\vec{k}) + t(\vec{i} + \vec{j} - \vec{k})$. **Eg. 6. 43**
10. Find the vector parametric, vector non-parametric and Cartesian formof the equation of the plane passing through the points $(-1,2,0), (2,2,-1)$ and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$. **(EG 6.44)**
11. Find the non-parametric form of vector equation, and cartesian equation of the plane passing through the point $(2,3,6)$ and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$ **EX. 6. 7 – 1**
12. Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points $(2,2,1), (9,3,6)$ and perpendicular to the plane $2x + 6y + 6z = 9$. **EX. 6. 7 – 2**
13. Find parametric form of vector equation and Cartesian equations of the plane passing through the points $(2, 2,1), (1,-2,3)$ and parallel to the straight line passing through the points $(2,1,-3)$ and $(-1,5,-8)$. **EX. 6. 7 – 3**
14. Find the non-parametric form of vector equation of the plane passing through the point $(1,-2, 4)$ and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$. **EX. 6. 7 – 4**
15. Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r} = (\vec{i} - \vec{j} + 3\vec{k}) + t(2\vec{i} - \vec{j} + 4\vec{k})$ and perpendicular to plane $\vec{r}.(\vec{i} + 2\vec{j} + \vec{k}) = 8$. **EX. 6. 7 – 5**
16. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points $(3,6,-2), (-1,-2,6)$ and $(6,-4,-2)$. **EX. 6. 7 – 6**

17. Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r} = (6\vec{i} - \vec{j} + \vec{k}) + s(-\vec{i} + 2\vec{j} + \vec{k}) + t(-5\vec{i} - 4\vec{j} - 5\vec{k})$ **EX. 6. 7 – 7**

CHAPTER 7 - DIFFERENTIALS AND PARTIAL DERIVATIVES

1. A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 $cubic\ m/min$, how fast is the depth of the water increases when the water is 8 metres deep? **(EX. 7.1 - 8)**
2. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high? **(EG 7.9)**
3. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall. (i) How fast is the top of the ladder moving down the wall? (ii) At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing? **(EX. 7.1 - 9)**
4. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car? **(EX. 7.1 - 10)**
5. A road running north to south crosses a road going east to west at the point P . Car A is driving north along the first road and car B is driving east along the second road. At a particular time car A is 10 kilometres to the north of P and travelling at 80 km/hr, while car B is 15 kilometres to the east of P and travelling at 100 km/hr. How fast is the distance between the two cars changing? **(EG 7.10)**

CHAPTER 8 - DIFFERENTIALS AND PARTIAL DERIVATIVES

1. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$. (EG 8.22) **June '23**
2. If $v(x,y) = \log\left(\frac{x^2+y^2}{x=y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$. **(EX 8.7 – 5)**
3. If $w(x,y,z) = \log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$, find $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$. **(EX 8.7 – 6)**
4. Using Euler's theorem $u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.**(creative)**

CHAPTER 10 - ORDINARY DIFFERENTIAL EQUATIONS

1. The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple? **(EG 10.27)**
2. The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours? **(EX. 10.8 - 1)**
3. Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000. **(EX. 10.8 - 2)**
4. Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years? **(EX 10.8 – 6)**
5. Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later? **(EX 10.8 – 5)**

CHAPTER 11 - PROBABILITY DISTRIBUTIONS LAPLACE

1. A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws. (i) Find the probability mass function. (ii) Find the cumulative distribution function. (iii) Find $P(3 \le X < 6)$ (iv) Find $P(X \ge 4)$. **(EG 11.8)**
2. A six sided die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find (i) the probability mass function (ii) the cumulative distribution function (iii) $P(4 \le X < 10)$ (iv) $P(X \ge 6)$ **(EX 11.2 – 2)**
3. A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find (i) $P(2 < X < 6)$ (ii) $P(2 \le X \le 5)$ (iii) $P(X \le 4)$ (iv) $P(3 < X)$ **(EG 11.10)**

4. A random variable X has the following probability mass function.

x	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2 \le X < 5)$ (iii) $P(3 < X)$ **(EX 11.2 – 6)**

5. Suppose that $f(x)$ given below represents a probability mass function,

x	1	2	3	4	5	6
$f(x)$	c^2	$2c^2$	$3c^2$	$4c^2$	c	$2c$

Find (i) the value of c (ii) Mean and variance. **(EG 11.16)**

6. If X is the random variable with probability density function $f(x)$ given by,
$$f(x) = \begin{cases} x-1, & 1 \le x < 2 \\ -x+3, & 2 \le x < 3 \\ 0, & Otherwise \end{cases}$$
find (i) the distribution function $F(x)$ (ii) $P(1.5 \le X \le 2.5)$ **(EG 11.12)**
7. If X is the random variable with probability density function $f(x)$ given by,
$$f(x) = \begin{cases} x+1, & -1 \le x < 0 \\ -x+1, & 0 \le x < 1 \\ 0, & Otherwise \end{cases}$$
then find (i) the distribution function $F(x)$ (ii) $P(-0.5 \le X \le 0.5)$ **(EX 11.3 – 5)**
8. The probability density function of random variable X is given by
$f(x) = \begin{cases} k, & 1 \le x \le 5 \\ 0, & Otherwise \end{cases}$. Find (i) Distribution function
(ii) $P(X < 3)$ (iii) $P(2 < X < 4)$ (iv) $P(3 \le X)$ **(EG 11.14)**
9. Let X be a random variable denoting the life time of an electrical equipment having probability density function $f(x) = \begin{cases} ke^{-2x}, & for\ x > 0 \\ 0, & for\ x \le 0 \end{cases}$
Find (i) the value of k (ii) Distribution function (iii) $P(X < 2)$
(iv) calculate the probability that X is at least for four unit of time (v) $P(X = 3)$. **(EG 11.15)**
10. The probability density function of X is given by $f(x) = \begin{cases} ke^{\frac{-x}{3}}, & for\ x > 0 \\ 0, & for\ x \le 0 \end{cases}$ Find
(i) the value of k (ii) the distribution function (iii) $P(X < 3)$ (iv) $P(5 \le X)$ (v) $P(X \le 4)$. **(EX 11.3 – 4)**
11. If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights
(i) exactly 10 will have a useful life of at least 600 hours;

(ii) at least 11 will have a useful life of at least 600 hours;
(iii) at least 2 will not have a useful life of at least 600 hours. **(EX 11.5 – 6)**

12. On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denote the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective. **(EG 11.22)**

CHAPTER 12 -DISCRETE MATHEMATICS

1. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5. **(Eg. 12.9)**
2. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation X_{11} on a subset $A = \{1,3,4,5,9\}$ of the set of remainders $\{0,1,2,3,4,5,6,7,8,9,10\}$. **(Eg. 12.10)**
3. Define an operation $*$ on $\mathbb{Q}$ as follows: $(a * b) = \frac{a+b}{2}, a, b \in \mathbb{Q}$. Examine the closure,commutative, and associative properties satisfied by $*$ on $\mathbb{Q}$. (ii) Define an operation$*$ on $\mathbb{Q}$ as follows: $(a * b) = \frac{a+b}{2}, a, b \in \mathbb{Q}$. Examine the existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$. **(Ex. 12.1-5)**
4. Define an operation$*$ on $\mathbb{Q}$ as follows: $(a * b) = \frac{ab}{3}, a, b \in \mathbb{Q}$. Examine the closure, commutative,, associative ,examine the existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$. **(similar creative)**
5. Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ be any three Boolean matrices. Find $(i)\ A \vee B$ $(ii)\ A \wedge B$ $(iii)\ (A \vee B) \wedge C$ $(iv)\ (A \wedge B) \vee C$. **(Ex. 12.1-8)**
6. Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in \mathbb{R} - \{0\} \right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine commutative and associative and examine the existence of identity, existence of inverse properties for the operation $*$ on M . **(Ex. 12.1-9)**
7. Let $M = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in \mathbb{R} - \{0\} \right\}$ and let $*$ be the matrix multiplication. Determine whether M closed under $*$. If so, examine commutative and associative ,examine the existence of identity, existence of inverse properties for the operation $*$ on M . **(similar creative)**
8. Let A be $\mathbb{Q}\backslash\{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is $*$ binary on A? If so, examine the commutative and associative properties satisfied by $*$ on A. (ii) Let A be $\mathbb{Q}\backslash\{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is $*$ binary on A? If so, examine the existence of identity, existence of inverse properties for the operation $*$ on A. **(Ex. 12.1-10)**
9. Let A be $\mathbb{Q}\backslash\{-1\}$. Define $*$ on A by $a * b = a + b + ab$. Is $*$ binary on A? If so, examine the Commutative, examine the existence of identity, existence of inverse properties for the operation $*$ on A. **(similar creative))**
10. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for following operation on the given set. $m * n = m + n - mn;\ m, n \in \mathbb{Z}$. **(EG 12.7**
11. Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table. **(Ex. 12.2-15)**

2 MARKS

CHAPTER 1

1. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}. **(EG 1.2)**
2. If A is a non-singular matrix of odd order, prove that $|adj\ A|$ is positive. **(EG 1.4)**
3. If A is symmetric, prove that then $adj\ A$ is also symmetric. **(EG 1.7)**
4. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal. **(EG 1.11)**
5. If $adj A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}. **(EG 1.6)**
6. If $adj\ (A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ find A^{-1}. **(EX 1.1 -9) June '23**
7. Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$. **(EG 1.8)**

CHAPTER 2

1. Simplify (i) $\sum_{n=1}^{102} i^n$ (ii) $\sum_{n=1}^{10} i^{n+50}$ (iii) $i^{59} + \frac{1}{i^{59}}$ **(EX 2.1 - 6)**
2. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$. **(EG 2.4)**
3. If $z_1 = 2 + 5i$ find the additive and multiplicative inverse of z_1. **(EX 2.3 - 3)**
4. Which one of the points $i, -2 + i$,and 3 is farthest from the origin? **(EG 2.11)**
5. Find the square root of $6 - 8i$. **(EG 2.17)**
6. Show that $|3z - 5 + i|$ represents a circle, and, find its centre and radius. **(EG 2.19)**
7. If $z_1 = 2 + 5i$ find the additive and multiplicative inverse of z_1. **(EX 2.3 - 3)**
8. If $\omega \neq 1$ is a cube root of unity, then show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1$. **(EX 2.8 - 1)**
9. If $\omega \neq 1$ is a cube root of unity, show that $(i)(1 - \omega + \omega^2)^6(1 + \omega - \omega^2)^6 = 128$ **(EX 2.8 - 8)**

CHAPTER 3

1. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients. **(EG 3.3)**
2. Construct a cubic equation with roots 1,1, and -2 **(EX 3.1 - 2)**
3. A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away. **(EX 3.1 - 12)**
4. Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}i$ as a root. **(EX 3.2 - 1)**
5. Find a polynomial equation of minimum degree with rational coefficients, having $2i + 3$ as a root. **(EX 3.2 - 3)**
6. Solve the equation $x^4 - 9x^2 + 20 = 0$. **(EG 3.16)**
7. Discuss the nature of the roots of polynomials: $(i) x^{2018} + 1947x^{1950} + 15x^8 + 26x^6 + 2019$
8. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$. **(EX 3.6 - 1)**
9. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions. **(EX 3.6 - 3)**

10. Find the exact number of real roots and imaginary of the equation $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$. **(EX 3.6 - 5)**

CHAPTER 4

1. Find the principal value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$. **(EG 4.3)**
2. Sketch the graph of $y = sin\left(\frac{1}{3}x\right)$ for $0 \leq x < 6\pi$. **(EX 4.1 - 3)**
3. State the reason for $cos^{-1}\left(\cos\left(\frac{-\pi}{6}\right)\right) \neq \frac{-\pi}{6}$ **(EX 4.2 - 2)**
4. Find the principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$(in radians and degrees). **(EG 4.1)**
5. Find the principal value of $\sin^{-1}(2)$, if it exists **(EG 4.2)**
6. Find (i) $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ **(EG 4.6)**
7. Find the period and amplitude of
   (i) $y = \sin 7x$ (ii) $y = -\sin\left(\frac{1}{3}x\right)$ $(iii) y = 4\sin(-2x)$. **(EX 4.1 - 2)**
8. For what value of x does $\sin x = \sin^{-1} x$? **(EX 4.1 - 5)**
9. Find the value of $sec^{-1}\left(\frac{-2\sqrt{3}}{3}\right)$. **(EG 4.13)**

CHAPTER 5

1. Find the general equation of a circle with centre $(-3,-4)$ and radius 3 units. **(EG 5.1)**
2. Find the general equation of the circle whose diameter is the line segment joining the points $(-4,-2)$ and $(1,1)$. **(EG 5.4)**
3. Examine the position of the point $(2,3)$ with respect to the circle $x^2 + y^2 - 6x - 8y + 12 = 0$. **(EG 5.5)**
4. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c. **(EG 5.12)**
5. If $y = 2\sqrt{2}x + c$ is a tangent to the circle $x^2 + y^2 = 16$, find the value of c. **(EX 5.1 - 8)**
6. Find the equation of the tangent to the circle $x^2 + y^2 - 6x + 6y - 8 = 0$ at $(2,2)$. **(EX 5.1 - 9)**

CHAPTER 6

1. If $\vec{a} = -3\vec{i} - \vec{j} + 5\vec{k}$, $\vec{b} = \vec{i} - 2\vec{j} + \vec{k}$, $\vec{c} = 4\vec{j} - 5\vec{k}$, find $\vec{a}.(\vec{b} \times \vec{c})$. **(EG 6.12)**
2. Find the volume of the parallelepiped whose coterminus edges are given by the vectors $2\vec{i} - 3\vec{j} + 4\vec{k}$, $\vec{i} + 2\vec{j} - \vec{k}$ and $3\vec{i} - \vec{j} + 2\vec{k}$. **(EG 6.13)**
3. Show that the vectors $\vec{i} + 2\vec{j} - 3\vec{k}$, $2\vec{i} - \vec{j} + 2\vec{k}$ and $3\vec{i} + \vec{j} - \vec{k}$ are coplanar. **(EG 6.14)**
4. Show that the points $(2,3,4)$, $(-1,4,5)$ –and $(8,1,2)$ are collinear. **(EX 6.4 – 9)**
5. For any vector $\vec{a}$, prove that $\vec{i} \times (\vec{a} \times \vec{i}) + \vec{j} \times (\vec{a} \times \vec{j}) + \vec{k} \times (\vec{a} \times \vec{k}) = 2\vec{a}$. **(EX 6.3 - 2)**
6. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar vectors, then show that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$ **(EX 6.3 - 6)**
7. Find the distance between the parallel planes $x + 2y - 2z + 1 = 0$ and $2x + 4y - 4z + 5 = 0$. **(EG 6.51)**

CHAPTER 7

1. The temperature in celsius in a long rod of length 10 m, insulated at both ends, is a function of length x given by $T = x(10 - x)$. Prove that the rate of change of temperature at the midpoint of the rod is zero. **(Eg. 7.2)**
2. A person learnt 100 words for an English test. The number of words the person remembers in t days after learning is given by $W(t) = 100 \times (1 - 0.1t^2), 0 \leq t \leq 10$. What is the rate at which the person forgets the words 2 days after learning? **(EG 7.3)**
3. If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when $x = 5$ units. **(EX. 7.1 - 4)**

4. If the mass $m(x)$ (in kilograms) of a thin rod of length x (in metres) is given by, $m(x) = \sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is $x = 3$ and $x = 27$ metres. **(EX 7.1 – 5)**
5. Evaluate the limit $\lim_{x\to 0}\left(\frac{\sin mx}{x}\right)$. **(Eg. 7.35)**
6. Evaluate: $\lim_{x\to\infty}\left(\frac{e^x}{x^m}\right), m \in \mathbb{N}$. **(Eg. 7.42)**

CHAPTER 8

1. Find df for $f(x) = x^2 + 3x$ and evaluate it for $(i)\ x = 2$ and $dx = 0.1$ $(ii)\ x = 3$ and $dx = 0.02$ **(EX 8.2 – 2)**
2. Show that $F(x,y) = \frac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1. **(EG 8.21)**
3. In each of the following cases, determine whether the following function is homogeneous or not. If it is so, find the degree.
   $(i)\ f(x,y) = x^2y + 6x^3 + 7$ $\quad(ii)\ h(x,y) = \frac{6x^2y^3-\pi y^5+9x^4y}{2020x^2+2019y^2}$
   $(iii)\ g(x,y,z) = \frac{\sqrt{3x^2+5y^2+z^2}}{4x+7y}$ $\quad(iv)\ U(x,y,z) = xy + \sin\left(\frac{y^2-2z^2}{xy}\right)$. **(EX 8.7 – 1)**
4. If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$. **(EX 8.7 – 4)**

CHAPTER 9

1. Evaluate: $\int_0^{\frac{\pi}{2}}(\sin^2 x + \cos^4 x)dx$ **(Eg. 9.37)**
2. Evaluate: $\int_0^{\frac{\pi}{2}}\begin{vmatrix}\cos^4 x & 7\\ \sin^5 x & 3\end{vmatrix}dx$. **(Eg. 9.38)**
3. Evaluate: $\int_0^1 x^3(1-x)^4dx$. **(EG 9.42)**
4. Evaluate: $\int_0^1 x^2(1-x)^3dx$ **(EX 9.6 – 1)**
5. Evaluate the following: $(i)\ \int_0^{\frac{\pi}{2}}\sin^{10} x\,dx$ $\quad(ii)\ \int_0^{\frac{\pi}{2}}\cos^7 x\,dx$ **(EX 9.6 – 1)**
6. Evaluate t $(i)\ \int_0^{\infty} x^5e^{-3x}dx$ **(EX 9.7 – 1)**

CHAPTER 10

1. Determine the order and degree (if exists) of the following differential equations:
   $(i)\ \frac{dy}{dx} = x + y + 5$ $\quad(ii)\ \left(\frac{d^4y}{dx^4}\right)^3 + 4\left(\frac{dy}{dx}\right)^7 + 6y = 5\cos 3x$ $\quad(iii)\ \frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2\log\left(\frac{d^2y}{dx^2}\right)$
   $(iv)\ 3\left(\frac{d^2y}{dx^2}\right) = \left(4 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}$ $\quad(v)\ dy + (xy - \cos x)dx = 0$ **(EG 10.1)**
2. Express each of the following physical statements in the form of differential equation.
   (i) Radium decays at a rate proportional to the amount Q present. **(EX 10.2 – 1(i))**
   (ii) The population P of a city increases at a rate proportional to the product of population and to the difference between 5,00,000 and the population. **(EX 10.2 – 1(ii))**
   (iii) For a certain substance, the rate of change of vapor pressure P with respect to temperature T is proportional to the vapor pressure and inversely proportional to the square of the temperature. **(EX 10.2 – 1(iii))**
   (iv) A saving amount pays 8% interest per year, compounded continuously. In addition, the income from another investment is credited to the amount continuously at the rate of ` 400 per year. **(EX 10.2 – 1(iv))**
3. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. Form a differential equation involving the rate of change of the radius of the rain drop. **(EX 10.2 – 2)**

CHAPTER 11

1. Two fair coins are tossed simultaneously (equivalent to a fair coin istossed twice). Find the probability mass function for number of heads occurred. **(EG 11.5)**
2. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred. **(Eg. 11.2 - 1) .**
3. Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred. **(EX 11.4 – 4)**

CHAPTER 12

1. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$. **(Eg. 12.8)**
2. Determine whether $*$ is a binary operation on the sets given below. (ii) $a * b = min\,(a, b)$ on $A = \{1,2,3,4,5\}$ (iii) $a * b = a\sqrt{b}$ is binary on $\mathbb{R}$. **(Ex. 12.1-1)**
3. On $\mathbb{Z}$, define $\otimes$ by $(m \otimes n) = m^n + n^m$: $m, n \in \mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$? **(Ex. 12.1-2)**
4. Let $A = \{a + \sqrt{5}b : a, b \in \mathbb{Z}\}$. Check whether the usual multiplication is a binary operation on A. **(Ex. 12.1-4)**
5. How many rows are needed for following statement formulae? (i) $p \vee \neg t \wedge (p \vee \neg s)$ $(ii)\big((p \wedge q) \vee (\neg r \vee \neg s)\big) \wedge (\neg t \wedge v)$ **(Eg. 12.12)**
6. Establish the equivalence property: $p \to q \equiv \neg p \vee q$ **(Eg. 12.17)**
7. Using the equivalence property, S.T. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$. **(Eg. 12.19)**
8. Construct the truth table for the following statements. (i) $\neg p \wedge \neg q$ (ii) $\neg(p \wedge \neg q)$ **(Ex. 12.2-6)**
9. Check whether the statement $p \to (q \to p)$ is a tautology or a contradiction without using the truth table. **(Ex. 12.2-12)**
10. Verify compound propositions are tautologies or contradictions or contingency $(p \wedge q) \wedge \neg(p \vee q)$**(Ex. 12.2-7)**

3 MARKS

CHAPTER 1

1. Verify $(AB)^{-1} = B^{-1}A^{-1}$with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$. **(EG 1.9)**
2. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(adj\ A) = (adj\ A)A = |A|I_2$. **(EX 1.1 -6)**
3. $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$. **(EX 1.1 -11)**
4. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form. **(EG 1.18)**

CHAPTER 2

1. Find the value of the real numbers $x$and y, if the complex number $(2 + i)x + (1 - i)y + 2i - 3$ and $x + (-1 + 2i)y + 1 + i$are equal. **(EG 2.2)**
2. If $|z| = 2$ show that $3 \leq |z + 3 + 4i| \leq 7$. **(EG 2.13)**
3. Show that the equation $z^3 + 2\bar{z} = 0$has five solutions. **(EX 2.5 - 9)**
4. Obtain the Cartesian form of the locus of z in each of the following cases. (i) $|z| = |z - i|$(ii) ($|2z - 3 - i| = 3$**(Eg 2.21)**
5. If $z = x + iy$is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$. Show that locus of z real axis**(EX 2.6 - 1)**

CHAPTER 3

1. If $p$and $q$are the roots of equation $lx^2 + nx + n = 0$,show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$. **(EX 3.1 - 9)**
2. If the equations $x^2 + px + q = 0$,and $x^2 + p'x + q' = 0$,have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$. **(EX 3.1 - 10)**
3. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$as a root. **(Eg.3.10)**
4. If $\alpha, \beta, \gamma$and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$,find a quadratic equation with integer coefficients whose roots $\alpha + \beta + \gamma + \delta$and $\alpha\beta\gamma\delta$. **(EX 3.1 - 8)**

CHAPTER 4

1. Find the value of $\sin^{-1}\left(sin\frac{5\pi}{9}cos\frac{\pi}{9} + cos\frac{5\pi}{9}sin\frac{\pi}{9}\right)$. **(EX 4.1 - 7)**
2. For what value of x , the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$holds? **(EX 4.2 - 7)**
3. Prove that $\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}, -1 < x < 1$. **(EG 4.11)**
4. Find the value of (iii) $cot^{-1}(1) + \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) - sec^{-1}(-\sqrt{2})$ **(EX 4.4 - 2)**

5.Find the value of $tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + sin^{-1}\left(\frac{-1}{2}\right)$**(EG 4.10)**

CHAPTER 5

1. Find the centre and radius of the circle $3x^2 + (a+1)y^2 + 6x - 9y + a + 4 = 0$. **(EG 5.9)**
2. If the equation $3x^2 + (3-p)xy + qy^2 - 2px = 8pq$ represents a circle, Find p and q . Also determine the centre and radius of the circle. **(EX 5.1 - 12)**

CHAPTER 6

1. With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. **(EG 6.4)**
2. Prove by vector method that an angle in a semi-circle is a right angle. **(EX 6.1 - 3)**
3. Prove by vector method that the area of the quadrilateral $ABCD$ having diagonals AC and BD is $\frac{1}{2}\left|\vec{AC} \times \vec{BD}\right|$. **(EX 6.1 - 6)**
4. Forces of magnitudes $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\vec{i} + 4\vec{j} + 5\vec{k}$ and $10\vec{i} + 6\vec{j} - 8\vec{k}$ respectively, act on a particle which is displaced from the point with position vector $4\vec{i} - 3\vec{j} - 2\vec{k}$ to the point with position vector $6\vec{i} + \vec{j} - 3\vec{k}$. Find the work done by the forces. **(EX 6.1 - 12)**
5. If the straight lines $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{\lambda}$ are coplanar, find λ and equations of the planes containing these two lines. **(EX 6.8 – 4)**

CHAPTER 7

1. If we blow air into a balloon of spherical shape at a rate of 1000 cm^3 per second. At what rate the radius of the balloon changes when the radius is $7cm$? Also compute the rate at which the surface area changes. **Eg. 7.7**
2. Find the acute angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection (0,0), (1,1). **(EG 7.15)**
3. Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally. **(EX. 7.2 - 10)**
4. Prove that among all the rectangles of the given area square has the least perimeter. **(Eg. 7.65)**
5. Find two positive numbers whose product is 20 and their sum is minimum. **Ex. 7.8 - 2**

CHAPTER 8

1. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$. **(EX. 8.2 – 4)**
2. The trunk of a tree has diameter 30 cm. During the following year, the circumference grew 6 cm. (i) Approximately, how much did the tree's diameter grow?
   (ii) What is the percentage increase in area of the tree's cross-section? **(EX. 8.2 – 5)**
3. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l. **(EX 8.1 – 6)**
4. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number. **(EX 8.1 – 7)**

CHAPTER 11

1. Two balls are chosen randomly from an urn containing 6 white and 4 black balls. Suppose that we win ` 30 for each black ball selected and we lose ` 20 for each white ball selected. If X denotes the winning amount, then find the values of X and number of points in its inverse images. **(EG 11.4)**
2. The cumulative distribution function of a discrete random variable is given by

$$F(x) = \begin{cases} 0, & -\infty < x < -1 \\ 0.15, & -1 \le x < 0 \\ 0.35, & 0 \le x < 1 \\ 0.60, & 1 \le x < 2 \\ 0.85, & 2 \le x < 3 \\ 1, & 3 \le x < \infty \end{cases}$$

. Find (i) the probability mass function (ii) $P(X < 1)$ and (iii) $P(X \ge 2)$. **(Eg. 11.9)**

13. If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X+3) = 10$ and $E(X+3)^2 = 116$, find μ and σ^2. **(EX 11.4 – 3)**

CHAPTER 12

1. Verify the (i) closure property, (ii) commutative property, (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation $+$ on $\mathbb{Z}_e =$ the set of all even integers. **(Eg. 12.4)**
2. Construct the truth table for $(p \,\bar{\vee}\, q) \wedge (p \,\bar{\vee}\, \neg q)$. **(Eg. 12.16)**
3. Show that (i) $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (ii) $\neg(p \to q) \equiv p \wedge \neg q$. **(Ex. 12.2-8)**
4. Prove that $q \to p \equiv \neg p \to \neg q$(**Ex. 12. 2 – 9**)

BOOK BACK ONE WORDS WITH ANSWER

CHAPTER 1

APPLICATIONS OF MATRICES AND DETERMINANTS

1. If $|adj\,(adj\ A)| = |A|^9$, then the order of the square matrix A is
(a) 3 **(b) 4** (c) 2 (d) 5

2. If A is a 3×3 non-singular matrix such that $AA^T = A^T$ and $B = A^{-1}A^T$, then $BB^T =$
(a) A (b) B **(c) I** (d) B^T

3. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = adj\ A$ and $C = 3A$, then $\frac{|adj\,B|}{|C|} =$
(a) $\frac{1}{3}$ **(b) $\frac{1}{9}$** (c) $\frac{1}{4}$ (d) 1

4. If $A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then $A =$
(a) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ **(c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$** (d) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

5. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I - A =$
(a) A^{-1} (b) $\frac{A^{-1}}{2}$ (c) $3A^{-1}$ **(d) $2A^{-1}$**

6. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adj\ (AB)| =$
(a) -40 **(b) -80** (c) -60 (d) -20

7. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is
(a) 15 (b) 12 (c) 14 **(d) 11**

8. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is
(a) 0 (b) -2 (c) -3 **(d) -1**

9. If A, B and C are invertible matrices of some order, then which one of the following is not true?
(a) $adj\ A = |A|A^{-1}B$ **(b) $adj(AB) = (adjA)(adjB)$**
(c) $det A^{-1} = (det A)^{-1}$ (d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

10. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$ then B^{-1}
(a) $\begin{bmatrix} 2 & -5 \\ 3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

11. If $A^T A^{-1}$ is symmetric, then $A^2 =$
(a) A^{-1} **(b) $(A^T)^2$** (c) A^T (d) $(A^{-1})^2$

12. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ then $(A^T)^{-1}$
(a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ **(d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$**

13. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{4}{5} \end{bmatrix}$ and $A^T = A^{-1}$ then the value of x is
(a) $\frac{-4}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

14. If $A = \begin{bmatrix} 1 & tan\frac{\theta}{2} \\ -tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I$, then $B =$

(a) $\left(\cos^2\frac{\theta}{2}\right)A$ **(b) $\left(\cos^2\frac{\theta}{2}\right)A^T$** (c) $(\cos^2\theta)I$ (d) $\left(\sin^2\frac{\theta}{2}\right)A$

15. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(adj\ A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$

(a) 0 (b) $\sin\theta$ (c) $\cos\theta$ **(d) 1**

16. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

(a) 17 (b) 14 **(c) 19** (d) 21

17. If $adjA = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $adjB = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$, then $adj(AB)$ is

(a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ **(b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$** (c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

18. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 4 (d) 3

19. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then the values of x and y are respectively,

(a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (b) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$ 3

(c) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ **(d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$**

20. Which of the following is/are correct?

(i) Adjoint of a symmetric matrix is also a symmetric matrix.

(ii) Adjoint of a diagonal matrix is also a diagonal matrix.

(iii) If $A$is a square matrix of order $n$and λ is a scalar, then $adj(\lambda A) = \lambda^n adj(A)$.

(iv) $A(adj\ A) = A(adj\ A) = |A|I$

(a) Only (i) (b) (ii) and (iii) (c) (iii) and (iv) **(d) (i), (ii) and (iv)**

21. If $\rho(A) = \rho([A|B])$, then the system $AX = B$ of linear equations is

(a) Consistent and has a unique solution **(b) consistent**

(c) Consistent and has infinitely many solution (d) inconsistent

22. If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0$, $(\cos\theta)x - y + z = 0$, $(\sin\theta)x - y + z = 0$ has a non-trivial solution then θ is

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ **(d) $\frac{\pi}{4}$**

23. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$. The system has infinitely many solutions if

(a) $\lambda = 7, \mu \ne -5$ (b) $\lambda = -7, \mu = 5$ (c) $\lambda \ne 7, \mu \ne -5$, **(d) $\lambda = 7, \mu = -5$**

24. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

(a) 2 (b) 4 (c) 3 **(d) 1**

25. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then $adj(adjA)$ is

(a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

CHAPTER 2

COMPLEX NUMBERS

1. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is
   (a) 0 (b) 1 (c) -1 (d) i
2. The value of $\sum_{i=1}^{13}(i^n + i^{n-1})$ is
   (a) $1 + i$ (b) i (c) 1 (d) 0
3. The area of the triangle formed by the complex numbers z, iz and $z + iz$ in the Argand's diagram is
   (a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2|z|^2$
4. The conjugate of a complex number is $\frac{1}{i-2}$. Then, the complex number is
   (a) $\frac{1}{i+2}$ **(b) $\frac{-1}{i+2}$** (c) $\frac{-1}{i-2}$ (d) $\frac{1}{i-2}$
5. If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to
   (a) 0 (b) 1 **(c) 2** (d) 3
6. If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is
   (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3
7. If $|z - 2 + i| \le 2$, then the greatest value of $|z|$ is
   (a) $\sqrt{3} - 2$ (b) $\sqrt{3} + 2$ (c) $\sqrt{5} - 2$ **(d) $\sqrt{5} + 2$**
8. If $\left|z - \frac{3}{z}\right| = 2$, then the least value of $|z|$ is
   (a) 1 (b) 2 (c) 3 (d) 5
9. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is
   (a) z (b) $\bar{z}$ (c) $\frac{1}{z}$ (d) 1
10. The solution of the equation $|z| - z = 1 + 2i$ is
    (a) $\frac{3}{2} - 2i$ (b) $\frac{-3}{2} + 2i$ (c) $2 - \frac{3}{2}i$ (d) $2 + \frac{3}{2}i$
11. If $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is
    (a) 1 **(b) 2** (c) 3 (d) 4
12. If z is a complex number such that $z \in C\backslash R$ and $z + \frac{1}{z} \in R$, then $|z|$ is
    (a) 0 **(b) 1** (c) 2 (d) 3
13. Let z_1, z_2 and z_3 be complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$, then ${z_1}^2 + {z_2}^2 + {z_3}^2$ is
    (a) 3 (b) 2 (c) 1 **(d) 0**
14. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is
    (a) $\frac{1}{2}$ **(b) 1** (c) 2 (d) 3
15. If $z = x + iy$ is a complex number such that $|z + 2| = |z - 2|$, then the locus of z is
    (a) real axis **(b) imaginary axis** (c) ellipse (d) circle
16. The principal argument of $\frac{3}{-1+i}$ is
    (a) $\frac{-5\pi}{6}$ (b) $\frac{-2\pi}{3}$ **(c) $\frac{-3\pi}{4}$** (d) $\frac{-\pi}{2}$
17. The principal argument of $(\sin 40° + i\cos 40°)$ is
    (1) $-110°$ (b) $-70°$ (c) $70°$ (d) $110°$
18. If $(1 + i)(1 + 2i)(1 + 3i) \ldots (1 + ni) = x + iy$, then $2.5.10 \ldots (1 + n^2)$ is
    (a) 1 (b) i **(c) $x^2 + y^2$** (d) $1 + n^2$
19. If $\omega \ne 1$ is a cubic root of unity and $(1 + \omega)^7 = A + B\omega$, then (A, B) equals

(*a*) (1,0) (*b*) (−1,1) (*c*) (0,1) **(*d*) (1, 1)**

20. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

(*a*) $\frac{2\pi}{3}$ (*b*) $\frac{\pi}{6}$ (*c*) $\frac{5\pi}{6}$ **(*d*) $\frac{\pi}{2}$**

21. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

(*a*) -2 **(*b*) -1** (*c*) 1 (*d*) 2

22. The product of all four values of $\left(\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

(*a*) -2 (*b*) -1 **(*c*) 1** (*d*) 2

23. If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to

(*a*) 1 (*b*) -1 (*c*) $\sqrt{3}i$ **(*d*) $-\sqrt{3}i$**

24. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(*a*) $cis\frac{2\pi}{3}$ (*b*) $cis\frac{4\pi}{3}$ (*c*) $-cis\frac{2\pi}{3}$ (*d*) $-cis\frac{4\pi}{3}$

25. If $\omega = cis\frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

CHAPTER 3
THEORY OF EQUATIONS

1. A zero of $x^3 + 64$ is

(*a*) 0 (*b*) 4 (*c*) $4i$ **(*d*) -4**

2. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f \circ g)(x)$, then the degree of h is

(*a*) mn (*b*) $m + n$ (*c*) m^n (*d*) n^m

3. A polynomial equation in x of degree n always has

(*a*) n distinct roots (*b*) n real roots **(*c*) n imaginary roots** (*d*) at most one root.

4. If α, β and γ are the roots of $x^3 + px^2 + qx + r = 0$, then $\sum\frac{1}{\alpha}$ is

(*a*) $\frac{-q}{r}$ (*b*) $\frac{-p}{r}$ (*c*) $\frac{q}{r}$ (*d*) $\frac{-q}{p}$

5. According to the rational root theorem, which number is not possible rational root of $4x^7 + 2x^4 - 10x^3 - 5$?

(*a*) -1 (*b*) 54 **(*c*) 45** (*d*) 5

6. The polynomial $x^3 - kx^2 + 9x$ has three real roots if and only if, k satisfies

(*a*) $|k| \leq 6$ (*b*) $k = 0$ (*c*) $|k| > 6$ **(*d*) $|k| \geq 6$**

7. The number of real numbers in $[0,2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

(*a*) 2 (*b*) 4 (*c*) 1 (*d*) ∞

8. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive root, if and only if

(*a*) $a \geq 0$ (*b*) $a > 0$ **(*c*) $a < 0$** (*d*) $a \leq 0$

9. The polynomial $x^3 + 2x + 3$ has

(*a*) one negative and two real roots (*b*) one positive and two imaginary roots

(*c*) three real roots (*d*) no solution

10. The number of positive roots of the polynomial $\sum_{j=0}^{n} {}^nC_r(-1)^r x^r$ is

(*a*) 0 **(*b*) n** (*c*) $< n$ (*d*) r

CHAPTER 4
INVERSE TRIGONOMETRIC FUNCTIONS

1. The value of $\sin^{-1}(\cos x), 0 \le x \le \pi$ is
(a) $\pi - x$ (2) $x - \frac{\pi}{2}$ **(3) $\frac{\pi}{2} - x$** (4) $x - \pi$

2. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$ then $\cos^{-1} x + \cos^{-1} y$ is equal to
(a) $\frac{2\pi}{3}$ **(b) $\frac{\pi}{3}$** (c) $\frac{\pi}{6}$ (d) π

3. $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} + \sec^{-1}\frac{5}{3} - \text{cosec}^{-1}\frac{13}{12}$ is equal to
(a) 2π (b) π **(c) 0** (d) $\tan^{-1}\frac{12}{65}$

4. If $\sin^{-1} x = 2\sin^{-1}\alpha$ has a solution, then
(a) $|\alpha| \le \frac{1}{\sqrt{2}}$ (b) $|\alpha| \ge \frac{1}{\sqrt{2}}$ (c) $|\alpha| < \frac{1}{\sqrt{2}}$ (d) $|\alpha| > \frac{1}{\sqrt{2}}$

5. $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for
(a) $-\pi \le x \le 0$ **(b) $0 \le x \le \pi$** (c) $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$ (d) $\frac{-\pi}{4} \le x \le \frac{3\pi}{4}$

6. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is
(a) 0 (b) 1 (c) 2 (d) 3

7. If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1} x$ is
(a) $\frac{-\pi}{10}$ (b) $\frac{\pi}{5}$ **(c) $\frac{\pi}{10}$** (d) $\frac{-\pi}{5}$

8. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is
(a) [1,2] (b) [−1,1] (c) [0,1] (d) [−1,0]

9. If $x = \frac{1}{5}$, the value of $\cos(\cos^{-1} x + 2\sin^{-1} x)$ is
(a) $-\sqrt{\frac{24}{25}}$ (b) $\sqrt{\frac{24}{25}}$ (c) $\frac{1}{5}$ **(d) $\frac{-1}{5}$**

10. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to
(a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ **(d) $\cos^{-1}\left(\frac{1}{2}\right)$**

11. If the function $f(x) = \sin^{-1}(x^2 - 3)$, then x belongs to
(a) [−1,1] (b) $[\sqrt{2}, 2]$
(c) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ (d) $[-2, -\sqrt{2}] \cap [\sqrt{2}, 2]$

12. If $\cot^{-1} 2$ and $\cot^{-1} 3$ are two angles of a triangle, then the third angle is
(a) $\frac{\pi}{4}$ **(b) $\frac{3\pi}{4}$** (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

13. $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$. Then x is a root of the equation
(a) $x^2 - x - 6 = 0$ **(b) $x^2 - x - 12 = 0$**
(c) $x^2 + x - 12 = 0$ (d) $x^2 + x - 6 = 0$

14. $\sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1 - 2\sin^2 x) =$
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

15. If $\cot^{-1}(\sqrt{\sin\alpha}) + \tan^{-1}(\sqrt{\sin\alpha})$, then $\cos 2u$ is equal to
(1) $\tan^2\alpha$ (b) 0 **(c) −1** (d) $\tan 2\alpha$

16. If $|x| \le 1$, then $2\tan^{-1} x - \sin^{-1}\frac{2x}{1+x^2}$ is equal to
(a) $\tan^{-1} x$ (b) $\sin^{-1} x$ **(c) 0** (d) π

17. The equation $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ xhas

(*a*) no solution (***b***) **unique solution**
(*c*) two solutions (*d*) infinite number of solutions

18. If $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to
(*a*) $\frac{1}{2}$ (***b***) $\mathbf{\frac{1}{\sqrt{5}}}$ (*c*) $\frac{2}{\sqrt{5}}$ (*d*) $\frac{\sqrt{3}}{2}$

19. If $\sin^{-1}\frac{x}{5} + \mathrm{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the value of x is
(*a*) 4 (*b*) 5 (*c*) 2 (***d***) **3**

20. $\sin(\tan^{-1} x), |x| < 1$ is equal to
(*a*) $\frac{x}{\sqrt{1-x^2}}$ (*b*) $\frac{1}{\sqrt{1-x^2}}$ (*c*) $\frac{1}{\sqrt{1+x^2}}$ (***d***) $\mathbf{\frac{x}{\sqrt{1+x^2}}}$

CHAPTER 5

TWO DIMENSIONAL ANALYTICAL GEOMETRY-II

1. The equation of the circle passing through (1,5) and (4,1) and touching y − axis is $x^2 + y^2 - 5x - 6y + 9 + \lambda(4x + 3y - 19) = 0$ where λ is equal to
(***a***) $\mathbf{0, \frac{-40}{9}}$ (*b*) 0 (*c*) $\frac{40}{9}$ (*d*) $\frac{-40}{9}$

2. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is
(*a*) $\frac{4}{3}$ (*b*) $\frac{4}{\sqrt{3}}$ (***c***) $\mathbf{\frac{2}{\sqrt{3}}}$ (*d*) $\frac{3}{2}$

3. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if
(*a*) $15 < m < 65$ (*b*) $35 < m < 85$
(*c*) $-85 < m < -35$ (***d***) $\mathbf{-35 < m < -15}$

4. The length of the diameter of the circle which touches the x −axis at the point (1,0) and passes through the point (2,3).
(*a*) $\frac{6}{5}$ (*b*) $\frac{5}{3}$ (***c***) $\mathbf{\frac{10}{3}}$ (*d*) $\frac{3}{5}$

5. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is
(*a*) 1 (*b*) 3 (***c***) $\mathbf{\sqrt{10}}$ (*d*) $\sqrt{11}$

6. The centre of the circle inscribed in a square formed by the lines $x^2 - 8 - 12 = 0$and $y^2 - 14y + 45 = 0$ is
(***a***) **(4, 7)** (*b*) (7,4) (*c*) (9,4) (*d*) (4,9)

7. The equation of the normal to the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ which is parallel to the line $2x + 4y = 3$ is
(**1**) $\mathbf{x + 2y = 3}$ (2) $x + 2y + 3 = 0$ (3)$2x + 4y + 3 = 0$ (4)$x - 2y + 3 = 0$

8. If $P(x, y)$ be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then $PF_1 + PF_2$ is
(*a*) 8 (*b*) 6 (***c***) **10** (*d*) 12

9. The radius of the circle passing through the point (6,2) two of whose diameter are $x + y = 6$ and $x + 2y = 4$ is
(*a*) 10 (***b***) $\mathbf{2\sqrt{5}}$ (*c*) 6 (*d*) 4

10. The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is
(*a*) $4(a^2 + b^2)$ (***b***) $\mathbf{2(a^2 + b^2)}$ (*c*) $a^2 + b^2$ (*d*) $\frac{1}{2}(a^2 + b^2)$

11. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x - 3)^2 + (y + 2)^2 = r^2$, then the value of r^2 is
(***a***) **2** (*b*) 3 (*c*) 1 (*d*) 4

12. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is

$(a)\ 3$ $(b)\ -1$ $(c)\ 1$ $(\boldsymbol{d})\ \mathbf{9}$

13. The ellipse $E_1: \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point $(0,4)$ circumscribes the rectangle R. The eccentricity of the ellipse is

$(a)\ \frac{\sqrt{2}}{2}$ $(b)\ \frac{\sqrt{3}}{2}$ $(\boldsymbol{c})\ \mathbf{\frac{1}{2}}$ $(d)\ \frac{3}{4}$

14. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the straight line $2x - y = 1$. One of the points of contact of tangents on the hyperbola is

$(a)\ \left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ $(b)\ \left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $(\boldsymbol{c})\ \left(\frac{\mathbf{9}}{\mathbf{2\sqrt{2}}}, \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right)$ $(d)\ (3\sqrt{3}, 2\sqrt{2})$

15. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ having centre at $(0,3)$ is

$(\boldsymbol{a})\ \boldsymbol{x^2 + y^2 - 6y - 7 = 0}$ $(b)\ x^2 + y^2 - 6y + 7 = 0$

$(c)\ x^2 + y^2 - 6y - 5 = 0$ $(d)\ x^2 + y^2 - 6y + 5 = 0$

16. Let C be the circle with centre at $(1,1)$ and radius $= 1$. If T is the circle centered at $(0, y)$ $0y$passing through the origin and touching the circle C externally, then the radius of T is equal to

$(a)\ \frac{\sqrt{3}}{\sqrt{2}}$ $(b)\ \frac{\sqrt{3}}{2}$ $(c)\ \frac{1}{2}$ $(\boldsymbol{d})\ \mathbf{\frac{1}{4}}$

17. Consider an ellipse whose centre is of the origin and its major axis is along x −axis. If itseccentrcity is 35 and the distance between its foci is 6, then the area of the quadrilateral inscribed in the ellipse with diagonals as major and minor axis of the ellipse is

$(a)\ 8$ $(b)\ 32$ $(c)\ 80$ $(\boldsymbol{d})\ \mathbf{40}$

18. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$is

$(\boldsymbol{a})\ \boldsymbol{2ab}$ $(b)\ ab$ $(c)\ \sqrt{ab}$ $(d)\ \frac{a}{b}$

19. An ellipse has OB as semi minor axes, F and F' its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

$(\boldsymbol{a})\ \mathbf{\frac{1}{\sqrt{2}}}$ $(b)\ \frac{1}{2}$ $(c)\ \frac{1}{4}$ $(d)\ \frac{1}{\sqrt{3}}$

20. The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is

$(a)\ \frac{\sqrt{3}}{2}$ $(\boldsymbol{b})\ \mathbf{\frac{1}{3}}$ $(c)\ \frac{1}{3\sqrt{2}}$ $(d)\ \frac{1}{\sqrt{3}}$

21. If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is

$(a)\ 2x + 1 = 0$ $(\boldsymbol{b})\ \boldsymbol{x = -1}$ $(c) 2x - 1 = 0$ $(d) x = 1$

22. The circle passing through $(1, -2)$ and touching the axis of x at $(3,0)$ passing through the point

$(a)\ (-5,2)$ $(b)\ (2,-5)$ $(\boldsymbol{c})\ \mathbf{(5,-2)}$ $(d)\ (-2,5)$

23. The locus of a point whose distance from $(-2,0)$ is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$ is

(a) a parabola (b) a hyperbola $(\boldsymbol{c})$ **an ellipse** (d) a circle

24. The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of $x^2 - (a+b)x - 4 = 0$ then the value of $(a+b)$ is

$(a)\ 2$ $(b)\ 4$ $(\boldsymbol{c})\ \mathbf{0}$ $(d)\ -2$

25. If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are $(11,2)$ the coordinates of the other end are

$(a)(-5,2)$ $(\boldsymbol{b})\mathbf{(2,-5)}$ $(c)(5,-2)$ $(d)(-2,5)$

CHAPTER 6

APPLICATIONS OF VECTOR ALGEBRA

1. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a},\vec{c},\vec{b}]$ is equal to
   (a) 2 $(b) -1$ (c) 1 **(d) 0**
2. If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then
   (a) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=1$ (b) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=-1$ **(c) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=0$** (d) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=2$
3. If $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$, then the value of $[\vec{a},\vec{b},\vec{c}]$ is
   (a) $|\vec{a}||\vec{b}||\vec{c}|$ (b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ (c) 1 $(d) -1$
4. If $\vec{a},\vec{b},\vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$, and is parallel to $\vec{c}$ then $\vec{a}\times(\vec{b}\times\vec{c})$ is equal to
   (a) $\vec{a}$ **(b) $\vec{b}$** (c) $\vec{c}$ (d) $\vec{0}$
5. If $[\vec{a},\vec{b},\vec{c}]=1$, then the value of $\frac{\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}}+\frac{\vec{b}.(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b}).\vec{c}}+\frac{\vec{c}.(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b}).\vec{a}}$ is
   (a) 1 $(b) -1$ (c) 2 (d) 3
6. The volume of the parallelepiped with its edges represented by the vectors $\vec{i}+\vec{j},\ \vec{i}+2\vec{j},\ \vec{i}+\vec{j}+\pi\vec{k}$ is
   (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ **(c) π** (d) $\frac{\pi}{4}$
7. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a},\vec{b},\vec{a}\times\vec{b}]=\frac{\pi}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is
   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$
8. If $\vec{a}=\vec{i}+\vec{j}+\vec{k},\ \vec{b}=\vec{i}+\vec{j},\ \vec{c}=\vec{i}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}\times\mu\vec{b}$, then the value of $\lambda+\mu$ is
   (a) 0 (b) 1 (c) 6 (d) 3
9. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a},\vec{b},\vec{c}]=3$, then $\{[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}]\}^2$ is equal to
   (a) 81 (b) 9 (c) 27 (d)18
10. If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is
    (a) $\frac{\pi}{2}$ **(b) $\frac{3\pi}{4}$** (c) $\frac{\pi}{4}$ (d) π
11. If the volume of the parallelepiped with $\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}$ as coterminous edges is 8 cubic units, then the volume of the parallelepiped with $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c}),(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})$ and $(\vec{c}\times\vec{a})(\vec{a}\times\vec{b})$ as coterminous edges is,
    (a) 8 cubic units (b) 512 cubic units **(c) 64 cubic units** (d) 24 cubic units
12. Consider the vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ such that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$ Let P_1 and P_2 be the planes determined by the pairs of vectors $\vec{a},\vec{b}$ and $\vec{c},\vec{d}$ respectively. Then the angle between P_1 and P_2 is
    (a) 0° (b) 45° (c) 60° (d) 90°
13. If $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$, where $\vec{a},\vec{b},\vec{c}$ are any three vectors such that $\vec{b}.\vec{c}\neq0$ and $\vec{a}.\vec{b}\neq0$, then $\vec{a}$ and $\vec{c}$ are
    (a) perpendicular **(b) parallel**
    (c) inclined at an angle $\frac{\pi}{3}$ (d) inclined at an angle $\frac{\pi}{6}$

14. If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} - 5\vec{k}$, $\vec{c} = 3\vec{i} + 5\vec{j} - \vec{k}$, then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is
(a) $-17\vec{i} + 21\vec{j} - 97\vec{k}$ (b) $17\vec{i} + 21\vec{j} - 123\vec{k}$
(c) $-17\vec{i} - 21\vec{j} + 97\vec{k}$ **(d) $-17\vec{i} - 21\vec{j} - 97\vec{k}$**

15. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ **(d) $\frac{\pi}{2}$**

16. If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$, then (α, β) is
(a) $(-5,5)$ **(b) $(-6,7)$** (c) $(5,-5)$ (d) $(6,-7)$

17. The angle between the line $\vec{r} = (\vec{i} + 2\vec{j} - 3\vec{k}) + t(2\vec{i} + \vec{j} - 2\vec{k})$ and the plane $\vec{r}.(\vec{i} + \vec{j}) + 4 = 0$ is
(a) $0°$ (b) $30°$ **(c) $45°$** (d) $90°$

18. The coordinates of the point where the line $\vec{r} = (6\vec{i} - \vec{j} + 3\vec{k}) + t(-\vec{i} + 4\vec{k})$ meets the plane $\vec{r}.(\vec{i} + \vec{j} - \vec{k}) = 3$ are
(a) $(2,1,0)$ (b) $(7,-1,-7)$ (c) $(1,2,-6)$ **(d) $(5,-1,1)$**

19. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is
(a) 0 **(b) 1** (c) 2 (d) 3

20. The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is
(a) $\frac{\sqrt{7}}{2\sqrt{2}}$ (b) $\frac{7}{2}$ (c) $\frac{\sqrt{7}}{2}$ (d) $\frac{7}{2\sqrt{2}}$

21. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then
(a) $c = \pm 3$ **(b) $c = \pm\sqrt{3}$** (c) $c > 0$ (d) $0 < c < 1$

22. The vector equation $\vec{r} = (\vec{i} - 2\vec{j} - \vec{k}) + t(6\vec{i} - \vec{k})$ represents a straight line passing through the points
(a) $(0,6,-1)$ and $(1,-2,-1)$ **(b) $(0,6,-1)$ and $(-1,-4,-2)$**
(c) $(1,-2,-1)$ and $(1,4,-2)$ (d) $(1,-2,-1)$ and $(0,-6,1)$

23. If the distance of the point $(1,1,1)$ from the origin is half of its distance from the plane $x + y + z + k = 0$, then the values of k are
(a) ± 3 (b) ± 6 (c) $-3,9$ **(d) $3,-9$**

24. If the planes $\vec{r}.(2\vec{i} - \lambda\vec{j} + \vec{k}) = 3$ and $\vec{r}.(4\vec{i} + \vec{j} - \mu\vec{k}) = 5$ are parallel, then the value of λ and μ are
(a) $\frac{1}{2}, -2$ (b) $\frac{-1}{2}, 2$ **(c) $\frac{-1}{2}, -2$** (d) $\frac{1}{2}, 2$

25. If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1, \lambda > 0$ is $\frac{1}{5}$, then the value of λ is
(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) 0 (d) 1

CHAPTER 7

APPLICATION OF DIFFERENTIAL CALCULUS

1. The volume of a sphere is increasing in volume at the rate of $3\pi\ cm^3/sec$. The rate of change of its radius when radius is $\frac{1}{2} cm$
(a) $3\ cm/s$ (b) $2\ cm/s$ (c) $1\ cm/s$ (d) $12 cm/s$

2. A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

(a) $\frac{3}{25}$ radians/sec (**b**) $\frac{4}{25}$ **radians/sec**
(c) $\frac{1}{5}$ radians/sec (d) $\frac{1}{3}$ radians/sec

3. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is
(a) $t = 0$ (**b**) $\boldsymbol{t = 1}$ (c) $t = 1$ (d) $t = 3$
4. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by
(a) 2 (**b**) **2.5** (c) 3 (d) 3.5
5. Find the point on the curve $6y = x^3 + 2$ at which y −coordinate changes 8 times as fast as x −coordinate is
(**a**) **(4, 11)** (b) $(4, -11)$ (c) $(-4, 11)$ (d) $(-4, -11)$
6. The abscissa of the point on the curve $f(x) = \sqrt{8 - 2x}$ at which the slope of the tangent is -0.25 ?
(a) -8 (**b**) $\boldsymbol{-4}$ (c) -2 (d) 0
7. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is
(a) $-4\sqrt{3}$ (b) -4 (**c**) $\boldsymbol{\frac{\sqrt{3}}{12}}$ (d) $4\sqrt{3}$
8. The tangent to the curve $y^2 + xy + 9 = 0$ is vertical when
(a) $y = 0$ (b) $y = \pm 3$ (c) $y = 1$ (**d**) $\boldsymbol{y = \pm 3}$
9. Angle between $y^2 = x$ and $x^2 = y$ at the origin is
(a) $\tan^{-1}\frac{3}{4}$ (b) $\tan^{-1}\frac{4}{3}$ (**c**) $\boldsymbol{\frac{\pi}{2}}$ (d) $\frac{\pi}{4}$
10. What is the value of the limit $\lim_{x \to 0}\left(\cot x - \frac{1}{x}\right)$ is
(**a**) **0** (b) 1 (c) 2 (d) ∞
11. The function $\sin^4 x + \cos^4 x$ is increasing in the interval
(a) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (b) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (**c**) $\boldsymbol{\left[\frac{\pi}{4}, \frac{\pi}{2}\right]}$ (d) $\left[0, \frac{\pi}{4}\right]$
12. The number given by the Rolle 's Theorem for the function $x^3 - 3x^2, x \in [0,3]$ is
(a) 1 (b) $\sqrt{2}$ (c) $\frac{3}{2}$ (**d**) **2**
13. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1,9]$ is
(a) 2 (b) 2.5 (**c**) **3** (d) 3.5
14. The minimum value of the function $|3 - x| + 9$ is
(a) 0 (b) 3 (c) 6 (**d**) **9**
15. The maximum slope of the tangent to the curve $y = e^x \sin x, x \in [0, 2\pi]$ is at
(a) $x = \frac{\pi}{4}$ (**b**) $\boldsymbol{x = \frac{\pi}{2}}$ (c) $x = \pi$ (d) $x = \frac{3\pi}{2}$
16. The maximum value of the function $x^2 e^{-2x}, x > 0$ is
(a) $\frac{1}{e}$ (b) $\frac{1}{2e}$ (**c**) $\boldsymbol{\frac{1}{e^2}}$ (d) $\frac{4}{e^4}$
17. One of the closest points on the curve $x^2 - y^2 = 4$ to the point (6,0) is
(a) (2,0) (b) $(\sqrt{5}, 1)$ (**c**) $\boldsymbol{(3, \sqrt{5})}$ (d) $(\sqrt{13}, -\sqrt{3})$
18. The maximum product of two positive numbers, when their sum of the squares is 200, is
(**a**) **100** (b) $25\sqrt{7}$ (c) 28 (d) $24\sqrt{14}$
19. The curve $y = ax^4 + bx^2$ with $ab > 0$
(a) has no horizontal tangent (b) is concave up
(c) is concave down (**d**) **has no points of inflection**
20. The point of inflection of the curve $y = (x - 1)^3$ is
(a) (0,0) (b) (0,1) (**c**) **(1, 0)** (d) (1,1)

CHAPTER 8

DIFFERENTIALS AND PARTIAL DERIVATIVES

1. A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is
(a) 0.2% **(b) 0.4%** (c) 0.04% (d) 0.08%

2. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?
(a) $\frac{1}{31}$ **(b) $\frac{1}{5}$** (c) 5 (d) 31

3. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to
(a) $e^{x^2+y^2}$ **(b) $2xu$** (c) x^2u (d) y^2u

4. If $v(x,y) = \log(e^x + e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to
(a) $e^x + e^y$ (b) $\frac{1}{e^x+e^y}$ (c) 2 **(d) 1**

5. If $(x,y) = x^y, x > 0,$, then $\frac{\partial w}{\partial x}$ is equal to
(a) $x^y \log x$ (b) $y \log x$ **(c) yx^{y-1}** (d) $x \log y$

6. If $f(x,y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to
(a) xye^{xy} **(b) $(1+xy)e^{xy}$** (c) $(1+y)e^{xy}$ (d) $(1+x)e^{xy}$

7. If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is
(a) 0.4 $cu.cm$ (b) 0.45 $cu.cm$ (c) 2 $cu.cm$ **(d) 4.8 $cu.cm$**

8. The change in the surface area $S = 6x^2$ of a cube when the edge length varies from x_0 to $x_0 + dx$ is
(a) $12x_0 + dx$ **(b) $12x_0dx$** (c) $6x_0dx$ (d) $6\,x_0 + dx$

9. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is
(a) $0.3xdx\ m^3$ (b) $0.3x\ m^3$ (c) $0.3\,x^2m^3$ **(d) $0.03\,x^3m^3$**

10. If $g(x,y) = 3x^2 - 5y + 2y^2$, $x(t) = e^t$ and $y(t) = \cos t$, then $\frac{dg}{dt}$ is equal to
(a) $6e^{2t} + 5\sin t - 4\cos t \sin t$ (b) $6e^{2t} - 5\sin t + 4\cos t \sin t$
(c) $3e^{2t} + 5\sin t + 4\cos t \sin t$ (d) $3e^{2t} - 5\sin t + 4\cos t \sin t$

11. If $f(x) = \frac{x}{x+1}$, then its differential is given by
(a) $\frac{-1}{(x+1)^2}dx$ **(b) $\frac{1}{(x+1)^2}dx$** (c) $\frac{1}{x+1}dx$ (d) $\frac{-1}{x+1}dx$

12. If $u(x,y) = x^2 + 3xy + y - 2019$, then $\left(\frac{\partial u}{\partial x}\right)_{(4,5)}$ is equal to
(a) -4 (b) -3 **(c) -7** (d) 13

13. Linear approximation for $g(x) = \cos x$, at $x = \frac{\pi}{2}$ is
(a) $x + \frac{\pi}{2}$ **(b) $-x + \frac{\pi}{2}$** (c) $x - \frac{\pi}{2}$ (d) $-x - \frac{\pi}{2}$

14. If $w(x,y,z) = x^2(y-z) + y^2(z-x) + z^2(x-y)$, then is
(a) $xy + yz + zx$ (b) $x(y+z)$ (c) $y(z+x)$ **(d) 0**

15. If $f(x,y,z) = xy + yz + zx$, then $f_x - f_z$ is equal to
(a) $z - x$ (b) $y - z$ (c) $x - z$ (d) $y - x$

CHAPTER 9

APPLICATIONS OF INTEGRATION

1. The value of $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$ is

(a) $\boldsymbol{\frac{\pi}{6}}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) π

2. The value of $\int_{-1}^{2} |x|\, dx$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\boldsymbol{\frac{5}{2}}$ (d) $\frac{7}{2}$

3. For any value of $n \in \mathbb{Z}$, $\int_0^{\pi} e^{\cos^2 x} \cos^3[(2n+1)x]\, dx$ is

(a) $\frac{\pi}{2}$ (b) π (c) $\mathbf{0}$ (d) 2

4. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x\, dx$ is

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) 0 (d) $\boldsymbol{\frac{2}{3}}$

5. The value of $\int_{-4}^{4} \left[\tan^{-1}\left(\frac{x^2}{x^4+1}\right) + \tan^{-1}\left(\frac{x^4+1}{x^2}\right)\right] dx$ is

(a) π (b) 2π (c) 3π (d) $\boldsymbol{4\pi}$

6. The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left[\frac{2x^7-3x^5+7x^3-x+1}{\cos^2 x}\right] dx$ is

(a) 4 (b) 3 (c) $\mathbf{2}$ (d) 0

7. If $f(x) = \int_0^x t\cos t\, dt$, then $\frac{df}{dx} =$

(a) $\cos x - x\sin x$ (b) $\sin x + x\cos x$ (c) $x\cos x$ (d) $x\sin x$

8. The area between $y^2 = 4x$ and its latus rectum is

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\boldsymbol{\frac{8}{3}}$ (d) $\frac{5}{3}$

9. The value of $\int_0^1 x(1-x)^{99} dx$ is

(a) $\frac{1}{11000}$ (b) $\frac{1}{10100}$ (c) $\boldsymbol{\frac{1}{10010}}$ (d) $\frac{1}{10001}$

10. The value of $\int_0^1 \frac{dx}{1+5^{\cos x}}$ is

(a) $\boldsymbol{\frac{\pi}{2}}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π

11. If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then n is

(a) 10 (b) 5 (c) 8 (d) $\mathbf{9}$

12. The value of $\int_0^{\frac{\pi}{6}} \cos^3 3x\, dx$ is

(a) $\frac{2}{3}$ (b) $\boldsymbol{\frac{2}{9}}$ (c) $\frac{1}{9}$ (d) $\frac{1}{3}$

13. The value of $\int_0^{\pi} \sin^4 x\, dx$ is

(a) $\frac{3\pi}{10}$ (b) $\boldsymbol{\frac{3\pi}{8}}$ (c) $\frac{3\pi}{4}$ (d) $\frac{3\pi}{2}$

14. The value of $\int_0^{\infty} e^{-3x} x^2 dx$ is

(a) $\frac{7}{27}$ (b) $\frac{5}{27}$ (c) $\frac{4}{27}$ (d) $\boldsymbol{\frac{2}{27}}$

15. If $\int_0^a \frac{1}{4+x^2} dx$ then a is

(a) 4 (b) 1 (c) 3 (d) $\mathbf{2}$

16. The volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about x −axis is
(a) πa^3 (b) $\frac{\pi a^3}{4}$ (c) $\frac{\pi a^3}{5}$ **(d) $\frac{\pi a^3}{6}$**

17. If $f(x) = \int_1^x \frac{e^{\sin u}}{u} du, x > 1$ and $\int_1^3 \frac{e^{\sin x^2}}{x} dx = \frac{1}{2}[f(a) - f(1)]$, then one of the possible value of a is
(a) 3 (b) 6 **(c) 9** (d) 5

18. The value of $\int_0^1 (\sin^{-1} x)^2 dx$ is
(a) $\frac{\pi^2}{4} - 1$ (b) $\frac{\pi^2}{4} + 2$ (c) $\frac{\pi^2}{4} + 1$ **(d) $\frac{\pi^2}{4} - 2$**

19. The value of $\int_0^a \left(\sqrt{a^2 - x^2}\right)^3 dx$ is
(a) $\frac{\pi a^3}{16}$ **(b) $\frac{3\pi a^4}{16}$** (c) $\frac{3\pi a^2}{8}$ (d) $\frac{3\pi a^4}{8}$

20. If $\int_0^x f(t)dt = x + \int_x^1 tf(t)dt$, then the value of $f(1)$ is
(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{3}{4}$

CHAPTER 10

ORDINARY DIFFERENTIAL EQUATIONS

1. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^{\frac{1}{4}} = 0$ are respectively
(a) 2, 3 (b) 3, 3 (c) 2, 6 (d) 2, 4

2. The differential equation representing the family of curves $y = A\cos(x + B)$, where A and B are parameters, is
(a) $\frac{d^2y}{dx^2} - y = 0$ **(b) $\frac{d^2y}{dx^2} + y = 0$** (c) $\frac{d^2y}{dx^2} = 0$ (d) $\frac{d^2x}{dy^2} = 0$

3. The order and degree of the differential equation $\sqrt{\sin x}\ (dx + dy) = \cos x\,(dx - dy)$ is
(a) 1, 2 (b) 2, 2 (c) 1, 1 **(d) 2, 1**

4. The order of the differential equation of all circles with centre at (h, k) and radius 'a' is
(a) 2 **(b) 3** (c) 4 (d) 1

5. The differential equation of the family of curves $y = Ae^x + Be^{-x}$, where A and B are arbitrary constants is
(a) $\frac{d^2y}{dx^2} + y = 0$ **(b) $\frac{d^2y}{dx^2} - y = 0$** (c) $\frac{dy}{dx} + y = 0$ (d) $\frac{dy}{dx} + y = 0$

6. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is
(a) $xy = k$ (b) $y = k\log x$ **(c) $y = kx$** (d) $\log y = kx$

7. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents
(a) straight lines (b) circles **(c) parabola** (d) ellipse

8. The solution of $\frac{dy}{dx} + p(x)y = 0$ is
(a) $y = ce^{\int pdx}$ **(b) $y = ce^{-\int pdx}$** (c) $x = ce^{-\int pdy}$ (d) $x = ce^{\int pdy}$

9. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{\lambda}$ is
(a) $\frac{x}{e^{\lambda}}$ **(b) $\frac{e^{\lambda}}{x}$** (c) λe^x (d) e^x

10. The integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is x, then $P(x)$
(a) x (b) $\frac{x^2}{2}$ **(c) $\frac{1}{x}$** (d) $\frac{1}{x^2}$

11. The degree of the differential equation $y(x) = 1 + \frac{dy}{dx} + \frac{1}{1.2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{1.2.3}\left(\frac{dy}{dx}\right)^3 + \ldots\ldots$ is

(*a*) 2 (*b*) 3 **(*c*) 1** (*d*) 4

12. If p and q are the order and degree of the differential equation $y\frac{dy}{dx}+x^3\frac{d^2y}{dx^2}+xy=\cos x$, when
(*a*) $p<q$ (*b*) $p=q$ **(*c*) $p>q$** (*d*) p exists, q does not exist

13. The solution of the differential equation $\frac{dy}{dx}+\frac{1}{\sqrt{1-x^2}}=0$ is
(*a*) $y+\sin^{-1}x=c$ (*b*) $x+\sin^{-1}y=0$
(*c*) $y^2+2\sin^{-1}x=C$ (*d*) $x^2+2\sin^{-1}y=0$

14. The solution of the differential equation $\frac{dy}{dx}=2xy$ is
(*a*) $y=Ce^{x^2}$ (*b*) $y=2x^2+C$ (*c*) $y=Ce^{-x^2}+C$ (*d*) $y=x^2+C$

15. The general solution of the differential equation $\log\left(\frac{dy}{dx}\right)=x+y$ is
(*a*) $e^x+e^y=C$ **(*b*) $e^x+e^{-y}=C$** (*c*) $e^{-x}+e^y=C$ (*d*) $e^{-x}+e^{-y}=C$

16. The solution of $\frac{dy}{dx}=2^{y-x}$ is
(*a*) $2^x+2^y=C$ (*b*) $2^x-2^y=C$ **(*c*) $\frac{1}{2^x}-\frac{1}{2^y}=C$** (*d*) $x+y=C$

17. The solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}+\frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is
(*a*) $x\phi\left(\frac{y}{x}\right)=k$ **(*b*) $\phi\left(\frac{y}{x}\right)=kx$** (*c*) $y\phi\left(\frac{y}{x}\right)=k$ (*d*) $\phi\left(\frac{y}{x}\right)=ky$

18. If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx}+Py=Q$, then P is
(*a*) $\log\sin x$ (*b*) $\cos x$ (*c*) $\tan x$ **(*d*) $\cot x$**

19. The number of arbitrary constants in the general solutions of order n and $n+1$ are respectively
(*a*) $n-1, n$ **(*b*) $n, n+1$** (*c*) $n+1, n+2$ (*d*) $n+1, n$

20. The number of arbitrary constants in the particular solution of a differential equation of third order is
(*a*) 3 (*b*) 2 (*c*) 1 **(*d*) 0**

21. **Integrating factor of the differential equation $\frac{dy}{dx}=\frac{x+y+1}{x+1}$ is**
(*a*) $\frac{1}{x+1}$ (*b*) $x+1$ (*c*) $\frac{1}{\sqrt{x+1}}$ (*d*) $\sqrt{x+1}$

22. The population P in any year t is such that the rate of increase in the population is proportional to the population. Then
(*a*) $P=Ce^{kt}$ (*b*) $P=Ce^{-kt}$ (*c*) $P=Ckt$ (*d*) $P=C$

23. P is the amount of certain substance left in after time t. If the rate of evaporation of the substance is proportional to the amount remaining, then
(*a*) $P=Ce^{kt}$ **(*b*) $P=Ce^{-kt}$** (*c*) $P=Ckt$ (*d*) $P=C$

24. If the solution of the differential equation $\frac{dy}{dx}=\frac{ax+3}{2y+f}$ represents a circle, then the value of a is
(*a*) 2 **(*b*) -2** (*c*) 1 (*d*) -1

25. The slope at any point of a curve $y=f(x)$ is given by $\frac{dy}{dx}=3x^2$ and it passes through $(-1,1)$. Then the equation of the curve is
(*a*) $y=x^3+2$ (*b*) $y=3x^2+4$ (*c*) $y=3x^3+4$ (*d*) $y=x^3+5$

CHAPTER 11

PROBABILITY DISTRIBUTIONS LAPLACE

1. Let X be random variable with probability density function $f(x) = \begin{cases} \frac{2}{x^3}, & x \geq 1 \\ 0, & x < 1 \end{cases}$. Which of the following statement is correct
(a) both mean and variance exist
(b) mean exists but variance does not exist
(c) both mean and variance do not exist
(d) variance exists but Mean does not exist.

2. A rod of length $2l$ is broken into two pieces at random. The probability density function of the shorter of the two pieces is $f(x) = \begin{cases} \frac{1}{l}, & 0 < x < l \\ 0, & l \leq x \leq 2l \end{cases}$.
The mean and variance of the shorter of the two pieces are respectively
(a) $\frac{l}{2}, \frac{l^2}{3}$ (b) $\frac{l}{2}, \frac{l^2}{6}$ (c) $l, \frac{l^2}{12}$ **(d) $\frac{l}{2}, \frac{l^2}{12}$**

3. Consider a game where the player tosses a six-sided fair die. If the face that comes up is 6, the player wins ` 36, otherwise he loses ` $k2$, where k is the face that comes up $k = \{1, 2, 3, 4, 5\}$. The expected amount to win at this game in ` is
(a) $\frac{19}{6}$ **(b) $\frac{-19}{6}$** (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

4. A pair of dice numbered 1, 2, 3, 4, 5, 6 of a six-sided die and 1, 2, 3, 4 of a four-sided die is rolled and the sum is determined. Let the random variable X denote this sum. Then the number of elements in the inverse image of 7 is
(a) 1 (b) 2 (c) 3 **(d) 4**

5. A random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is
(a) 6 (b) 4 (c) 3 **(d) 2**

6. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible values of X are
(a) $i + 2n, i = 0,1,2\ldots n$
(b) $2i - n, i = 0, 1, 2\ldots n$
(c) $n - i, i = 0,1,2\ldots n$
(d) $2i + 2n, i = 0,1,2\ldots n$

7. If the function $f(x) = \frac{1}{12}$ $for\ a < x < b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?
(a) 0 and 12 (b) 5 and 17 (c) 7 and 19 **(d) 16 and 24**

8. Four buses carrying 160 students from the same school arrive at a football stadium. The buses carry, respectively, 42, 36, 34, and 48 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on that bus. Then $E[X]$ and $E[Y]$ respectively are
(a) 50, 40 (b) 40,50 **(c) 40.75, 40** (d) 41,41

9. Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with Probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result. The value of $E[X]$ is
(a) 0.11 (b) 1.1 **(c) 11** (d) 1

10. On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is
(a) $\frac{11}{243}$ (b) $\frac{3}{8}$ (c) $\frac{1}{243}$ (d) $\frac{5}{243}$

11. If $P(X = 0) = 1 - P(X = 1)$. If $E[X] = 3Var(X)$, then $P(X = 0)$.
(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ **(d) $\frac{1}{3}$**

12. If X is a binomial random variable with expected value 6 and variance 2.4, Then $P(X = 5)$ is
(a) $\binom{10}{5}\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^4$ (b) $\binom{10}{5}\left(\frac{3}{5}\right)^{10}$ (c) $\binom{10}{5}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^6$ **(d) $\binom{10}{5}\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^5$**

13. The random variable X has the probability density function $f(x) = \begin{cases} ax+b, 0<x<1 \\ 0, \quad Otherwise \end{cases}$ and $E(X) = \frac{7}{12}$, then a and b are respectively
(a) 1 and $\frac{1}{2}$ (b) $\frac{1}{2}$ and 1 (c) 2 and 1 (d) 1 and 2

14. Suppose that X takes on one of the values 0, 1, and 2. If for some constant k, $P(X=i) = kP(X=i-1)$ for $i = 1,2$ and $P(X=0) = \frac{1}{7}$. Then the value of k is
(a) 1 **(b) 2** (c) 3 (d) 4

15. Which of the following is a discrete random variable?
I. The number of cars crossing a particular signal in a day.
II. The number of customers in a queue to buy train tickets at a moment.
III. The time taken to complete a telephone call.
(a) I and II (b) II only (c) III only (d) II and III

16. If $f(x) = \begin{cases} 2x, 0 \le x \le a \\ 0, \quad Otherwise \end{cases}$ is a probability density function of a random variable, then the value of a is
(a) 1 (b) 2 (c) 3 (d) 4

17. The probability function of a random variable is defined as:

x	-2	-1	0	1	2
$f(x)$	k	$2k$	$3k$	$4k$	$5k$

Then $E(X)$ is equal to:
(a) $\frac{1}{15}$ (b) $\frac{1}{10}$ (c) $\frac{1}{3}$ **(d) $\frac{2}{3}$**

18. Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X-3)$ is
(a) 0.24 (b) 0.48 (c) 0.6 **(d) 0.96**

19. If in 6 trials, X is a binomial variate which follows the relation $9P(X=4) = P(X=2)$, then the probability of success is
(a) 0.125 **(b) 0.25** (c) 0.375 (d) 0.75

20. A computer salesperson knows from h is past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers?
(a) $\frac{57}{20^3}$ (b) $\frac{57}{20^2}$ (c) $\frac{19^3}{20^3}$ (d) $\frac{57}{20}$

CHAPTER 12

DISCRETE MATHEMATICS

1. A binary operation on a set S is a function from
(a) $S \to S$ **(b) $(S \times S) \to S$** (c) $S \to (S \times S)$ (d) $(S \times S) \to (S \times S)$

2. Subtraction is not a binary operation in
(a) $\mathbb{R}$ (b) $\mathbb{Z}$ **(c) $\mathbb{N}$** (d) $\mathbb{Q}$

3. Which one of the following is a binary operation on $\mathbb{N}$?
(a) Subtraction **(b) Multiplication** (c) Division (d) All the above

4. In the set $\mathbb{R}$ of real numbers '$*$' is defined as follows. Which one of the following is not a binary operation on $\mathbb{R}$?
(a) $a * b = min(a.b)$ (b) $a * b = max(a, b)$
(c) $a * b = a$ **(d) $a * b = a^b$**

5. The operation $*$ defined by $a * b = \frac{ab}{7}$ is not a binary operation on

(*a*) $\mathbb{Q}^+$ (**b**) $\mathbb{Z}$ (*c*) $\mathbb{R}$ (*d*) $\mathbb{C}$

6. In the set $\mathbb{Q}$ define $a \odot b = a + b + ab$. For what value of y, $3 \odot (y \odot 5) = 7$?
(*a*) $y = \frac{2}{3}$ (**b**) $\boldsymbol{y = \frac{-2}{3}}$ (*c*) $y = \frac{-3}{2}$ (*d*) $y = 4$

7. If $a * b = a^2 + b^2$ on the real numbers then $*$ is
(*a*) commutative but not associative (*b*) associative but not commutative
(**c**) **both commutative and associative** (*d*) neither commutative nor associative

8. Which one of the following statements has the truth value T ?
(*a*) $\sin x$ is an even function. (*b*) Every square matrix is non-singular
(*c*) The product of complex number and its conjugate is purely imaginary
(**d**) **5 is an irrational number**

9. Which one of the following statements has truth value F ?
(*a*) Chennai is in India or 2 is an integer
(*b*) Chennai is in India or 2 is an irrational number
(**c**) **Chennai is in China or 2 is an integer**
(*d*) Chennai is in China or 2 is an irrational number

10. If a compound statement involves 3 simple statements, then the number of rows in the truth table is
(*a*) 9 (**b**) **8** (*c*) 6 (*d*) 3

11. Which one is the inverse of the statement $(p \vee q) \rightarrow (p \wedge q)$?
(*a*) $(p \wedge q) \rightarrow (p \vee q)$ (*b*) $\neg(p \vee q) \rightarrow (p \wedge q)$
(*c*) $(\neg p \vee \neg q) \rightarrow (\neg p \wedge \neg q)$ (**d**) $\boldsymbol{(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)}$

12. Which one is the contra positive of the statement $(p \vee q) \rightarrow r$?
(**a**) $\boldsymbol{\neg r \rightarrow (\neg p \wedge \neg q)}$ (*b*) $\neg r \rightarrow (p \vee q)$ (*c*) $r \rightarrow (p \wedge q)$ (*d*) $p \rightarrow (q \vee r)$

13. The truth table for $(p \wedge q) \vee \neg q$ is given below

p	q	$(p \vee q) \vee (\neg q)$
T	T	(1)
T	F	(2)
F	T	(3)
F	F	(4)

Which one of the following is true?

	(1)	(2)	(3)	(4)
(*a*)	T	T	T	T
(*b*)	T	F	T	T
(**c**)	$\boldsymbol{T}$	$\boldsymbol{T}$	$\boldsymbol{F}$	$\boldsymbol{T}$
(*d*)	T	F	F	F

14. In the last column of the truth table for $\neg(p \vee \neg q)$ the number of final outcomes of the truth value $'F'$ are
(*a*) 1 (*b*) 2 (**c**) **3** (*d*) 4

15. Which one of the following is incorrect? For any two propositions p and q, we have
(*a*) $\neg(p \vee q) \equiv \neg p \wedge \neg q$ (*b*) $\neg(p \wedge q) \equiv \neg p \vee \neg q$
(**c**) $\boldsymbol{\neg(p \vee q) \equiv \neg p \vee \neg q}$ (*d*) $\neg(\neg p) \equiv p$

16.

p	q	$(p \wedge q) \rightarrow (\neg P)$
T	T	(1)
T	F	(2)
F	T	(3)
F	F	(4)

Which one of the following is correct for the truth value of $(p \wedge q) \rightarrow (\neg P)$

	(1)	(2)	(3)	(4)

(a)	T	T	T	T
$(\boldsymbol{b})$	$\boldsymbol{F}$	$\boldsymbol{T}$	$\boldsymbol{T}$	$\boldsymbol{T}$
(c)	F	F	T	T
(d)	T	T	T	F

17. The dual of $\neg(p \vee q) \vee [\, p \vee (\, p \wedge \neg r)]$ is
    (a) $\neg(p \wedge q) \wedge [\, p \vee (\, p \wedge \neg r)]$
    (b) $(p \vee q) \wedge [\, p \wedge (\, p \vee \neg r)]$
    (c) $\neg(p \wedge q) \wedge [\, p \wedge (\, p \wedge r)]$
    $(\boldsymbol{d})$ $\boldsymbol{\neg\, (p \wedge q) \wedge [p \wedge (\, p \vee \neg r)]}$
18. The proposition $p \wedge (\neg p \vee q)$ is
    (a) a tautology
    (b) a contradiction
    $(\boldsymbol{c})$ logically equivalent to $\boldsymbol{p \wedge q}$
    (d) logically equivalent to $p \vee q$
19. Determine the truth value of each of the following statements:
    $(\boldsymbol{a})$ $\boldsymbol{4 + 2 = 5}$ and $\boldsymbol{6 + 3 = 9}$
    (b) $3 + 2 = 5$ and $6 + 1 = 7$
    (c) $4 + 5 = 9$ and $1 + 2 = 4$
    (d) $3 + 2 = 5$ and $4 + 7 = 11$

	(1)	(2)	(3)	(4)
(a)	F	T	F	T
(b)	T	F	T	F
(c)	T	T	F	F
(d)	F	F	T	T

20. Which one of the following is not true?
    (a) Negation of a negation of a statement is the statement itself.
    (b) If the last column of the truth table contains only T then it is a tautology.
    (c) If the last column of its truth table contains only F then it is a contradiction
    $(\boldsymbol{d})$ If $\boldsymbol{p}$ and $\boldsymbol{q}$ are any two statements then $\boldsymbol{p \leftrightarrow q}$ is a tautology.