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MATHEMATICS

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AND

PTA

QUESTION PAPERS

- UPTO-2024

Important 2M,3M,5Marks Question Bank

&

Minimum Material

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CHAPTER 1
APPLICATIONS OF MATRICES AND DETERMINANTS

2 MARKS

Example 1.4 If A is a non-singular matrix of odd order, prove that $|adj\ A|$ is positive. ***(PTA-4)***

Example 1.11 Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal ***(MAR-23) (PTA-1)***

EXERCISE 1.1

9. If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ find A^{-1}. ***(JUN-24) (JUN-23) (PTA-6)***

EXERCISE 1.2

1.Find the rank of the following matrices by minor method:

(ii) $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$ ***(MAR-24)*** (iii) $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$ ***(PTA-5)***

iv) $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$ ***(JUL-22)***

CQ: To find the number of coins, in each category, write the suitable system of equations forgiven situations: "a bag contains 3types of coins namely Rs.1,Rs.2 and Rs.5.There are 30coins amounting to Rs 100 in total." ***(MAR-19)***

CQ: If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$ find $adj(AB)$. ***(PTA-3)***

3 MARKS

Example 1.3 If $A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$ then find $|adj(adjA)|$ (CQ) ***(MAR-24)***

Example 1.8 Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$ ***(MAR-20)***

Example 1.9 Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$ ***(SEP-20) (JUL-22)***

EXERCISE 1.1

3. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$. ***(MAR-23)***

6. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(adj\ A) = (adj\ A)A = |A|I_2$ ***(SEP-20) (AUG-21)***

EXERCISE 1.2

1. Find the rank of the following matrices by minor method:

(iv) $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0, \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0, \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0,$ ***(MAR-24)***

EXERCISE 1.5

2. If ax^2+bx+c is divided by $x+3, x-5$, and $x-1$, the remainders are 21,61 and 9 respectively. Find a, b and c (Use Gaussian elimination method.) ***(PTA-3)***

4. A boy is walking along the path $y=ax^2+bx+c$ through the points$(-6,8)(-2,-12)$, and $(3,8)$. He wants to meet his friend at $(7,60)$. Will he meet his friend? (Use Gaussian elimination method.) ***(MAR-23)***

Example 1.34 Investigate for what values of λ and μ the system of linear equations

$x+2y+z=7, x+y+\lambda z=\mu, x+3y-5z=5$

has (i) no solution (ii) a unique solution (iii) an infinite number of solutions. ***(PTA-2)***

EXERCISE 1.6

1. Test for consistency and if possible, solve the following systems of equations by rank method.

(iii) $2x+2y+z=5, x-y+z=1, 3x+y+2z=4$ ***(SEP-20)***

(iv) $2x-y+z=2, 6x-3y+3z=6, 4x-2y+2z=4.$ ***(PTA-5)***

EXERCISE 1.7

2. Determine the values of λ for which the following system of equations

$x+y+3z=0, 4x+3y+\lambda z=0, 2x+y+2z=0$ has (i) a unique solution (ii) a non-trivial solution. ***(MAR-19) (PTA-4)***

CQ: Why Cramer's rule is not applicable to solve the system $3x+y+z=2,\ x-3y+2z=1,\ 7x-y+4z=5$ ***(MAY-22)***

CQ: . Solve the system by Cramer's rule

$x-y+2z=2,\ 2x+y+4z=7,\ 4x-y+z=4$ ***(AUG-21)***

CQ: Examine the consistency of the system of equations

$4x+3y+6z=25, x+5y+7z=13, 2x+9y+z=1.$ If it is consistent then solve. ***(PTA-1)***

CHAPTER2

COMPLEX NUMBERS

2 MARKS

EXERCISE 2.1

Simplify the following: 3. $\sum_{n=1}^{12} i^n$ ***(MAR-24)***

Example 2.4 Simplify$\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3$. into rectangular form ***(MAR-20)***

Example 2.7 Find z^{-1}, if $z=(2+3i)(1-i)$ ***(AUG-21) (JUNE-24) (PTA-2)***

6. If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$ show that $x^2 + y^2 + 3x - 3y + 2 = 0$.*(JUNE-23) (PTA-1)*

EXERCISE 2.8

4. If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$ show that

(i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$ (ii) $xy - \frac{1}{xy} = 2i\sin(\alpha + \beta)$

(iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$ (iv) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$ *(MAR-24)*

* **PROPERTY**: State and prove triangle inequality *(JUNE-23)*

***Prove that** $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$ *(SEP-20)*

CQ: solve $z^4 = 1 - \sqrt{3}i$.*(PTA-2)*

CHAPTER3

THEORY OF EQUATIONS

2 MARKS

Example 3.2 If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2 . *(MAR-24)*

EXERCISE 3.1

8. If α, β, γ and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$, find a quadratic Equation with integer coefficients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$. *(SEP-20)*

9. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$. *(MAR-23)*

Example 3.9 Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}$ as a root *(MAY-22)*

Example 3.11 Show that the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x. *(JUNE-24)*

CQ: If α and β are the roots of $x^2 - 5x + 6 = 0$ then prove that $\alpha^2 - \beta^2 = \pm 5$.*(AUG-21)*

CQ: If α and β are the roots of $x^2 + 5x + 6 = 0$ then prove that $\alpha^2 + \beta^2 = 13$. *(JULY-22)*

3 MARKS

Example 3.2 If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2 . ***(PTA-4)***

Example 3.7 If p is real, discuss the nature of the roots of the equation
$4x^2 + 4px + p + 2 = 0$,in terms of p . ***(MAR-20)***

Example 3.18 Solve the equation $2x^3 + 11x^2 - 9x - 18 = 0$. ***(PTA-5)***

Example 3.19 Obtain the condition that the roots of $x^3 + px^2 + qx + r = 0$ are in A.P. ***(SEP-20)***
Example 3.25 Solve the equation $x^3 - 5x^2 - 4x + 20 = 0$. ***(PTA-1)***

EXERCISE 3.1

1.If the sides of a cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid. ***(AUG-21)***

EXERCISE 3.2

3.Find a polynomial equation of minimum degree with rational coefficients, having $2i + 3$ as a root. ***(JUNE-24)***

Example 3.30 Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary roots. ***(JUNE-23)***

EXERCISE 3.3

6. Solve the cubic equations: (i)$2x^3 - 9x^2 + 10x = 3$. ***(MAY-22)***

EXERCISE 3.5

5. Solve the equations (i) $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$. ***(PTA-1)***

EXERCISE 3.6

3. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions. ***(PTA-2)***

CQ: If $a+b+c=0$ and a, b, c are rational numbers then prove that the roots of the equation $(b+c-a)x^2+(c+a-b)x+(a+b-c)=0$ are rational numbers. ***(MAR-23)***

CQ: Prove that the roots of the equation $x^4-3x^2-4=0$ are $\pm 2, \pm i$. ***(JULY-22)***

5 MARKS

EXERCISE 3.1

5. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$. ***(PTA-2)***

EXERCISE 3.2

4. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root. ***(PTA-5)***

Example 3.18 Solve the equation $2x^3 + 11x^2 - 9x - 18 = 0$. ***(JUNE-23)***

Example 3.28 Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$. ***(PTA-3)***

EXERCISE 3.5

7. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution. ***(MAR-23) (PTA-6)***

CQ: Solve $x^{11} - x^6 + x^5 - 1 = 0$ ***(MAR-19)***

CHAPTER 4

INVERSE TRIGONOMETRIC FUNCTIONS

2 MARKS

EXERCISE 4.1

4. Find the value of (ii) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$ ***(MAR-20) (PTA-3)***

5. For what value of x does $\sin x = \sin^{-1} x$? ***(AUG-21)***

EXERCISE 4.2

2. State the reason for $\cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right) \neq -\frac{\pi}{6}$. ***(PTA-5)***

Example 4.8 Find the principal value of $\tan^{-1}(\sqrt{3})$ ***(MAY-22) (SEP-20)***

EXERCISE 4.3

2. Find the value of (i) $\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$ ***(JUNE-24)*** (ii) $\tan^{-1}\left(\tan\left(-\frac{\pi}{6}\right)\right)$ ***(PTA-1)***

EXERCISE 4.4

1. Find the principal value of (i) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (iii) $\operatorname{cosec}^{-1}(-\sqrt{2})$ ***(PTA-2)***

CQ: Find the value of $\sin^{-1}(1) + \cos^{-1}(1)$ ***(JULY-22)***

CQ: Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$. ***(PTA-4)***

CQ: Find the value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$. ***(PTA-6)***

3 MARKS

EXERCISE 4.1

7. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$ ***(JUNE-23)***

EXERCISE 4.2

7. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds? ***(MAR-23)***

Example 4.15 Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x, |x| > 1$ ***(MAR-24)***

5 MARKS

EXERCISE 4.1

7. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$ ***(JULY-22)***

EXERCISE 4.2

8. Find the value of (ii) $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right) + \cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$ ***(AUG-21)***

Example 4.20 Evaluate $\sin\left[\sin^{-1}\left(\frac{3}{5}\right) + \sec^{-1}\left(\frac{5}{4}\right)\right]$. ***(PTA-2)***

Example 4.22 If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$ ***(MAR-23) (PTA-1)***

Example 4.27: Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ if $6x^2 < 1$. ***(PTA-6)***

Example 4.28 Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ ***(JUNE-23) (MAR-24)***

EXERCISE 4.4

2. Find the value of (iii) $\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$ ***(MAY-22)***

EXERCISE 4.5

6. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$. ***(JUNE-24)***

*Draw the graph of $\tan x$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan^{-1} x$ in $(-\infty, \infty)$ ***(SEP-20)***

*Draw the graph of $\cos x$ in $[0, \pi]$ and $\cos^{-1} x$ in $[-1, 1]$ ***(MAR-20)***

* Draw the graph of $\sin x$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin^{-1} x$ in $[-1, 1]$ ***(PTA-4)***

CQ: Solve $\cos\left(\tan^{-1} x\right) = \sin\left(\cot^{-1}\frac{3}{4}\right)$. ***(PTA-5)***

CQ: Solve for x: $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$. ***(PTA-3)***

CHAPTER 5

TWO-DIMENSIONAL ANALYTICAL GEOMETRY-II

2 MARKS

Eg.5.1 Find the general equation of a circle with centre (-3,-4) and radius 3 units. ***(MAR-24)***

CQ: An elliptical whispering room has height 5m and width 26m. Where should two persons stand if they would like to whisper back and forth and be heard. ***(PTA-6)***

CHAPTER6

APPLICATIONS OF VECTOR ALGEBRA

2 MARKS

EXERCISE 6.2

3.The volume of the parallelepiped whose coterminous edges are $7\hat{\imath} + \lambda\hat{\jmath} - 3\hat{k}, \hat{\imath} + 2\hat{\jmath} - \hat{k}, -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$ is 90 cubic units. Find the value of λ . ***(JUNE-24)***

6. Determine whether the three vectors $2\hat{\imath} + 3\hat{\jmath} + \hat{k}, \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $3\hat{\imath} + \hat{\jmath} + 3\hat{k}$. are coplanar. ***(AUG-21)***

EXERCISE 6.3

8. If $\hat{a}, \hat{b}, \hat{c}$ are three-unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a} \times (\hat{b} \times \hat{c}) = \frac{1}{2}\hat{b}$, find the angle between $\hat{a}$ and $\hat{c}$. ***(SEP-20)***

EXERCISE 6.4

5. Find the acute angle between the following lines.
(iii) $2x = 3y = -z$ and $6x = -y = -4z$ ***(PTA-4)***

EXERCISE 6.6

1. Find the vector equation of a plane which is at a distance of 7 units from the origin having $3, -4, 5$ as direction ratios of a normal to it. ***(MAR-23)***

5. Find the intercepts cut off by the plane $\vec{r}.(6\hat{\imath} + 4\hat{\jmath} - 3\hat{k}) = 12$ on the coordinate axes. ***(PTA-3)***

Example 6.30 Find the acute angle between the straight lines $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$ and state whether they are parallel or perpendicular. ***(JULY-22)***

Example 6.45 Verify whether the line $\frac{x-3}{-4} = \frac{y-4}{-7} = \frac{z+3}{12}$ lies in the plane $5x - y + z = 8$. ***(PTA-1)***

Example 6.48 Find the angle between the straight line $\vec{r} = (2\hat{\imath} + 3\hat{\jmath} + \hat{k}) + t(\hat{\imath} - \hat{\jmath} + \hat{k})$and the plane $2x - y + z = 5$. ***(PTA-6)***

Example 6.49 Find the distance of a point $(2,5,-3)$ from the plane $\vec{r}.(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) = 5$. ***(JUNE-23)***

CQ: Show that the vectors $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$, $\hat{\imath} - \hat{\jmath}$ and $3\hat{\imath} - \hat{\jmath} + 6\hat{k}$ are coplanar. ***(MAR-24)***

CQ: show that the distance from the origin to the plane $3x+6y+2z+7=0$ is 1. ***(MAR-22)***

CQ: Find the vector and cartesian equations of the plane containing $\frac{x-2}{2}=\frac{y-2}{3}=\frac{z-1}{3}$ and parallel to the line $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z+1}{1}$. ***(MAR-24)***

CQ: Find the shortest distance between the straight lines $\frac{x-6}{1}=\frac{2-y}{2}=\frac{z-2}{2}$ and $\frac{x+4}{3}=\frac{y}{-2}=\frac{1-z}{2}$. ***(PTA-3)***

CQ: If $\vec{a}=\hat{\imath}+\hat{\jmath}-\hat{k},\ \vec{b}=2\hat{\imath}+3\hat{\jmath},\ \vec{c}=\hat{\jmath}-\hat{k}$ verify that $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$. ***(PTA-4)***

CHAPTER 7

APPLICATIONS OF DIFFERENTIAL CALCULUS

2 MARKS

Example 7.5 A particle is fired straight up from the ground to reach a height of *s* feet in *t* seconds, where $S(t)=128t-16t^2$. (i) Compute the maximum height of the particle reached. ***(PTA-2)***

Example 7.11 Find the equations of tangent and normal to the curve $y=x^2+3x-2$ at the point (1,2). ***(MAR-23)***

Example 7.12 Find the points on the curve $y=x^3-3x^2+x-2$ at which the tangent is parallel to the line $y=x$. ***(MAR-22)***

Example 7.20 Find the value in the interval $\left(\frac{1}{2},2\right)$ satisfied by the Rolle's theorem for the function $f(x)=x+\frac{1}{x}, x\in\left[\frac{1}{2},2\right]$. ***(MAR-20)***

EXERCISE 7.2

5. Find the tangent and normal to the following curves at the given points on the curve.

(i) $y=x^2-x^4\ at\ (1,0)$ ***(JULY-22)***

EXERCISE 7.3

1. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.

(i) $f(x)=\left|\frac{1}{x}\right|, x\in[-1,1]$. ***(PTA-6)***

Example 7.35 Evaluate the limit $\lim_{x\to 0}\left(\frac{\sin mx}{x}\right)$***(SEP-20)***

Example 7.36 Evaluate the limit $\lim_{x\to 0}\left(\frac{\sin x}{x^2}\right)$. ***(PTA-5)***

Example 7.40 Evaluate $\lim_{x\to 0^+} x\log x$. ***(JULY-22)***

Example 7.42 Evaluate: $\lim_{x\to\infty}\left(\frac{e^x}{x^m}\right), m\in\mathbb{N}$ ***(AUG-21)***

CQ: A car A is travelling from west at 50 km/hr and car B is travelling towards north at 60 km/hr. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when Car A is 0.3 kilometres and Car B is 0.4 kilometres from the intersection?

(SEP-20)

EXERCISE 7.9

2. Sketch the graphs of $(i)\ y = -\frac{1}{3}(x^3 - 3x + 2)$. ***(PTA-6)***

CQ: Find the maximum value of $\frac{\log x}{x}$. ***(MAR-23)***

CQ: Prove that the area of the largest rectangle that can be inscribed in a circle of radius $r\ cm$. Is $2r^2$. ***(MAR-19)***

CQ: A missile fired from the ground level rises x meters vertically upwards in t seconds and $x = 100t - \frac{25}{2}t^2$. Find

(i) The initial velocity of the missile

(ii) The time when the height of the missile maximum

(iii) The maximum height reached

(iv) The velocity with which the missile strikes the ground. ***(MAR-19)***

CQ: A square shaped thin material with area 196 sq. Units to make into an open box by cutting small squares from the four corners and folding the sides upward . Prove that the length of the side of removed square is $\frac{7}{3}$ when the volume of the box is maximum. ***(MAR-20)***

CHAPTER 8

DIFFERENTIALS AND PARTIAL DERIVATIVES

2 MARKS

Example 8.6 Let $g(x) = x^2 + \sin x$ .Calculate the differential dg. ***(AUG-21)***

EXERCISE 8.1

7. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number. ***(JUNE-23)***

Example 8.7 If the radius of a sphere, with radius $10\ cm$, has to decrease by $0.1cm$, approximately how much will its volume decrease? . ***(MAR-23)***

EXERCISE 8.2

2. Find df for $f(x) = x^2 + 3x$ and evaluate it for (i) $x = 2$ and $dx = 0.1$. ***(MAR-22) (MAR-20)***

5 MARKS

EXERCISE 8.4

2. For each of the following functions find the f_x , f_y and show that $f_{xy} = f_{yx}$

$(i) f(x,y) = \frac{3x}{y+\sin x}$ ***(PTA-3)***

6.Let $w(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}, (x,y,z) \neq (0,0,0)$. Show that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0$. ***(PTA-4)***

EXERCISE 8.6

4. Let $U(x,y,z) = xyz, x = e^{-t}, y = e^{-t}\cos t, z = \sin t, t \in \mathbb{R}$. Find $\frac{dU}{dt}$. ***(PTA-1)***

Example 8.22 If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$. ***(JUNE-23)***

CQ: If $w = x + 2y + z^2$ and $x = \cos t; y = \sin t; z = t$ find $\frac{dw}{dt}$ by using chain rule. Also find $\frac{dw}{dt}$ by substitution of *x, y* and *z* in *w* and hence verify the result. ***(MAR-19)***

CQ: If $f(x,y) = \log\sqrt{x^2+y^2}$, show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.***(PTA-2)***

CQ: Let $z(x,y) = xe^y + ye^{-x}, x = e^{-t}, y = st^2, s,t \in R$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.***(PTA-5)***

CQ: If $u = \sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$.***(PTA-6)***

CHAPTER9
APPLICATIONS OF INTEGRATION

2 MARKS

Example 9.25 Evaluate: $\int_{-\log 2}^{\log 2} e^{-|x|}dx$. ***(PTA-1)***

EXERCISE 9.3

1. Evaluate the following definite integrals: $(i) \int_3^4 \frac{dx}{x^2-4}$ ***(SEP-20)***

Example 9.31 Evaluate $\int_0^{\pi} x^2 \cos nx\, dx$, where *n* is a positive integer

Example 9.35 Evaluate $\int_b^{\infty} \frac{1}{a^2+x^2}\, dx$, $a > 0, b \in \mathbb{R}$. ***(MAR-23)***

EXERCISE 9.6

1.Evaluate the following: $(i) \int_0^{\frac{\pi}{2}} \sin^{10} x\, dx$ ***(PTA-3)*** ***(MAR-24)***

EXERCISE 9.7

CQ: Show that the area of the region bounded between the parabola $y^2 = 16x$ and its latus rectum is $\frac{128}{3}$. *(JULY-22)*

CQ: Find the area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its latus rectums. *(SEP-20)*

CQ: Find the area of the region common the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$. *(MAR-23)*

CQ: Show that the area of the region bounded by $3x - 2y = 0$, $x = -3, x = 1$ and x-axis is $\frac{15}{2}$ *(AUG-21)*

CQ: Find the area enclosed by the curve $y = -x^2$ and straight line $x + y + 2 = 0$. *(PTA-1)*

CHAPTER10

ORDINARY DIFFERENTIAL EQUATIONS

2 MARKS

EXERCISE 10.1

1. For each of the following differential equations, determine its order, degree (if exists)

(iii) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{d^2y}{dx^2}\right)$ *(PTA-1)* (v) $y\left(\frac{dy}{dx}\right) = \frac{x}{\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^3}$. *(PTA-4)*

EXERCISE 10.2

2. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. Form a differential equation involving the rate of change of the radius of the rain drop. *(JUNE-23)*

Example 10.3 Form the differential equation by eliminating the arbitrary constants A and B from $y = A\cos x + B\sin x$ .. *(JUNE-24)*

Example10.5 Find the differential equation of the family of parabolas $y^2 = 4ax$ where a is an arbitrary constant. *(MAR-20) (PTA-6)*

EXERCISE 10.3

8. Find the differential equation of the curve represented by $xy = ae^x + be^{-x} + x^2$. *(PTA-2)*

EXERCISE 10.4

1.Show that each of the following expressions is a solution of the corresponding given differential equation.

(ii) $y = ae^x + be^{-x}$; $y'' - y = 0$. *(JULY-22)*

4. Show that $y = e^{-x} + mx + n$ is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right) - 1 = 0$.*(PTA-3)*

EXERCISE 10.5

4. Solve the following differential equations: (i)$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ *(MAR-22) (AUG-21) (PTA-6)*

CQ: An equation relating to the stability of an aircraft is given by $\frac{dv}{dt} = g\cos\alpha - kv$,where g, α, k are constants and v is the velocity. Obtain an expression in terms of v if $v = 0$ when $t = 0$.*(PTA-6)*

CHAPTER 11
PROBABILITY DISTRIBUTIONS

2 MARKS

EXERCISE 11.1

1. Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images. ***(MAR-22)***

Example 11.10 A random variable X has the following probability mass function.

Find k. ***(MAR-22)***

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	10k

Example 11.13 If X is the random variable with distribution function $F(x)$ given by,

$F(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x < 1 \\ 1 & 1 \le x \end{cases}$ then find (i) the probability density function $f(x)$ ***(JUNE-24)***

Example 11.6 A pair of fair dice is rolled once. Find the probability mass function to get the number of fours. ***(JUNE-23)***

EXERCISE 11.2

4. Suppose a discrete random variable can only take the values 0, 1, and 2. The probability mass function is defined by $f(x) = \begin{cases} \frac{x^2+1}{k}, & \text{for } x = 0,1,2 \\ 0, & \text{for otherwise} \end{cases}$ Find (i) the value of k. ***(PTA-1)***

6. A random variable X has the following probability mass function.

x	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k. ***(JULY-22)***

EXERCISE11.3

3. The probability density function of X is given by. $f(x) = \begin{cases} kxe^{-2x}, & x > 0 \\ 0, & x \le 0 \end{cases}$

Find the value of k. ***(AUG-21)***

1. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function

$f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & \text{otherwise} \end{cases}$ Find (i) the value of k ***(JULY-22)***

6. If X is the random variable with distribution function $F(x)$ given by,

Find (i) the value of k (ii) the distribution function (iii) $P(X < 3)$ (iv)$P(5 \leq X)$ (iv)$P(X \leq 4)$ ***(PTA-2)***

Example 11.20 A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (iii) the probability of getting at least one correct answer. ***(PTA-1)***

Example 11.21 The mean and variance of a binomial variate X are respectively 2 and 1.5. Find (i) $P(X) = 0$ (ii) $P(X) = 1$ (iii) $P(X \geq 1)$ ***(JUNE-24) (JUNE-23)***

EXERCISE 11.4

4. Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred. ***(PTA-6)***

7. The probability density function of the random variable X is given by

$f(x) = \begin{cases} 16e^{-4x} & for\ x > 0 \\ 0 & for\ x \leq 0 \end{cases}$ find the mean and variance of X. ***(PTA-4)***

EXERCISE 11.5

8. If $X \sim B(n, p)$ such that $4P(X = 4) = P(X = 2)$ and $n = 6$. Find the distribution, mean and standard deviation of X. ***(SEP-20)***

CQ: The distribution function of a continuous random variable X is :

$F(x) = \begin{cases} 0 & , x < 1 \\ \dfrac{x-1}{4} & , 1 \leq x \leq 5 \\ 1 & , x > 5 \end{cases}$ Find (i) $P(x < 3)$ (ii) $P(2 < x < 4)$ (iii) $P(3 \leq x)$ ***(MAR-22)***

CQ: The mean score of 1000 students for an examination are 34 and the standard deviation is 16. Determine the limit of the marks of the central 70% of the candidates by assuming the distribution is normal. $P[0 < Z < 1.04] = 0.35$ ***(MAR-19)***

CQ: Three fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred. Verify the results by binomial distribution. ***(MAR-20)***

CQ: The sum of mean and variance of a binomial distribution for five trails is 1.8. Find the distribution. ***(PTA-5)***

CHAPTER12
DISCRETE MATHEMATICS

2 MARKS

Example12.1 Examine the binary operation (closure property) of the following operations on the respective sets (if it is not, make it binary): $a * b = \left(\frac{a-1}{b-1}\right); \forall a, b \in Q. \forall a, b$ not equal to 1. ***(SEP-20)***

Theorem 12.1: Uniqueness of identity. ***(MAR-20)***

EXERCISE 12.1

2. On Z , define $\otimes$ by $(m \otimes n) = m^n + n^m : \forall m, n \in Z.$. Is $\otimes$ binary on Z ? ***(PTA-3)***

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Important - 2 & 3 Mark Question

Std-XII Volume - 1 & 2 (2024-2025)

CHAPTER-1 APPLICATIONS OF MATRICES AND DETERMINANTS

1 EXERCISE 1.1

8. If $\text{adj}(A)=\begin{bmatrix}2 & -4 & 2\\ -3 & 12 & -7\\ -2 & 0 & 2\end{bmatrix}$, find A.

Example 1.5

Find a matrix A if $\text{adj}(A)=\begin{bmatrix}7 & 7 & -7\\ -1 & 11 & 7\\ 11 & 5 & 7\end{bmatrix}$.

2 EXERCISE 1.1

9. If $\text{adj}(A)=\begin{bmatrix}0 & -2 & 0\\ 6 & 2 & -6\\ -3 & 0 & 6\end{bmatrix}$, find A^{-1}.

Example 1.6

If $\text{adj}\, A=\begin{bmatrix}-1 & 2 & 2\\ 1 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, find A^{-1}.

3 EXERCISE 1.1

7. If $A=\begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix}$ and $B=\begin{bmatrix}-1 & -3\\ 5 & 2\end{bmatrix}$, verify that $(AB)^{-1}=B^{-1}A^{-1}$.

Example 1.9

Verify $(AB)^{-1}=B^{-1}A^{-1}$ with $A=\begin{bmatrix}0 & -3\\ 1 & 4\end{bmatrix}, B=\begin{bmatrix}-2 & -3\\ 0 & -1\end{bmatrix}$.

4 EXERCISE 1.2

2. Find the rank of the following matrices by row reduction method:

(i) $\begin{bmatrix}1 & 1 & 1 & 3\\ 2 & -1 & 3 & 4\\ 5 & -1 & 7 & 11\end{bmatrix}$ (iii) $\begin{bmatrix}3 & -8 & 5 & 2\\ 2 & -5 & 1 & 4\\ -1 & 2 & 3 & -2\end{bmatrix}$

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CHAPTER-7 APPLICATIONS OF DIFFERENTIAL CALCULUS

1 EXERCISE 7.1

2. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.
   (i) How long does the camera fall before it hits the ground?
   (ii) What is the average velocity with which the camera falls during the last 2 seconds?
   (iii) What is the instantaneous velocity of the camera when it hits the ground?

4. If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when $x = 5$ units.
5. If the mass $m(x)$ (in kilograms) of a thin rod of length x (in metres) is given by, $m(x) = \sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is $x = 3$ and $x = 27$ metres.

2 EXERCISE 7.3

2. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions :
   (i) $f(x) = x^2 - x,\ x \in [0,1]$
4. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval:
   (i) $f(x) = x^3 - 3x + 2,\ x \in [-2,2]$

3 EXERCISE 7.3

6. A race car driver is kilometer stone 20. If his speed never exceeds 150 km/hr, what is the maximum kilometer he can reach in the next two hours.

7. Suppose that for a function $f(x), f'(x) \le 1$ for all $1 \le x \le 4$. Show that $f(4) - f(1) \le 3$.

10. Using mean value theorem prove that for, $a > 0, b > 0, |e^{-a} - e^{-b}| < |a - b|$.

4 EXERCISE 7.4

1. Write the Maclaurin series expansion of the following functions:
   (i) e^x (ii) $\sin x$ (iii) $\cos x$
   (iv) $\log(1-x)$; $-1 \le x < 1$ (v) $\tan^{-1}(x)$; $-1 \le x \le 1$

2. Write down the Taylor series expansion, of the function $\log x$ about $x = 1$ upto three non-zero terms for $x > 0$.

Example 11.20

A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (iii) the probability of getting at least one correct answer.

CHAPTER- 12 DISCRETE MATHEMATICS

1

EXERCISE 12.1

2. On $\mathbb{Z}$, define $\otimes$ by $(m \otimes n) = m^n + n^m : \forall m, n \in \mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$?

3. Let $*$ be defined on $\mathbb{R}$ by $(a * b) = a + b + ab - 7$. Is $*$ binary on $\mathbb{R}$? If so, find $3 * \left(\frac{-7}{15}\right)$.

Example12.1

Examine the binary operation (closure property) of the following operations on the respective sets (if it is not, make it binary):

(i) $a * b = a + 3ab - 5b^2; \forall a, b \in \mathbb{Z}$ (ii) $a * b = \left(\frac{a-1}{b-1}\right), \forall a, b \in \mathbb{Q}$

2

8. Show that (i) $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (ii) $\neg(p \rightarrow q) \equiv p \wedge \neg q$.

9. Prove that $q \rightarrow p \equiv \neg p \rightarrow \neg q$

10. Show that $p \rightarrow q$ and $q \rightarrow p$ are not equivalent

11. Show that $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

13. Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent.

Example 12.16

Construct the truth table for $(p \overline{\vee} q) \wedge (p \overline{\vee} \neg q)$.

Example 12.17

Establish the equivalence property: $p \rightarrow q \equiv \neg p \vee q$

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IMPORTANT FIVE MARK QUESTIONS (TENTATIVE)

12TH MATHS

Sun Tuition Center - Villupuram (9629216361)

Note : * IMPORTANT AND EASY PROBLEM ONLY HERE. FIRST PRACTICE THE FOLLOWING AND THEN THE OTHER PROBLEMS.

CHAPTER-1 APPLICATIONS OF MATRICES AND DETERMINANTS

1 EXERCISE 1.1

3. If $F(\alpha)=\begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ 0 & 1 & 0\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}$, show that $[F(\alpha)]^{-1}=F(-\alpha)$.

2 EXERCISE1.1

5. If $A=\frac{1}{9}\begin{bmatrix}-8 & 1 & 4\\ 4 & 4 & 7\\ 1 & -8 & 4\end{bmatrix}$, prove that $A^{-1}=A^T$.

3 **Example 1.24**

If $A=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x-y+z=4, x-2y-2z=9, 2x+y+3z=1$.

4 EXERCISE1.3

2. If $A=\begin{bmatrix}-5 & 1 & 3\\ 7 & 1 & -5\\ 1 & -1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1 & 2\\ 3 & 2 & 1\\ 2 & 1 & 3\end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x+y+2z=1, 3x+2y+z=7, 2x+y+3z=2$.

5 **EXERCISE 1.4**

1. Solve the following systems of linear equations by Cramer's rule:

(iv) $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0, \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0, \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$

6 **Example 1.34**

Investigate for what values of λ and μ the system of linear equations

$x+2y+z=7,\ x+y+\lambda z=\mu, x+3y-5z=5$

has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

7 EXERCISE1.6

2. Find the value of k for which the equations $kx-2y+z=1,\ \ x-2ky+z=-2,\ \ x-2y+kz=1$ have

(i) no solution (ii) unique solution (iii) infinitely many solution

8 EXERCISE1.6

3. Investigate the values of λ and μ the system of linear equations $2x+3y+5z=9$, $7x+3y-5z=8,\ 2x+3y+\lambda z=\mu$, have

(i) no solution (ii) a unique solution (iii) an infinite number of solutions.

CHAPTER -12 DISCRETE MATHEMATICS

1 EXERCISE 12.1

9. (i) Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\} \right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the commutative and associative properties satisfied by $*$ on M.

(ii) Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\} \right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the existence of identity, existence of inverse properties for the operation $*$ on M.

2 EXERCISE 12.1

10. (i) Let A be $\mathbb{Q} \setminus \{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is $*$ binary on A? If so, examine the commutative and associative properties satisfied by $*$ on A.

(ii) Let A be $\mathbb{Q} \setminus \{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is $*$ binary on A? If so, examine the existence of identity, existence of inverse properties for the operation $*$ on A.

3 **Example 12.9**

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5.

4 **Example 12.10**

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subset $A = \{1, 3, 4, 5, 9\}$ of the set of remainders $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

5 **Example 12.7**

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for following operation on the given set.

$m * n = m + n - mn;\ m, n \in \mathbb{Z}$

6 **Example 12.19**

Using the equivalence property, show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.

7 EXERCISE 12.2

15. Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

8 EXERCISE 12.2

13. Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent.

$\therefore x=3, y=-2, =-1$

Do it yourself:

Exercise: 1.3 – 2,1(iv),(iii)
Exercise: 1.4 – 1(iii),5
Example: 1.23,1.27,1.12

3) The upward speed v(t) of a rocket at a time is approximated by $v(t)=at^2+bt+c$, $0 \le t \le 100$ where a, b, and c are constants. It has been found the speed at times t=3,t=6 and t=9 seconds are respectively,64,133and 208 miles per second respectively. Find the speed at time t=15 seconds.

Sol:

$v(3)=64 \Rightarrow 9a+3b+c$
$v(6)=133 \Rightarrow 36a+6b+c$
$v(9)=208 \Rightarrow 81a+9b+c$

$$[A|B]=\left[\begin{array}{ccc|c}9&3&1&64\\36&6&1&133\\81&9&1&208\end{array}\right]$$

$$\xrightarrow[R_2\to R_2-4R_1]{R_3\to R_3-9R_1}\left[\begin{array}{ccc|c}9&3&1&64\\0&-6&-3&-123\\0&-18&-8&-368\end{array}\right]$$

$$\xrightarrow[R_3\to -R_3]{R_3\to R_3-9R_1}\left[\begin{array}{ccc|c}9&3&1&64\\0&2&1&-123\\0&-18&-8&-368\end{array}\right]$$

$$\xrightarrow{R_3\to R_3-9R_2}\left[\begin{array}{ccc|c}9&3&1&64\\0&2&1&41\\0&0&-1&-1\end{array}\right]$$

$c=1$
$2b+c=41$
$2b=40$
$b=20$
$9a+3b+c=64 \Rightarrow 9a+60+1=64$
$9a=3 \Rightarrow a=\frac{1}{3}$

By Cramer's rule, $a=\frac{\Delta_a}{a}=\frac{100}{-6000}=-\frac{1}{60}$

$b=\frac{\Delta_b}{b}=\frac{-7800}{-6000}=\frac{78}{60}=\frac{13}{10}$

$c=\frac{\Delta_c}{c}=\frac{20,000}{-6000}=-\frac{20}{6}=-\frac{10}{3}$

The equation of path is

$y=-\frac{1}{60}x^2+\frac{13}{10x}-\frac{10}{3}$

$x=70$ எனில் $y=-\frac{70^2}{60}+\frac{13}{10}(70)-\frac{10}{3}$

$y=-\frac{4900}{60}+\frac{910}{60}\times 6-\frac{10}{3}$

$=\frac{-4900+5460-200}{60}$

$=\frac{5460-5160}{60}$

$=\frac{360}{60}=6$

when $x=70$, we get $y=6$, CSK won.

4). A boy is walking along the path $y=ax^2+bx+c$ through the points (-6,8),(-2,-12) and (3,8).He wants to meet his friend at P(7,60).Will he meets his friend?

Sol:

$y=c+bx+ax^2$
It passes $(-6,8),(-2,-12), and\ (3,8)$
$c-6b+36a=8$
$c-2b+4a=-12$
$c+3b+9a=8$

$$[A|B]=\left[\begin{array}{ccc|c}1&-6&36&8\\1&-2&4&-12\\1&3&9&8\end{array}\right]$$

$$\xrightarrow[R_2\to R_2-R_1]{R_3\to R_3-R_1}\left[\begin{array}{ccc|c}1&-6&36&8\\0&4&-32&-20\\0&9&-27&0\end{array}\right]$$

$R_3\to\frac{1}{3}R_1$

$$\xrightarrow{R_2\to\frac{1}{4}R_2}\left[\begin{array}{ccc|c}1&-6&36&8\\0&1&-8&-4\\0&3&-3&0\end{array}\right]$$

$$\xrightarrow{R_3\to R_3-3R_2}\left[\begin{array}{ccc|c}1&-6&1&8\\0&1&-8&-4\\0&0&21&-50\end{array}\right]$$

$5c=-50 \Rightarrow c=-10$……………(i)
$-3b+2c=-29$…………………(ii)
$36a-6b+c=8$………………(iii)
$(ii) \Rightarrow -3b-20=-29$
$-3b=-29+20$
$=-9$
$b=+3$
$(iii) \Rightarrow 36a-18-10=8$
$36a=8+28$

∧க்கு ஒரு F இருந்தாலும் F வரவேண்டும்.

∨க்கு ஒரு T இருந்தாலும் T வரவேண்டும்.

$p \longrightarrow q$க்கு T F க்கு F .... மற்றதற்கு T

$p \leftrightarrow q$ க்கு T T க்கு T மேலும் F F க்கு T .... மற்றதற்கு F

Example: Construct the truth table (p∨q)∧(~q)

P	Q	$(p \vee p)$	$\sim q$	$(p \vee p) \wedge (\sim p)$
T	T	T	F	F
T	F	T	T	T
F	T	T	F	F
F	F	F	T	F

1) Show that $p \rightarrow q \equiv (\sim p) \vee q$

P	q	$P \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

P	q	$\sim p$	$(\sim p) \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

$p \rightarrow q \equiv (\sim p) \vee q$

4) Show that $q \rightarrow p \equiv \neg p \rightarrow \neg q$

P	q	$q \rightarrow p$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	T	T

5) Verify whether the compound statement $(p \wedge q) \wedge [\neg (p \vee q)]$ is a tautology or contradiction or contingency.

P	q	$p \wedge q$	$(p \vee q)$	$\neg(p \vee q)$	$[(p\wedge q) \vee [\neg(p \vee q)]]$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

$(p \wedge q) \wedge [\neg (p \vee q)]$ is contradiction.

6) Show that $\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$

p	q	$(p\wedge q)$	$\neg(p\wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

	Range: $y \geq -\frac{25}{4}$	
Intercepts	x - Intercept $=-2, 3$ y - Intercept$=-6$	x- Intercept$=3$ y - Intercept $=-9$
Critical Points	$f'(x) = 2x - 1$ $x = \frac{1}{2}$	$f'(x) = 3(x^2 - 2)$ $x = \pm\sqrt{2}$
Local extrema	For $x = 1/2$ Local minimum $f\left(\frac{1}{2}\right) = -\frac{25}{4}$	For $x = \sqrt{2}, f''(x) > 0$ Local minimum $f(\sqrt{2}) = -4\sqrt{2} - 9$ Local maximum $x = -\sqrt{2}, f''(x) < 0$ $f(\sqrt{2}) = 4\sqrt{2} - 9$