1/14/2025

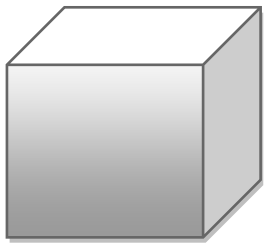
# 10th

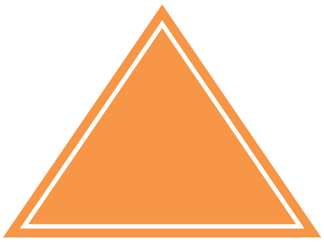
# MATHEMATICS

## UNIT EXERCISE COMPLETE SOLUTION


**NEW SYLLABUS 2024-2025**







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# 10th MATHS UNIT EXERCISE FULL SOLUTION NEW SYLLABUS EM (2024-2025)

## CHAPTER – 1 (RELATIONS AND FUNCTIONS)

**1. If the ordered pairs $(x^2 - 3x, y^2 + 4y)$ and $(-2, 5)$ are equal, then find $x$ and $y$.**

**Solution:**

Given: $(x^2 - 3x, y^2 + 4y)$ and $(-2,5)$ are equal

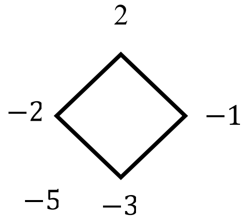
$(x^2 - 3x, y^2 + 4y) = (-2,5)$

$x^2 - 3x = -2$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$\boxed{x = 1, 2}$

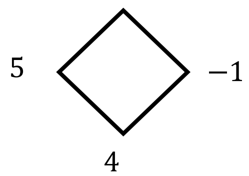



$y^2 + 4y = 5 - 5$

$y^2 + 4y - 5 = 0$

$(y+5)(y-1) = 0$

$\boxed{y = -5, 1}$




**2. The cartesian product $A \times A$ has 9 elements among which $(-1, 0)$ and $(0, 1)$ are found. Find the set $A$ and the remaining elements of $A \times A$.**

**Solution:**

Given $n(A \times A) = 9$ . Also the two ordered pairs

$A \times A = (-1,0)$ and $(0,1)$

$n(A) \times n(A) = 9, n(A) = 3$

From the given two ordered pairs of $(-1,0)$ and $(0,1)$

$A = \{-1,0,1\}, A \times A = \{-1,0,1\} \times \{-1,0,1\}$

$= \{(1,-1), (-1,0), (-1,1), (0,-1), (0,0), (0,1)$

$(1,-1), (1,0), (1,1)\}$

The remaining elements of

$A \times A = \{(1,-1), (-1,1), (0,-1), (0,0), (1,-1),$

$(1,0), (1,1)\ \}$

**3. Given that $f(x) = \begin{cases} \sqrt{x-1} & x \ge 1 \\ 4 & x < 1 \end{cases}$. Find (i) $f(0)$ (ii) $f(3)$ (iii) $f(a+1)$ in terms of $a$. (Given that $a \ge 0$ )**

**Solution:**

Given: $f(x) = \begin{cases} \sqrt{x-1}, & x \ge 1 \\ 4, & x < 1 \end{cases}$

(i) $\boxed{f(0) = 4}$ $0 < 1$, [It satisfies the $2^{nd}$ condition]

(ii) $f(3) = \sqrt{3-1} \rightarrow \boxed{f(3) = \sqrt{2}}$ $3 \ge 1$,

[it satisfies the 1st condition]

(iii) $f(a+1) = \sqrt{(a+1) - 1} \rightarrow \boxed{f(a+1) = \sqrt{a}}$

$a + 1 \ge 1$, [it satisfies the 1st condition]

**4. Let $A = \{9, 10, 11, 12, 13, 14, 15, 16, 17\}$ and let $f: A \rightarrow N$ be defined by $f(n) =$ the highest prime factor of $n \in A$. Write $f$ as a set of ordered pairs and find the range of $f$.**

**Solution:**

Given: $A = \{9,10,11,12,13,14,15,16,17\}$

$f(n) =$The highest prime factor of $n$

$f(9) = 3$, $[9 = 3 \times 3]$

$f(10) = 5$, $[10 = 2 \times 5]$

$f(11) = 11$, [11 is a prime number]

$f(12) = 3$, $[12 = 2 \times 2 \times 3]$

$f(13) = 13$, [13 is a prime number]

$f(14) = 7$, $[14 = 2 \times 7$

$f(15) = 5$, $[15 = 3 \times 5]$

$f(16) = 2$, $[16 = 2 \times 2 \times 2 \times 2]$

$f(17) = 17$, [17 is a prime number]

$f = \{(9,3), (10,5), (11,11), (12,3), (13,13), (14,7),$
$(15,5), (16,2), (17,17)\}$

Range of $f = \{2,3,5,7,11,13,17\}$

(Note: 1 is neither a prime nor a composite)

**5. Find the domain of the function**

$$f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$$

**Solution:**

Given: $f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$

When $x = 0; f(0) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 0}}} = 1$

When $x = 1$ ; $f(1) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 1}}} = \sqrt{2}$

When $x = -1$ ; $f(-1) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 1}}} = 1$

When $x = 2; f(2) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 4}}}$

$= \sqrt{1 + \sqrt{1 - \sqrt{-3}}}$ is an imaginary

When $x = -2; f(-2) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 4}}}$

$= \sqrt{1 + \sqrt{1 - \sqrt{-3}}}$ is an imaginary

From the above, except $(-1,0,1)$ the result for the other values of $x$ become imaginary.

The domain $= \{-1,0,1\}$

**6. If $f(x) = x^2, g(x) = 3x$ and $h(x) = x - 2$, Prove that $(f \circ g) \circ h = f \circ (g \circ h)$.**

**Solution:**

Given : $f(x) = x^2$ $g(x) = 3x$ and $h(x) = x - 2$ ;

To prove : $(f \circ g) \circ h = f \circ (g \circ h)$

$$(f \circ g) = f(g(x)) = f(3x) = (3x)^2 = 9x^2$$

$$(f \circ g) \circ h = f \circ g(h(x)) = f \circ g(x-2) = \boxed{9(x-2)^2} \text{ --------- } (1)$$

$$(g \circ h) = g(h(x)) = g(x-2) = 3(x-2)$$

$$f \circ (g \circ h) = f(g \circ h) = (3(\chi-2))^2 = \boxed{9(x-2)^2} \text{ --------- } (2)$$

From (1) & (2) $\boxed{(f \circ g) \circ h = f \circ (g \circ h)}$

**Hence Proved**

**7. Let $A = \{1,2\}$ and $B = \{1,2,3,4\}$, $C = \{5,6\}$ and $D = \{5,6,7,8\}$. Verify whether $A \times C$ is a subset of $B \times D$ ?**

**Solution:**

Given: $A = \{1,2\}$ $B = \{1,2,3,4\}$ $C = \{1,6\}$

$D = \{5,6,7,8\}$ To show that : $A \times C \subset B \times D$

$$A \times C = \{1,2\} \times \{5,6\} = \{(1,5),(1,6),(2,5),(2,6)\} \text{ ------ } (1)$$

$$B \times D = \{1,2,3,4\} \times \{5,6,7,8\} = \{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),(3,8),(4,5),(4,6),(4,7),(4,8)\} \text{ ------ } (2)$$

From (1) & (2) $\boxed{A \times C \subset B \times D}$

**Hence Proved**

**8. If $f(x) = \frac{x-1}{x+1}, x \neq -1$ show that $f(f(x)) = -\frac{1}{x}$ provided $x \neq 0$.**

**Solution:**

Given: If $f(x) = \frac{x-1}{x+1}$

show that $f(f(x)) = -\frac{1}{x}$

$$f(x) = \frac{x-1}{x+1}$$

$$f(f(x)) = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = \frac{\frac{x-1-x-1}{\cancel{x+1}}}{\frac{x-1+x+1}{\cancel{x+1}}}$$

$$f(f(x)) = \frac{-2}{2x} \rightarrow \boxed{f(f(x)) = -\frac{1}{x}} \text{ (Proved)}$$

**9. The functions $f$ and $g$ are defined by**

$$f(x) = 6x + 8;\ g(x) = \frac{x-2}{3}$$

**(i) Calculate the value of $gg\left(\frac{1}{2}\right)$**

**(ii) Write an expression for $gf(x)$ in its simplest form.**

**Solution:**

Given: $f(x) = 6x + 8$ ; $g(x) = \frac{x-2}{3}$

(i) $$gg(x) = g(g(x)) = \frac{\frac{x-2}{3} - 2}{3} = \frac{x-2-6}{3 \times 3} = \frac{x-8}{9}$$

$$gg(x) = \frac{x-8}{9}$$

$$gg\left(\frac{1}{2}\right) = \frac{\frac{1}{2} - 8}{9} = \frac{1-16}{2 \times 9} = -\frac{15}{2 \times 9} = -\frac{5}{6}$$

(ii) $$gf(x) = g(f(x)) = g(6x+8) = \frac{6x+8-2}{3} = \frac{6x+6}{3} = \frac{6(x+1)}{3} = 2(x+1)$$

**10. Write the domain of the following real functions**

**(i) $f(x) = \frac{2x+1}{x-9}$ (ii) $p(x) = \frac{-5}{4x^2+1}$**

**(iii) $g(x) = \sqrt{x-2}$ (iv) $h(x) = x + 6$**

**Solution:**

(i). $f(x) = \frac{2x+1}{x-9}$

According to the denominator all values are defined except $x = 9$

Domain of $\boxed{f = \mathbb{R} - \{9\}}$

(ii). $p(x) = \frac{-5}{4x^2+1}$

According to the denominator, all values $x$ are defined

Domain of $\boxed{p = \mathbb{R}}$

(iii). $g(x) = \sqrt{x-2}$

According to the Square root, when $x < 2$, it will become an imaginary.

Domain of $\boxed{g = [2, \infty)}$

(iv). $h(x) = x + 6$ $h(x)$ is defined for all values $x$

Domain of $\boxed{h = \mathbb{R}}$

## CHAPTER – 2 (NUMBERS AND SEQUENCES)

**1. Prove that $n^2 - n$ divisible by 2 for every positive integer $n$.**

**Solution:**

To Prove: $n^2 - n$ divisible by 2 for every positive integer $n$.

$$n^2 - n = n(n-1)$$

Here, when $n = Odd, n-1$ becomes even

when $n = Even, n-1$ becomes odd

The product of an odd and an even is always an even number which is divisible by 2.

$\therefore n^2 - n$ divisible by 2 for every positive integer $n$.

**2. A milk man has 175 litres of cow's milk and 105 litres of buffalow's milk. He wishes to sell the milk by filling the two types of milk in cans of equal capacity. Calculate the following (i) Capacity of a can (ii) Number of cans of cow's milk (iii) Number of cans of buffalow's milk.**

**Solution:**

Cow's milk = 175 litres;

Buffalow's milk = 105 litres

The milkman wants to separate them with equal sizes of can. The size of the can is the HCF of (175, 105)

$$175 = 5 \times 5 \times 7;$$

$$105 = 3 \times 5 \times 7.$$

The HCF of $(175,105) = 5 \times 7 = 35$

(i) The capacity of the each can = 35 litre

(ii) Number of cans required for cow's milk :

$$\frac{175}{35} = 5$$

(iii) Number of cans required for buffalow's milk :

$$\frac{105}{35} = 3$$

**3. When the positive integers $a, b$ and $c$ are divided by 13 the respective remainders are 9,7 and 10 . Find the remainder when $a + 2b + 3c$ is divided by 13 .**

**Solution:**

When $a$ is divided by 13, the remainder is 9

$$\boxed{a \equiv 9 (mod\ 13)} --- (1)$$

Similarly $b \equiv 7\ (mod\ 13) --- (2)$

Similarly $c \equiv 10\ (mod\ 13) --- (3)$

$(2) \times 2 \rightarrow 2b \equiv 14\ (mod\ 13)$

(Multiplication of Modulo arithmetic)

$$\boxed{2b \equiv 1 (mod\ 13)}\ [14 = 13 \times 1 + 1]$$

$(3) \times 3 \rightarrow 3c \equiv 30$ (mod 13)

$$\boxed{3c \equiv 4 (mod\ 13)} [30 = 13 \times 2 + 4]$$

$a + 2b + 3c \equiv (9 + 1 + 4)\ (mod\ 13)$

(Addition of Modulo arithmetic)

$a + 2b + 3c \equiv 14\ (mod\ 13)$

$$\boxed{a + 2b + 3c \equiv 1}\ (mod\ 13)\ [14 = 13 \times 1 + 1]$$

When $a + 2b + 3c$ is divided by 13,

The remainder is 1.

**4. Show that 107 is of the form $4q + 3$ for any integer $q$.**

**Solution:**

Let $107 = 4q + 3$

$$107 - 3 = 4q$$

$$104 = 4q$$

104 is divisible by 4 for any integer $q$.

$\therefore$ 107 is of the form $4q + 3$.

**5. If $(m+1)^{th}$ term of an A.P. is twice the $(n+1)^{th}$ term, then prove that $(3m+1)^{th}$ term is twice the $(m+n+1)^{th}$ term.**

**Solution:**

Let $a$ and $d$ be the 1st term and the common difference of an AP

It's $n^{th}$ term $\rightarrow t_n = a + (n-1)d$

The condition given is $t_{m+1} = 2(t_{n+1})$

$$a + (m+1-1)d = 2[a + (n+1-1)d]$$

$$\boxed{a + md = 2[a + nd]} ------ (1)$$

Also $t_{3m+1} = a + (3m+1-1)d$

$= a + 3md$

$= a + md + 2md$

$= 2(a + nd) + 2md$ [From(1) : $a + md = 2[a + nd]$]

$= 2(a + md + nd)$

$= 2[a + (m+n)d]$

$= 2[a + (m+n+1-1)d]$

$= 2t_{m+n+1}$

$$(3m+1)^{th} \text{ term} = \boxed{2 \times (m+n+1)^{th}} \text{ term}$$

**Hence proved**

**6. Find the $12^{th}$ term from the last term of the A. P $-2, -4, -6, \ldots -100$.**

**Solution:**

Given $A.P = -2, -4, -6, \ldots, -100$

By reversing the given A.P $= -100, , -6, -4, -2$.

Now $a = -100, d = -2 - (-4)$

$= -2 + 4 = 2$

$$\boxed{t_n = a + (n-1)d}$$

$12^{th}$ term $t_{12} = -100 + (12-1)2$

$t_{12} = -100 + 22$

$= -78$

The $12^{th}$ from the last term of the given AP is $-78$

**7. Two A.P.'s have the same common difference. The first term of one A.P. is 2 and that of the other is 7. Show that the difference between their $10^{th}$ terms is the same as the difference between their $21^{st}$ terms, which is the same as the difference between any two corresponding terms.**

**Solution:**

Given : $AP_1 = 2$ $AP_2 = 7$

1st term 2, 1st term 7.

The common difference is the same for both the AP's

Difference of the $n^{th}$ terms of the two AP's:

$$t_n \; of \; AP_1 - T_n \; of \; AP_2$$

$$[a_1 + (n-1)d] - [A_1 + (n-1)d]$$

$$2 + (n-1)d - 7 - (n-1)d = -5$$

The Difference between any corresponding terms of the two AP's is always $-5$

The Difference between their $10^{th}$ terms $= -5$

The Difference between their21st terms $= -5$

$$\therefore t_n \; of \; AP_1 - T_n \; of \; AP_2 = -5$$

**8. A man saved ₹ 16500 in ten years. In each year after the first he saved ₹ 100 more than he did in the preceding year. How much did he save in the first year?**

**Solution:**

Given : $S_{10} = 16500$

Let the 1st year savings $= a$

The $2^{nd}$ year savings $= a + 100$

The $3^{rd}$ year savings $= a + 100 + 100 = a + 200$

It forms an AP with a common difference $d = 100$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2a + (10-1)100] = 16500$$

$5\,[2a + 900] = 16500$

$$2a + 900 = \frac{16500}{5} = 3300$$

$$2a = 3300 - 900 = 2400$$

$$a = \frac{2400}{2} = 1200$$

In the 1st year he saved ₹ 1200.

**9. Find the G.P. in which the $2^{nd}$ term is $\sqrt{6}$ and the $6^{th}$ term is $9\sqrt{6}$.**

**Solution:**

Given: The $2^{nd}$ term of the GP i.e. $ar = \sqrt{6}$

The $6^{th}$ term of the GP i.e. $ar^5 = 9\sqrt{6}$

$$\frac{ar^5}{ar} = \frac{9\sqrt{6}}{\sqrt{6}}$$

$r^4 = 9$ (or) $r^2 = 3$ (or) $r = \pm\sqrt{3}$

When $r = \sqrt{3}$, $ar = \sqrt{6}$; $a = \frac{\sqrt{6}}{r} = \frac{\sqrt{3}\times\sqrt{2}}{\sqrt{3}} = \sqrt{2}$

When $r = -\sqrt{3}$, $ar = \sqrt{6}$; $a = \frac{\sqrt{6}}{r} = \frac{\sqrt{3}\times\sqrt{2}}{-\sqrt{3}} = -\sqrt{2}$

The general GP sequence: $a, ar, ar^2,$

The required GP with $a = \sqrt{2}$ $and$ $r = \sqrt{3}$ :

$$\sqrt{2}, \sqrt{2}\times\sqrt{3}, \sqrt{2}\times\sqrt{3}^2,$$

$$\boxed{\sqrt{2}, \sqrt{6}, 3\sqrt{2},}$$

The required GP with $= -\sqrt{2}$ $and$ $r = -\sqrt{3}$:

$$-\sqrt{2}, (-\sqrt{2})\times(-\sqrt{3}), (-\sqrt{2})\times(-\sqrt{3})^2, ..$$

$$\boxed{-\sqrt{2}, \sqrt{6}, -3\sqrt{2},}$$

**10. The value of a motor cycle depreciates at the rate of 15% per year. What will be the value of the motor cycle 3 year hence, which is now purchased for ₹ 45,000 ?**

**Solution:**

Given :

Present Value of the motor cycle: $a =$ ₹ 45000

Depreciation $= 15\%$ .

The depreciation constant ratio : $r = 1 - \frac{15}{100} = \frac{85}{100}$

To find the value of the motor cycle after 3 years means the value at the $4^{th}$ year. $n = 4$

Depreciated value after 3 year i.e. $t_n = a \times r^{n-1}$

$$= 45000 \times \left(\frac{85}{100}\right)^{4-1}$$

$$= 45000 \times \left(\frac{85}{100}\right)^{3}$$

$$= 45000 \times \frac{85}{100} \times \frac{85}{100} \times \frac{85}{100}$$

$$= 27635.625$$

The value of the motor cycle after 3 years = ₹ 27636

## CHAPTER – 3 (ALGEBRA)

**1. Solve $\frac{1}{3}(x + y - 5) = y - z = 2x - 11 = 9 - (x + 2z)$**

**Solution:**

Given:

$$\frac{1}{3}(x + y - 5) = y - z$$

$$x + y - 5 = 3y - 3z$$

$$\boxed{x - 2y + 3z = 5 ------(1)}$$

$y - z = 2x - 11$

$$\boxed{2x - y + z = 11 -------(2)}$$

$2x - 11 = 9 - (x + 2z)$

$$\boxed{3x + 2z = 20 ------(3)}$$

$(2) \times 2 \Rightarrow \boxed{4x - 2y + 2z = 22 -----(4)}$

$$\boxed{x - 2y + 3z = 5 -------(1)}$$

$(4)-(1) \Rightarrow \boxed{3x - z = 17 ----(5)}$

$$3x + 2z = 20 ------(3)$$
$$3x - z = 17 ------(5)$$

$(3)-(5) \Rightarrow 3z = 3$

$$\boxed{z = 1}$$

$(5) \Rightarrow 3x - 1 = 17$

$3x = 18$

$$\boxed{x = 6}$$

$(2) \Rightarrow 2(6) - y + 1 = 11$

$y = 13 - 11$

$$\boxed{y = 2}$$

$$\boxed{x = 6\ y = 2\ z = 1}$$

**2. One hundred and fifty students are admitted to a school. They are distributed over three sections $A, B$ and $C$. If 6 students are shifted from section A to section $C$, the sections will have equal number of students. If 4 times of students of section $C$ exceeds the number of students of section $A$ by the number of students in section $B$, find the number of students in the three sections.**

**Solution:**

Let x, $y, z$ be the number of students in the sections $A$, $B$ and C.

$$\boxed{x + y + z = 150 ----(1)}$$

$x - 6 = z + 6$

$$\boxed{x - z = 12 ------(2)}$$

$4z - x = y$

$$\boxed{x + y - 4z = 0 ------(3)}$$

$$x + y + z = 150 ------(1)$$
$$x + y - 4z = 0 ------(3)$$

$(1)-(3) \Rightarrow 5z = 150$

$$\boxed{z = 30}$$

$(2) \Rightarrow x - 30 = 12$

$$\boxed{x = 42}$$

$(1) \Rightarrow 42 + y + 30 = 150$

$$\boxed{y = 78}$$

The number of students in the sections $A = 42$;
The number of students in the sections $B = 78$;
The number of students in the sections $C = 30$.

**3. In a three-digit number, when the tens and the hundreds digit are interchanged the new number is 54 more than three times the original number. If 198 is added to the number, the digits are reversed. The tens digit exceeds the hundreds digit by twice as that of the tens digit exceeds the unit digit. Find the original number.**

**Solution:**

Let $x, y, z$ be the $100^{th}$, $10^{th}$ and the unit place of the 3 digit number.

Then the number becomes: $100x + 10y + z$

$$100y + 10x + x = 3(100x + 10y + z) + 54$$
$$100y + 10x + z = 300x + 30y + 3z + 54$$
$$290x - 70y + 2z = -54$$
$$\boxed{145x - 35y + z = -27 ----(1)}$$

$100x + 10y + z + 198 = 100z + 10y + x$

$$99x - 99z = -198$$
$$x - z = -2$$
$$\boxed{z = x + 2 ----(2)}$$
$$y - x = 2(y - z)$$
$$y - x = 2y - 2z$$
$$x + y - 2z = 0$$

$x + y - 2(x + 2) = 0$ [From $(2) z = x + 2$]

$x + y - 2x - 4 = 0$

$$\boxed{y = x + 4 ----(3)}$$

Substitute (1) in (2) and (3) Equation

$145x - 35(x + 4) + x + 2 = -27 ----(4)$

$145x - 35x - 140 + x + 2 = -27$

$$111x = 111$$
$$\boxed{x = 1}$$

$(3) \Rightarrow y = x + 4 = 1 + 4 = 5$

$(2) \Rightarrow z = x + 2 = 1 + 2 = 3$

$$\boxed{x = 1\ y = 5\ z = 3}$$

Required Number is $= 100(1) + 10(5) + 3 = 153$

**4. Find the least common multiple of $xy(k^2 + 1) + k(x^2 + y^2)$ and $xy(k^2 - 1) + k(x^2 - y^2)$**

**Solution:**

$$\begin{aligned} xy(k^2+1) + k(x^2+y^2) &= xyk^2 + xy + kx^2 + ky^2 \\ &= xyk^2 + kx^2 + ky^2 + xy \\ &= kx(ky + x) + y(ky + x) \\ &= (kx + y)(ky + x) \end{aligned}$$

$$\begin{aligned} xy(k^2-1) + k(x^2-y^2) &= xyk^2 - xy + kx^2 - ky^2 \\ &= xyk^2 + kx^2 - ky^2 - xy \\ &= kx(ky + x) - y(ky + x) \\ &= (ky + x)(kx - y) \end{aligned}$$

Hence, LCM $= (ky + x)(kx + y)(kx - y)$

$$\boxed{LCM = (ky + x)(k^2x^2 - y^2)}$$

**5. Find the GCD of the following by division algorithm**
**$2x^4 + 13x^3 + 27x^2 + 23x + 7, x^3 + 3x^2 + 3x + 1, x^2 + 2x + 1$**

**Solution:**

$$f(x) = 2x^4 + 13x^3 + 27x^2 + 23x + 7$$
$$g(x) = x^3 + 3x^2 + 3x + 1$$
$$h(x) = x^2 + 2x + 1$$

$$\begin{array}{r|l} & \quad 2x + 7 \\ \hline x^3 + 3x^2 + 3x + 1 & 2x^4 + 13x^3 + 27x^2 + 23x + 7 \\ & 2x^4 + 6x^3 + 6x^2 + 2x \\ & \qquad\qquad\qquad (-) \\ \hline & 7x^3 + 21x^2 + 21x + 7 \\ & 7x^3 + 21x^2 + 21x + 7 \\ & \qquad\qquad\qquad (-) \\ \hline & \qquad 0 \end{array}$$

$$\begin{array}{r|l} & \quad x + 1 \\ \hline x^2 + 2x + 1 & x^3 + 3x^2 + 3x + 1 \\ & x^3 + 2x^2 + 1x \\ & \qquad\qquad (-) \\ \hline & x^2 + 2x + 1 \\ & x^2 + 2x + 1 \\ & \qquad\qquad (-) \\ \hline & \qquad 0 \end{array}$$

GCD of $\boxed{f(x), g(x), h(x) = x^2 + 2x + 1}$

**6. Reduce the given Rational expressions to its lowest form**

**(i)** $\dfrac{x^{3a} - 8}{x^{2a} + 2x^a + 4}$

**(ii)** $\dfrac{10x^3 - 25x^2 + 4x - 10}{-4 - 10x^2}$

**Solution:**

$$(i)\ \frac{x^{3a} - 8}{x^{2a} + 2x^a + 4} = \frac{(x^a)^3 - 2^3}{x^{2a} + 2x^a + 4}$$
$$= \frac{(x^a - 2)(x^{2a} + 2x^a + 4)}{x^{2a} + 2x^a + 4}$$
$$= x^a - 2$$

$$(ii)\ \frac{10x^3 - 25x^2 + 4x - 10}{-4 - 10x^2}$$
$$= \frac{5x^2(2x - 5) + 2(2x - 5)}{-2(2 + 5x^2)}$$
$$= \frac{(5x^2 + 2)(2x - 5)}{-2(5x^2 + 2)}$$
$$= \frac{2x - 5}{-2}$$
$$= -x + \frac{5}{2}$$

**7. Simplify:** $\dfrac{\frac{1}{p} + \frac{1}{q+r}}{\frac{1}{p} - \frac{1}{q+r}} \times \left(1 + \dfrac{q^2 + r^2 - p^2}{2qr}\right)$

**Solution:**

$$\frac{\frac{1}{p} + \frac{1}{q+r}}{\frac{1}{p} - \frac{1}{q+r}} \times \left(1 + \frac{q^2 + r^2 - p^2}{2qr}\right)$$
$$= \frac{\frac{q+r+p}{p(q+r)}}{\frac{q+r-p}{p(q+r)}} \times \left(\frac{2qr + q^2 + r^2 - p^2}{2qr}\right)$$
$$= \frac{q+r+p}{q+r-p} \times \frac{(q+r)^2 - p^2}{2qr}$$
$$= \frac{q+r+p}{q+r-p} \times \frac{(q+r+p)(q+r-p)}{2qr}$$
$$= \frac{(p+q+r)^2}{2qr}$$

**8. Arul, Madan and Ram working together can clean a store in 6 hours. Working alone, Madan takes twice as long to clean the store as Arul does. Ram needs three times as long as Arul does. How long would it take each if they are working alone?**

**Solution:**

Arul, Ravi and Ram together complete the work in 6 $hrs$. So their combined workmanship $= \frac{1}{6}$

Let Arul completes the works alone by himself

$= x\ hrs$

Ravi completes the works alone by himself

$= 2x\ hrs$

And Ram completes the works alone by himself

$= 3x\ hrs$

Their individual workmanships $= \frac{1}{x}, \frac{1}{2x}, \frac{1}{3x}$

Their combined workmanships

$$\frac{1}{x} + \frac{1}{2x} + \frac{1}{3x} = \frac{1}{6}$$
$$\frac{6 + 3 + 2}{6x} = \frac{1}{6}$$
$$\frac{11}{6x} = \frac{1}{6}$$
$$x = 11$$

Arul completes the works alone by himself in 11 $hrs$

Ravi completes the works alone by himself in

$2 \times 11 = 22\ hrs$

Ram completes the works alone by himself in

$3 \times 11 = 33\ hrs$

**9. Find the square root of $289x^4 - 612x^3 + 970x^2 - 684x + 361$.**

**Solution:**

To find Square root of

$$289x^4 - 612x^3 + 970x^2 - 684x + 361$$

| | $17x^2 - 18x + 19$ |
|---|---|
| 17 | $289x^4 - 612x^3 + 970x^2 - 684 + 361$ |
| | $289x^4$ |
| | $(-)$ |
| $34x - 18$ | $-612x^3 + 970x^2$ |
| | $-612x^3 + 374x^2$ |
| | $(-)$ |
| $34x^2 - 36 + 19$ | $646x^2 - 684 + 361$ |
| | $646x^2 - 684 + 361$ |
| | $(-)$ |
| | 0 |

$$\therefore \sqrt{289x^4 - 612x^3 + 970x^2 - 684x + 361} = \boxed{|17x^2 - 18x + 19|}$$

**10. Solve $\sqrt{y+1} + \sqrt{2y-5} = 3$**

**Solution:**

$$\sqrt{y+1} + \sqrt{2y-5} = 3$$
$$\sqrt{y+1} = 3 - \sqrt{2y-5}$$

Squaring on Both side

$$(\sqrt{y+1})^2 = (3 - \sqrt{2y-5})^2$$
$$y + 1 = 9 + 2y - 5 - 6\sqrt{2y-5}$$
$$y + 3 = 6\sqrt{2y-5}$$
$$(y+3)^2 = 36(2y-5)$$

Squaring on Both side

$$y^2 + 6y + 9 = 72y - 180$$
$$y^2 - 66y + 189 = 0$$
$$(y-3)(y-63) = 0$$
$$\boxed{y = 3;\quad 63}$$

**11. A boat takes 1.6 hours longer to go 36 kms up a river than down the river. If the speed of the water current is 4 km per hr , what is the speed of the boat in still water?**

**Solution:**

Given: The speed of the river $= 4\ kmph$

Distance travelled by the boat on the upstream and the downstream side $= 36\ km$

Let the speed of the boat $= x\ kmph$

The speed of the boat on the upstream side

$$= (x-4)\ kmph$$

The speed of the boat on the downstream side

$$= (x+4)\ kmph$$

The time taken for the upstream travel $= \frac{Distance}{Speed}$

$$= \frac{36}{(x-4)}$$

And the time taken for the downstream travel

$$\frac{Distance}{Speed} = \frac{36}{(x+4)}$$

The time taken on the upstream travel =The time taken on the downstream travel $+1.6hr$

Difference in time taken: $T_2 - T_1 = 1.6\ km/hr$

$$\frac{36}{x-4} - \frac{36}{x+4} = 1.6 = \frac{8}{5}$$
$$36\left[\frac{1}{x-4} - \frac{1}{x+4}\right] = \frac{8}{5}$$
$$\left[\frac{x+4-x+4}{(x-4)(x+4)}\right] = \frac{8}{5 \times 36}$$
$$\frac{8}{x^2-16} = \frac{8}{180}$$
$$x^2 - 16 = 180$$
$$x^2 = 196$$
$$\boxed{x = \pm 14}$$

Speed will never be negative, the speed of the boat

$$= 14\ km/hr.$$

**12. Is it possible to design a rectangular park of perimeter 320 m and area 4800 m$^2$? If so find its length and breadth.**

**Solution:**

Given:

Perimeter of the rectangular park $= 360\ m$ and its area $= 4800\ m^2$

Let the length and the breadth of the rectangular park $= l,$ Type equation here.

Perimeter of the Rectangle $= 3201\ m$

$$2(l+b) = 320$$
$$l + b = 160$$
$$\boxed{b = 160 - l}$$

Area of the Rectangle $= 4800\ m^2$

$$l \times b = 4800$$
$$l \times (160 - l) = 4800$$
$$160l - l^2 = 4800$$
$$l^2 - 160l + 4800 = 0$$
$$(l-120)(l-40) = 0$$
$$\boxed{l = 120{,}40}$$
$$b = 160 - 120z$$
$$\boxed{b = 40}$$

The length and the breadth of the Rectangular park $= 120\ m, 40\ m$

**13. At $t$ minutes past 2 pm , the time needed to 3 pm is 3 minutes less than $\frac{t^2}{4}$. Find $t$.**

**Solution:**

Given: The time past after $2\ pm = t$ minutes

And the time need to reach $3\ pm = \left(\frac{t^2}{4} - 3\right)$ minutes

The time between 2 pm and 3 $pm = 1\ hr\ (or)$ 60 minutes

$$t + \frac{t^2}{4} - 3 = 60$$
$$4t + t^2 - 12 = 240$$
$$t^2 + 4t - 252 = 0$$
$$(t+18)(t-14) = 0$$
$$t = -18\ (or)\ 14$$

Neglecting the negative value, $t = 14\ minutes$

**14. The number of seats in a row is equal to the total number of rows in a hall. The total number of seats in the hall will increase by 375 if the number of rows is doubled and the number of seats in each row is reduced by 5 . Find the number of rows in the hall at the beginning.**

**Solution:**

Let the number of rows $= x$;

As per the given condition the number of seats in each row also $= x$

The total number of seats in the hall $= x \times x = x^2$

If the rows are doubled and the seats are reduced by 5 in each row, then the total

seats are increased by 375 more than original.

$$\text{i.e. } 2x(x-5) = x^2 + 375$$
$$2x^2 - 10x = x^2 + 375$$
$$2x^2 - 10x - x^2 - 375 = 0$$
$$x^2 - 10x - 375 = 0$$
$$(x-25)(x+15) = 0$$
$$x = 25\ (or)\ -15$$

Neglecting the negative value, the number of rows in the hall at the beginning $= 25$

**15. If $\alpha$ and $\beta$ are the roots of the polynomial $f(x) = x^2 - 2x + 3$, find the polynomial whose roots are (i) $\alpha + 2, \beta + 2$**

**(ii) $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$.**

**Solution:**

Given: $f(x) = x^2 - 2x + 3$;

Here $a = 1, b = -2, c = 3$

Sum of the roots: $\alpha + \beta = \frac{-b}{a} = \frac{-(-2)}{1} = 2$

Product of the roots : $\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$

(i) To find the equation with the roots of $\alpha + 2, \beta + 2$

Sum of the roots:

$$(\alpha+2) + (\beta+2) = (\alpha+\beta) + 4 = 2 + 4 = 6$$

Product of the roots:

$$(\alpha+2)(\beta+2) = \alpha\beta + 2\alpha + 2\beta + 4$$
$$= \alpha\beta + 2(\alpha+\beta) + 4$$
$$= 3 + 2 \times 2 + 4 = 11$$

The required equation :

$x^2 - (Sum\ of\ \text{roots})x + Product\ of\ the\ roots = 0$

$$x^2 - 6x + 11 = 0$$

(ii) To find the equation with the roots of $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

Sum of the roots:

$$\frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1} = \frac{(\alpha-1)(\beta+1) + (\alpha+1)(\beta-1)}{(\alpha+1)(\beta+1)}$$
$$= \frac{\alpha\beta + \alpha - \beta - 1 + \alpha\beta - \alpha + \beta - 1}{\alpha\beta + \alpha + \beta + 1}$$
$$= \frac{2\alpha\beta - 2}{\alpha\beta + (\alpha+\beta) + 1}$$
$$= \frac{2 \times 3 - 2}{3 + 2 + 1} = \frac{4}{6} = \frac{2}{3}$$

Product of the roots :

$$\frac{\alpha-1}{\alpha+1} \times \frac{\beta-1}{\beta+1} = \frac{(\alpha-1)(\beta+1) + (\alpha+1)(\beta-1)}{(\alpha+1)(\beta+1)}$$
$$= \frac{\alpha\beta - \alpha - \beta + 1}{\alpha\beta + \alpha + \beta + 1}$$
$$= \frac{\alpha\beta - (\alpha+\beta) + 1}{\alpha\beta + (\alpha+\beta) + 1}$$
$$= \frac{3 - 2 + 1}{3 + 2 + 1} = \frac{2}{6} = \frac{1}{3}$$

The required equation :

$x^2 - (\text{ Sum } of\ roots)x + Product\ of\ the\ roots = 0$

$$x^2 - \frac{2}{3}x + \frac{1}{3} = 0$$

Multiplying it by 3 →: $\boxed{3x^2 - 2x + 1 = 0}$

**16. If -4 is a root of the equation $x^2 + px - 4 = 0$ and if the equation $x^2 + px + q = 0$ has equal roots, find the values of $p$ and $q$.**

**Solution:**

Given : $(-4)$ is the root of the eqn. $x^2 + px - 4$

$$f(x) = x^2 + px - 4 = 0$$
$$f(-4) = (-4)^2 + p(-4) - 4 = 0$$
$$16 - 4p - 4 = 0$$
$$4p = 12\ \text{(or)}\ p = 3$$

Also $x^2 + px + q = 0$ has equal roots.

For this equation $a = 1, b = p, c = q$

For equal roots, $b^2 - 4ac = 0$

$$p^2 - 4 \times 1 \times q = 0$$
$$3^2 - 4q = 0$$
$$9 - 4q = 0\ [p = 3]$$
$$4q = 9$$
$$q = \frac{9}{4}$$
$$\boxed{p = 3, q = \frac{9}{4}}$$

**17. Two farmers Thilagan and Kausigan cultivates three varieties of grains namely rice, wheat and ragi. If the sale (in ₹) of three varieties of grains by both the farmers in the month of April is given by the matrix and the May month sale (in ₹) is exactly twice as that of the April month sale for each variety.**
**(i) What is the average sales of the months April and May.**
**(ii) If the sales continues to increase in the same way in the successive months, what will be sales in the month of August?**

**April Sale in ₹**

Rice Wheat Ragi

$$A = \overbrace{\begin{pmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{pmatrix}}^{}\begin{matrix} \textit{Thilagan} \\ \textit{Kausigan} \end{matrix}$$

**Solution:**

Given: April sale in ₹

Rice Wheat Ragi

$$A = \begin{pmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{pmatrix}$$

The sales in the month of May is exactly twice the April.

The sales in the month of May $= 2A$

(i) The average sales of April and May

$$= \frac{A + 2A}{2} = \frac{3A}{2} = \frac{3}{2} \times A$$

$$= \frac{3}{2}\begin{pmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{2} \times 500 & \frac{3}{2} \times 1000 & \frac{3}{2} \times 1500 \\ \frac{3}{2} \times 2500 & \frac{3}{2} \times 1500 & \frac{3}{2} \times 500 \end{pmatrix}$$

$$= \begin{pmatrix} 750 & 1500 & 2250 \\ 3750 & 2250 & 750 \end{pmatrix}$$

(ii) Sales in April = A, May = 2A, June = 4A, July = 8A, August = 16A

Sales in August $= 16A$

$$= 16\begin{pmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{pmatrix}$$

$$= \begin{pmatrix} 16 \times 500 & 16 \times 1000 & 16 \times 1500 \\ 16 \times 2500 & 16 \times 1500 & 16 \times 500 \end{pmatrix}$$

$$= \begin{pmatrix} 8000 & 16000 & 24000 \\ 40000 & 24000 & 8000 \end{pmatrix}$$

**18. If $\cos\theta\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} + \sin\theta\begin{pmatrix} x & -\cos\theta \\ \cos\theta & x \end{pmatrix} = I_2$, find $x$.**

**Solution:**

$$\cos\theta\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} + \sin\theta\begin{pmatrix} x & -\cos\theta \\ \cos\theta & x \end{pmatrix} = I_2$$

$$\cos\theta\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} + \sin\theta\begin{pmatrix} x & -\cos\theta \\ \cos\theta & x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & \cos^2\theta \end{pmatrix} + \begin{pmatrix} x\sin\theta & -\cos\theta\sin\theta \\ \sin\theta\cos\theta & x\sin\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos^2\theta + x\sin\theta & \cos\theta\sin\theta - \cos\theta\sin\theta \\ -\cos\theta\sin\theta + \sin\theta\cos\theta & \cos^2\theta + x\sin\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos^2\theta + x\sin\theta & 0 \\ 0 & \cos^2\theta + x\sin\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\cos^2\theta + x\sin\theta = 1$$

$$x\sin\theta = 1 - \cos^2\theta$$

$$x\sin\theta = \sin^2\theta$$

$$\boxed{x = \sin\theta}$$

**19. Given $A = \begin{pmatrix} p & 0 \\ 0 & 2 \end{pmatrix}, B = \begin{pmatrix} 0 & -q \\ 1 & 0 \end{pmatrix}, C = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}$ and if $BA = C^2$, find $p$ and $q$.**

**Solution:**

$$A = \begin{pmatrix} p & 0 \\ 0 & 2 \end{pmatrix} B = \begin{pmatrix} 0 & -q \\ 1 & 0 \end{pmatrix} C = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}$$

$$BA = C^2$$

$$\begin{pmatrix} 0 & -q \\ 1 & 0 \end{pmatrix}\begin{pmatrix} p & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 + 0 & 0 - 2q \\ +p0 & 0 + 0 \end{pmatrix} = \begin{pmatrix} 4 - 4 & -4 - 4 \\ 4 + 4 & -4 + 4 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -2q \\ p & 0 \end{pmatrix} = \begin{pmatrix} 0 & -8 \\ 8 & 0 \end{pmatrix}$$

Equating Equal Elements

$$p = 8$$

$$-2q = -8$$

$$q = 4$$

$$\boxed{p = 8, q = 4}$$

**20. $A = \begin{pmatrix} 3 & 0 \\ 4 & 5 \end{pmatrix}, B = \begin{pmatrix} 6 & 3 \\ 8 & 5 \end{pmatrix}, C = \begin{pmatrix} 3 & 6 \\ 1 & 1 \end{pmatrix}$ find the matrix $D$, such that $CD - AB = 0$.**

**Solution:**

$$A = \begin{pmatrix} 3 & 0 \\ 4 & 5 \end{pmatrix}, B = \begin{pmatrix} 6 & 3 \\ 8 & 5 \end{pmatrix},$$

$$C = \begin{pmatrix} 3 & 6 \\ 1 & 1 \end{pmatrix} D = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

$$CD = AB \quad [CD = 0 + AB = AB]$$

$$\begin{pmatrix} 3 & 6 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 6 & 3 \\ 8 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 3a + 6c & 3b + 6d \\ a + c & b + d \end{pmatrix} = \begin{pmatrix} 18 + 0 & 9 + 0 \\ 24 + 40 & 12 + 25 \end{pmatrix}$$

$$\begin{pmatrix} 3a + 6c & 3b + 6d \\ a + c & b + d \end{pmatrix} = \begin{pmatrix} 18 & 9 \\ 64 & 37 \end{pmatrix}$$

$$3a + 6c = 18 \Rightarrow \boxed{a + 2c = 6 - - - - -(1)}$$

$$\boxed{a + c = 64 - - - - (2)}$$

$$3b + 6d = 9 \Rightarrow \boxed{b + 2d = 3 - - - - (3)}$$

$$\boxed{b + d = 37 - - - - (4)}$$

$$a + 2c = 6 - - - - (1)$$

$$a + c = 64 - - - - (2)$$

$$(1) - (2) \Rightarrow \boxed{c = -58}$$

$$(2) \Rightarrow a - 58 = 64$$

$$\boxed{a = 122}$$

$$b + 2d = 3 - - - - (3)$$

$$b + d = 37 - - - (4)$$

$$(3) - (4) \Rightarrow \boxed{d = -34}$$

$$(4) \Rightarrow b - 34 = 37$$

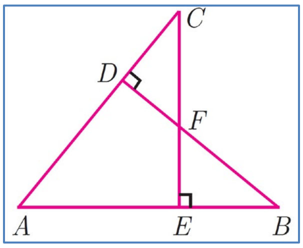
$$\boxed{b = 71}$$

$$\boxed{D = \begin{pmatrix} 122 & 71 \\ -58 & -34 \end{pmatrix}}$$

## CHAPTER – 4 (GEOMETRY)

**1. In the figure, if $BD \perp AC$ and $CE \perp AB$, prove that**

**(i) $\triangle AEC \sim \triangle ADB$ (ii) $\frac{CA}{AB} = \frac{CE}{DB}$**




**Solution:**

(i) Given: $BD \perp AC$ , $CE \perp AB$

In the $\Delta AEC$ and the $\Delta ADB$,

$\angle AEC = \angle ADB = 90^o$ (Given),

Also $\angle A$ is common for both the $\Delta AEC$ and $\Delta ADB$

When two angles are equal,

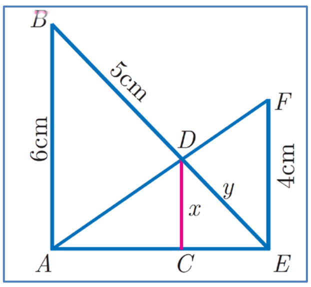
Then the $3^{rd}$ angles are also equal.

By AA Similarity, $\boxed{\Delta AEC \sim \Delta ADB}$ (Hence proved)

(ii) Since $\Delta AEC \sim \Delta ADB$,

$\boxed{\frac{CA}{AB} = \frac{CE}{DB}}$ (Hence proved)

**2. In the given figure $AB \| CD \| EF$. If $AB = 6$ cm, $CD = x$ cm, $EF = 4$ cm, $BD = 5$ cm and $DE = y$ cm. Find $x$ and $y$.**




**Solution:**

Given: $AB \| CD \| EF$

In $\Delta DAB$ and $\Delta DFE$,

$\angle ADB = \angle FDE$ (Vertically opposite angle)

$\angle DAB = \angle DFE$ (Alternate angles are equal $AB \| EF$)

Due to AA Similarity, $\Delta ADB \sim \Delta FDE$

$$\frac{DE}{DB} = \frac{FE}{AB} = \frac{DF}{DA}$$

$$\frac{y}{5} = \frac{4}{6} = \frac{DF}{AD}$$

$$y = \frac{4 \times 5}{6}$$

$$= \frac{10}{3}$$

$$\boxed{y = 3.33\ cm}$$

Let $AB = a = 6\ cm$,

And $EF = b = 4\ cm$,

Then $CD = x = \frac{ab}{a+b}$ (By Height of the Intersection )

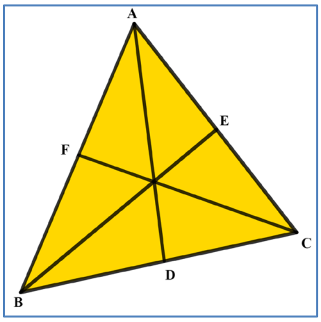
$$x = \frac{6 \times 4}{6 + 4}$$

$$x = \frac{24}{10}$$

$$\boxed{x = 2.4\ cm}$$

**3. O is any point inside a triangle $ABC$. The bisector of $\angle AOB$, $\angle BOC$ and $\angle COA$ meet the sides $AB, BC$ and $CA$ in point $D$, $E$ and $F$ respectively. Show that $AD \times BE \times CF = DB \times EC \times F$**

**Solution:**




In the $\Delta ABC$, mark a point $O$ inside to it at anywhere.

Joint OA, OB

According to the Angular bisector theorem,

OD is the angular bisector of the $\angle AOB$ in the $\Delta AOB$

In the $\Delta AOB$, $\boxed{\frac{AD}{DB} = \frac{AO}{BO} - - - - (1)}$

OE is the angular bisector of the $\angle BOC$ in the $\Delta BOC$

In the $\Delta BOC$, $\boxed{\frac{BE}{EC} = \frac{BO}{CO} - - - - - - (2)}$

OF is the angular bisector of the $\angle COA$ in the $\Delta COA$

In the $\Delta COA$, $\boxed{\frac{CF}{FA} = \frac{CO}{AO} - - - - - - (3)}$

$$(1) \times (2) \times (3)$$

$$\frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA} = \frac{AO}{BO} \times \frac{BO}{co} \times \frac{CO}{AO}$$

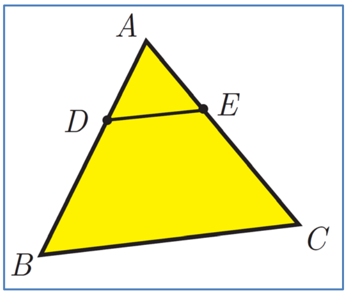
$$\frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA} = 1$$

$$\boxed{AD \times BE \times CF = DB \times EC \times FA}$$

**(Hence Proved)**

**4. In the figure, $ABC$ is a triangle in which $AB = AC$. Points $D$ and $E$ are points on the side $AB$ and $AC$ respectively such that $AD = AE$. Show that the points $B, C, E$ and $D$ lie on a same circle.**

**Solution:**




In the given fig. $AB = AC$ . The $\Delta ABC$ is an isosceles triangle.

$$\boxed{\angle DBC = \angle ECB ----(1)}$$

In the quadrilateral $BCED$, $DE \| BC$ $[AD = AE]$

BD is the transversal of BC and DE,

$$\boxed{\angle EDB + \angle DBC = 180^{\circ} --- (2)}$$

$CE$ is the transversal of BC and DE,

$$\boxed{\angle DEC + \angle ECB = 180^{\circ} ----(3)}$$

From (1) and (2) $\rightarrow \angle EDB + \angle ECB = 180^{\circ}$

From (1) and (3) $\rightarrow \angle DEC + \angle DBC = 180^{\circ}$

From the above two,

**The sum of the opposite angles are $180^{\circ}$**

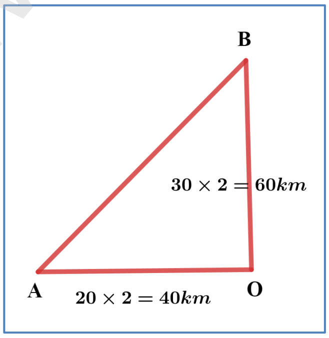
The quadrilateral BCED lies on a same circle.

∴BCED is a cyclic quadrilateral.

**(Hence Proved)**

**5. Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels at a speed of 20 km/hr and the second train travels at 30 k/hr. After 2 hours, what is the distance between them?**

**Solution:**




Let $O$ be the Railway Station.

From $O$, the train A departures towards (due) west at a speed of 20 $km/hr$

After 2 hour, the train A is at $20 \times 2 = 40$ $km$ from O.

From $O$, the Train $B$ departures towards (due) north at a speed of 30 $km/hr$

After 2 hour, the train $B$ is at $30 \times 2 = 60$ $km$ from O.

Now the points $A$, $O$, and $B$ are form a right $\Delta AOB$. $AB^2 = AO^2 + BO^2$ $AB = \sqrt{AO^2 + BO^2}$

$$AB = \sqrt{40^2 + 60^2}$$

$$= \sqrt{1600 + 3600}$$

$$= \sqrt{5200}$$

$$= \sqrt{400 \times 13}$$

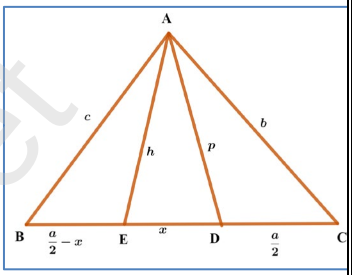
$$\boxed{AB = 20\sqrt{13}} \text{ km}$$

**6. $D$ is the mid point of side $BC$ and $AE \perp BC$. If $BC = a, AC = b, AB = c, ED = x, AD = p$ and $AE = h$, prove that**

**(i) $b^2 = p^2 + ax + \frac{a^2}{4}$ (ii) $c^2 = p^2 - ax + \frac{a^2}{4}$**

**(iii) $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$**

**Solution:**




In △ $ABC$ Midpoint of BC is $D$ and $AE \perp BC$.

$BC = a, AC = b, AB = c,$

$ED = x, AD = p, AE = h$

$BD = DC = \frac{a}{2}$ $ED = x,$

$BE = \frac{a}{2} - x.$

$AE \perp BC$ In $\Delta AED$ $AD^2 = AE^2 + ED^2$

$$p^2 = h^2 + x^2 \Rightarrow \boxed{h^2 = p^2 - x^2 ----(1)}$$

(i). In $\Delta AEC$ $AC^2 = AE^2 + EC^2$

$$b^2 = h^2 + \left(x + \frac{a}{2}\right)$$

$$b^2 = p^2 - x^2 + \left(x + \frac{a}{2}\right)^2 \text{ [From (1)]}$$

$$b^2 = p^2 - x^2 + x^2 + ax + \frac{a^2}{4}$$

$$\boxed{b^2 = p^2 + ax + \frac{a^2}{4} ----(2)}$$

(ii). In $\Delta AEB$ $AB^2 = AE^2 + EB^2$

$$c^2 = h^2 + \left(\frac{a}{2} - x\right)^2$$

$$c^2 = p^2 - x^2 + \left(\frac{a}{2} - x\right)^2 \text{ [ From (1) ]}$$

$$c^2 = p^2 - x^2 + \frac{a^2}{4} - ax + x^2$$

$$\boxed{c^2 = p^2 - ax + \frac{a^2}{4} ----(3)}$$

(iii). From (2) and (3)

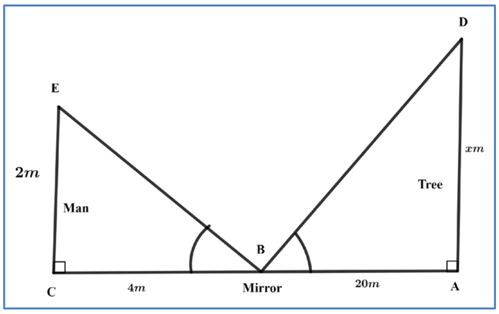
$$b^2 + c^2 = p^2 + ax + \frac{a^2}{4} + p^2 - ax + \frac{a^2}{4}$$

$$= 2p^2 + 2\left(\frac{a^2}{4}\right)$$

$$\boxed{b^2 + c^2 = 2p^2 + \frac{a^2}{2}}$$

**7. A man whose eye-level is 2 m above the ground wishes to find the height of a tree. He places a mirror horizontally on the ground 20 m from the tree and finds that if he stands at a point $C$ which is 4 m from the mirror $B$, he can see the reflection of the top of the tree. How height is the tree?**

**Solution:**




From the fig. Man's eyelevel $CE = 2\ m$;

Let the tree's height $AD = x\ m$

$B$ is the mirror point. Now DB is the incidental ray, BE is the reflected ray.

$$\angle ABD = \angle CBE$$

Also $\angle BAD = \angle BCE = 90^\circ$ ($\cdot\perp$ to the ground)

By AA similarity, $\Delta BAD \sim \Delta BCE$,

$$\frac{AD}{CE} = \frac{BA}{BC}$$

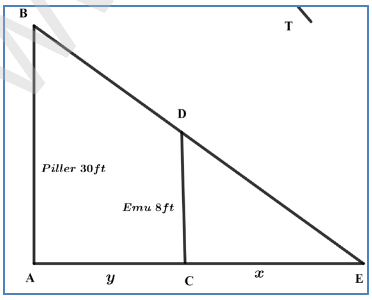
$$\frac{x}{2} = \frac{20}{4} \text{ (or)}$$

$$x = \frac{20 \times 2}{4} = 10m$$

The height of the tree $= 10\ m$

**8. An Emu which is 8 feet tall is standing at the foot of a pillar which is 30 feet high. It walks away from the pillar. The shadow of the Emu falls beyond Emu. What is the relation between the length of the shadow and the distance from the Emu to the pillar?**

**Solution:**




$AB = 30ft$ is the pillar with a light at top.

If the emu ($CD = 8ft$) is walking away from the foot of the pillar,

Then it's shadow is in front of it.

The shadow length of the emu is based on it's distance from the light pillar.

$AB$ and $CD$ are $\perp$ to ground and the $\angle E$ is common, the $\Delta ECD \sim \Delta EAB$

$$\frac{EC}{EA} = \frac{CD}{AB} \rightarrow \frac{x}{x+y} = \frac{8}{30}$$

$$30x = 8x + 8y$$

$$22x = 8y$$

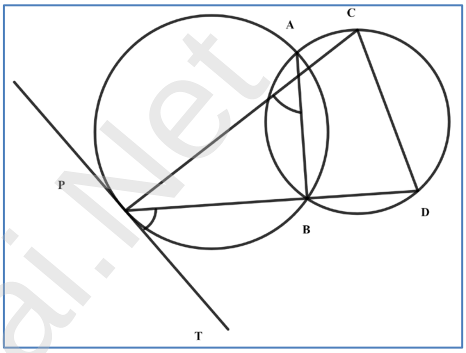
$$x = \frac{8y}{22}$$

$$= \frac{4}{11} \times y$$

Length of the shadow $= \frac{4}{11} \times$ Distance of the emu from the pillar.

**9. Two circles intersect at $A$ and $B$. From a point $P$ on one of the circles lines $PAC$ and $PBD$ are drawn intersecting the second circle at $C$ and $D$. Prove that $CD$ is parallel to the tangent at $P$**

**Solution:**




$PT$ is the tangent of the circle.

According to the Alternate segment theorem

$$\angle YPB = \angle PAB ----(1)$$

Since the quadrilateral ABCD is a cyclic on circle

The sum of the opposite angles $= 180^\circ$

Also The exterior angle $=$ The opposite interior angle

$$\angle PDC = \angle PAB ----(2)$$

Comparing (1) and (2),

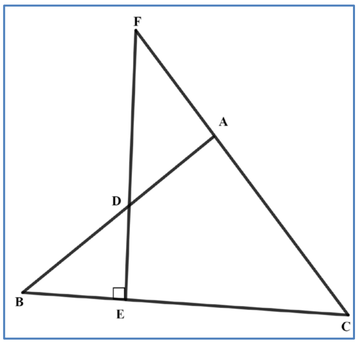
$\angle PCD = \angle YPB$ [Alternate angles are equal]

CD is parallel to the tangent $PT$ at P.

(Hence Proved)

**10. Let $ABC$ be a triangle and D, E, F are points on the respective sides $AB, BC, AC$ (or their extensions). Let $AD: DB = 5:3, BE: EC = 3:2$ and $AC = 21$. Find the length of the line segment $CF$.**

**Solution:**



Given: $AD: DB = 5:3$, $BE: EC = 3:2$, $AC = 21$ unit.

$$\frac{AD}{DB} = \frac{5}{3}, \frac{BE}{EC} = \frac{3}{2}$$

In the $\Delta ABC$, $D, E$ and $F$ are on the sides AB, BC and CA .

According to Menelaus theorem, for the collinearity of $D, E$ and $F$

$$\frac{BE}{EC} \times \frac{CF}{FA} \times \frac{AD}{DB} = -1$$

$$\frac{3}{2} \times \frac{CF}{FC - AC} \times \frac{5}{3} = -1$$

$$\frac{CF}{(-CF) - 21} \times \frac{5}{2} = -1$$

$$\frac{CF}{(-CF) - 21} = -\frac{2}{5}$$

$$5\,CF = 2CF + 42$$

$$5\,CF - 3CF = 42$$

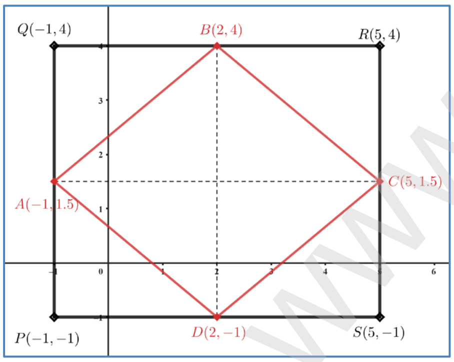
$$3\,CF = 42$$

$$\boxed{CF = 14}\text{ Units}$$

## CHAPTER – 5 (COORDINATE GEOMETRY)

**1. PQRS is a rectangle formed by joining the points $P(-1,-1), Q(-1,4), R(5,4)$ and $S(5,-1)$. $A, B, C$ and $D$ are the mid-points of $PQ, QR, RS$ and $SP$ respectively. Is the quadrilateral $ABCD$ a square, a rectangle or a rhombus? Justify your answer.**

**Solution:**




Given: PQRS is a rectangle,

Their points are $P(-1,-1), Q(-1,4), R(5,4)$ and $S(5,-1)$

$A, B, C$ and $D$ are the mid-points of PQ, $QR$, RS and SP respectively.

Mid Point of PQ $= A\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = A\left(-1, \frac{3}{2}\right)$

Mid Point of QR $= B\left(\frac{-1+5}{2}, \frac{4+4}{2}\right) = B(2, 4)$

Mid Point of $RS = C\left(\frac{5+5}{2}, \frac{4-1}{2}\right) = C\left(5, \frac{3}{2}\right)$

Mid Point of $PS = D\left(\frac{-1+5}{2}, \frac{-1-1}{2}\right) = D(2, -1)$

$$AB = \sqrt{(2+1)^2 + \left(4 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$BC = \sqrt{(5-2)^2 + \left(\frac{3}{2} - 4\right)^2} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$CD = \sqrt{(2-5)^2 + \left(-1 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$AD = \sqrt{(-1-2)^2 + \left(\frac{3}{2} + 1\right)} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$AC = \sqrt{(5+1)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)} = \sqrt{36 + 0} = 6$$

$$BD = \sqrt{(2-2)^2 + (-1-4)^2} = \sqrt{0 + 25} = 5$$

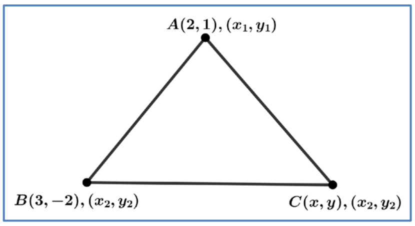
$$AB = BC = CD = AD = \frac{\sqrt{61}}{2}$$

Diagonal $AC \neq BD$

Hence ABCD is a Rhombus.

**2. The area of a triangle is 5 sq.units. Two of its vertices are $(2,1)$ and $(3,-2)$. The third vertex is $(x, y)$ where $y = x + 3$. Find the coordinates of the third vertex.**

**Solution:**




Vertices of the Triangle

$(2,1)\ (3,-2), (x, y)$

Given $\boxed{y = x + 3 - - - - (1)}$

Area of triangle $= 5$ sq.units

$$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} = 5$$

$$\frac{1}{2}\begin{vmatrix} 2 & 3 & x & 2 \\ 1 & -2 & y & 1 \end{vmatrix} = 5$$

$$[(-4 + 3y + x) - (3 - 2x + 2y)] = 10$$

$$[-4 + 3y + x - 3 + 2x - 2y] = 10$$

$$3x + y - 7 = 10$$

$$3x + y = 17$$

$$3x + x + 3 = 17$$

$$4x = 14$$

$$\boxed{x = \frac{7}{2}}$$

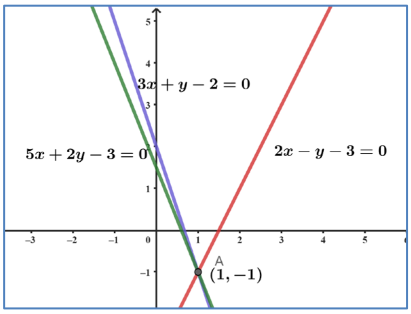
$$y = \frac{7}{2} + 3$$

$$\boxed{y = \frac{13}{2}}$$

Third Vertices of the Triangle $\left(\frac{7}{2}, \frac{13}{2}\right)$

**3. Find the area of a triangle formed by the lines $3x + y - 2 = 0, 5x + 2y - 3 = 0$ and $2x - y - 3 = 0$**

**Solution:**




$$3x + y = 2 ----(1)$$
$$5x + 2y = 3 ---(2)$$
$$2x - y = 3 ----(3)$$

Solve (1) and (2)

$$(1) \times 2 \Rightarrow 6x + 2y = 4 ----(4)$$
$$5x + 2y = 3 ----(2)$$

$(4) - (2) \Rightarrow \boxed{x = 1}$

$$(1) \Rightarrow 3(1) + y = 2 \Rightarrow \boxed{y = -1}$$

**Intersection Points $(1, -1)$**

Solve (2) and (3)

$$5x + 2y = 3 ----(2)$$
$$(3) \times 2 \Rightarrow 4x - 2y = 6 ----(5)$$

$(2) + (5) \Rightarrow 9x = 9 \Rightarrow \boxed{x = 1}$

$$(3) \Rightarrow 2(1) - y = 3 \Rightarrow \boxed{y = -1}$$

**Intersection Points $(1, -1)$**

Solve (1) and (3)

$$3x + y = 2 ----(1)$$
$$2x - y = 3 ----(3)$$

$(1) + (3) \Rightarrow 5x = 5 \rightarrow \boxed{x = 1}$

$$(1) \Rightarrow 3(1) + y = 2 \Rightarrow \boxed{y = -1}$$

**Intersection Points $(1, -1)$**

Three points are same line and Concurrent.

So, No triangle formed.

Area of the triangle $= 0\ sq.units$.

**4. If vertices of a quadrilateral are at $A(-5, 7), B(-4, k), C(-1, -6)$ and $D(4, 5)$ and its area is 72 sq. units. Find the value of $k$.**

**Solution:**

Given Vertices of a Quadrilateral are

$A(-5,7)$, $B(-4,k)$, $C(-1,-6)$ $D(4,5)$

Area of the Quadrilateral $= 72$ Sq.units

$$\frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix} = 72$$

$$\frac{1}{2}\begin{vmatrix} -5 + 1 & -4 - 4 \\ 7 + 6 & k - 5 \end{vmatrix} = 72$$

$$\begin{vmatrix} -4 & -8 \\ 13 & k - 5 \end{vmatrix} = 144$$

$$-4k + 20 + 104 = 144$$
$$-4k + 124 = 144$$
$$-4k = 20$$
$$\boxed{k = -5}$$

**5. Without using distance formula, show that the points $(-2, -1), (4, 0), (3, 3)$ and $(-3, 2)$ are vertices of a parallelogram.**

**Solution:**

$A(-2,-1)$, $B(4,0)$, $C(3,3)$ , $D(-3,2)$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Slope of AB} = \frac{0 + 1}{4 + 2} = \frac{1}{6}$$

$$\text{Slope of BC} = \frac{3 - 0}{3 - 4} = \frac{3}{-1} = -3$$

$$\text{Slope of CD} = \frac{2 - 3}{-3 - 3} = \frac{-1}{-6} = \frac{1}{6}$$

$$\text{Slope of AD} = \frac{2 + 1}{-3 + 2} = \frac{3}{-1} = -3$$

$$\text{Slope of AB} = \text{Slope of CD} = \frac{1}{6}$$

$$\text{Slope of BC} = \text{Slope of AD} = -3$$

$AB \parallel CD$ and $BC \parallel AD$.

Hence the vertices form a Parallelogram.

**6. Find the equations of the lines, whose sum and product of intercepts are 1 and -6 respectively.**

**Solution:**

Let $x$ intercept $a$ and $y$ intercept $b$.

Sum of two intercepts $a + b = 1$

Product of two intercepts $ab = -6$.

$$ab = -6$$
$$a(1 - a) = -6\ [\because b = 1 - a]$$
$$a - a^2 = -6$$
$$a^2 - a - 6 = 0$$
$$(a - 3)(a + 2) = 0$$
$$a = 3\ a = -2$$
$$\text{If } a = 3\ \ b = -2$$
$$\text{If } a = -2\ \ b = 3$$

The Equation of line Intercept form $\boxed{\frac{x}{a} + \frac{y}{b} = 1}$

If $a = 3$ and $y = -2$ Sub Eqn. of Straight Line

$$\frac{x}{3} + \frac{y}{-2} = 1$$
$$\frac{x}{3} - \frac{y}{2} = 1$$

$$\boxed{2x-3y-6=0}$$

If $a=-2$ and $y=3$ Sub Eqn. of Straight Line

$$\frac{x}{-2}+\frac{y}{3}=1$$
$$\frac{-x}{2}+\frac{y}{3}=1$$
$$\boxed{3x-2y+6=0}$$

**7. The owner of a milk store finds that, he can sell 980 litres of milk each week at ₹ 14 /litre and 1220 litres of milk each week at ₹16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at ₹17/litre?**

**Solution:**

Take a Points are $(14,980),(16,1220)$

The Eqn. of Straight Line (**Two Point Form**)

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$
$$\frac{y-980}{1220-980}=\frac{x-14}{16-14}$$
$$\frac{y-980}{240}=\frac{x-14}{2}$$
$$y-980=120(x-14)$$
$$y-980=120x-1680$$
$$y=120x-700$$

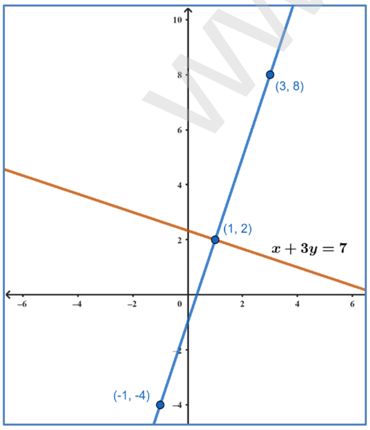
If $x=17$ $y=120(17)-700$

$$y=2040-300$$
$$\boxed{y=1340}$$

The milk Owner could sell 1340 lit milk at the rate of ₹ 17 weekly.

**8. Find the image of the point $(3,8)$ with respect to the line $x+3y=7$ assuming the line to be a plane mirror.**

**Solution:**




Given: The line of the mirror: $x+3y=7$.

The coordinate of the object Point: $A(3,8)$

The Eqn. of Mirror $x+3y=7---(1)$

Let the Image Point of P $Q(h,k)$

Object and it's image are always equidistant from the mirror perpendicularly.

The perpendicular line of the mirror: $3x-y+k=0$

It passes through the object point: $A(3,8)$

$3\times3-8+k=0,=k=-1$

The perpendicular eqn. is $3x-y-1=0$

$$3x-y=1---(2)$$
$$(1)\times3\to 3x+9y=21----(3)$$
$$3x-y=1----(2)$$
$$(3)-(2)\to 10y=20$$
$$\boxed{y=2}$$
$$(1)\to\ x+3(2)=7\Rightarrow \boxed{x=1}$$

AB and PQ Intersect points are $R(1,2),P(3,8),Q(h,k)$

Mid Point $(1,2)$

$$\left(\frac{3+h}{2}\ \frac{8+k}{2}\right)=(1,2)$$
$$\frac{3+h}{2}=1\Rightarrow h=-1$$
$$\frac{8+k}{2}=2\Rightarrow k=-4$$

$P(3,8)$ Image points is $\boxed{Q(-1,-4)}$

**9. Find the equation of a line passing through the point of intersection of the lines $4x+7y-3=0$ and $2x-3y+1=0$ that has equal intercepts on the axes.**

**Solution:**

$$4x+7y=3----(1)$$
$$2x-3y=-1---(2)$$
$$(2)\times2\to 4x-6y=-2---(3)$$
$$4x+7y=3---(1)$$
$$(3)-(1)\to -13y=-5$$
$$\boxed{y=\frac{5}{13}}$$
$$(2)\to 2x-3\left(\frac{5}{13}\right)=-1$$
$$2x-\frac{15}{13}=-1$$
$$2x=-1+\frac{15}{13}=\frac{2}{13}$$
$$\boxed{x=\frac{1}{13}}$$

So the line Intersect Point $\left(\frac{1}{13},\frac{5}{13}\right)$

$x$ and $y$ Intercept are Equal $a=b$

$$\frac{x}{a}+\frac{y}{b}=1$$
$$\frac{x}{a}+\frac{y}{a}=1\ [a=b]$$
$$\boxed{x+y=a----(1)}$$

Passing Through the Points $\left(\frac{1}{13}\frac{5}{13}\right)$

$$\frac{1}{13}+\frac{5}{13}=a$$

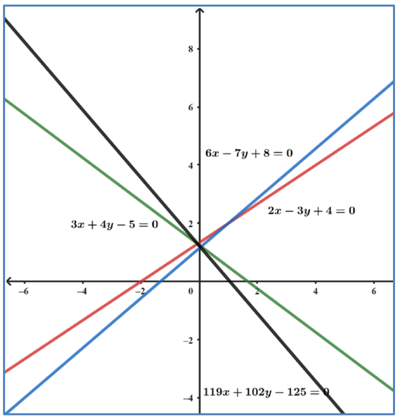
$$a = \frac{6}{13}$$

$$(1) \Rightarrow x + y = \frac{6}{13}$$

$$\boxed{13x + 13y - 6 = 0}$$

**10. A person standing at a junction (crossing) of two straight paths represented by the equations $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ seek to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. Find the equation of the path that he should follow.**

**Solution:**




$$2x - 3y = -4 - - - (1)$$

$$3x + 4y = 5 - - - (2)$$

$$(1) \times 3 \Rightarrow 6x - 9y = -12 - - - -(3)$$

$$(2) \times 2 \Rightarrow 6x + 8y = 10 - - - -(4)$$

$$(3) - (4) \Rightarrow -17y = -22$$

$$\left(y = \frac{22}{17}\right)$$

$$(1) \Rightarrow 2x - 3\left(\frac{22}{17}\right) = -4$$

$$2x - \frac{66}{17} = -4$$

$$2x = -4 + \frac{66}{17} = -\frac{2}{17}$$

$$\left(x = -\frac{1}{17}\right)$$

The Point of intersection $\left(-\frac{2}{17}, \frac{22}{17}\right)$

Given Eqn. $6x - 7y + 8 = 0$

The perpendicular of this *eqn.* which is passing through the point of intersection gives the shortest distance.

The perpendicular eqn. of the $3^{rd}$ *eqn.* is

$$-7x - 6y + k = 0$$

It passes through the point $\left(-\frac{1}{17}, \frac{22}{17}\right)$

$$7\left(\frac{-1}{17}\right) + 6\left(\frac{22}{17}\right) + k = 0$$

$$\frac{-7}{17} + \frac{132}{17} + k = 0$$

$$k = \frac{125}{17}$$

The Required Equation of the Path

$$7x + 6y - \frac{125}{17} = 0$$

$$\boxed{119x + 102y - 125 = 0}$$

## CHAPTER – 6 (TRIGONOMETRY)

**1. Prove that**

**(i) $\cot^2 A\left(\frac{\sec A - 1}{1 + \sin A}\right) + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right) = 0$**

**(ii) $\frac{\tan^2 \theta - 1}{\tan^2 \theta + 1} = 1 - 2\cos^2 \theta$**

**Solution:**

**(i)** $\cot^2 A\left(\frac{\sec A - 1}{1 + \sin A}\right) + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right) = 0$

LHS : $\cot^2 A\left(\frac{\sec A - 1}{1 + \sin A}\right) + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right)$

$$= \frac{1}{\tan^2 A}\left(\frac{\sec A - 1}{1 + \sin A}\right) + \frac{1}{\cos^2 A}\left(\frac{\sin A - 1}{1 + \sec A}\right)$$

$$= \frac{1}{\sec^2 A - 1}\left(\frac{\sec A - 1}{1 + \sin A}\right) + \frac{l}{1 - \sin^2 A}\left(\frac{\sin A - 1}{1 + \sec A}\right)$$

$$= \frac{1}{(\sec A + 1)(\sec A - 1)}\left(\frac{\sec A - 1}{1 + \sin A}\right) - \frac{1}{(1 + \sin A)(1 - \sin A)}\left(\frac{1 - \sin A}{1 + \sec A}\right)$$

$$= \frac{1}{(\sec A + 1)(1 + \sin A)} - \frac{1}{(1 + \sin A)(1 + \sec A)}$$

$= 0$

$=$ RHS

**(ii)** $\frac{\tan^2\theta - 1}{\tan^2\theta + 1} = 1 - 2\cos^2\theta$

LHS: $\frac{\tan^2\theta - 1}{\tan^2\theta + 1} = \frac{\tan^2\theta - 1}{\sec^2\theta}$

$$= (\tan^2\theta - 1)\cos^2\theta$$

$$= \left(\frac{\sin^2\theta}{\cos^2\theta} - 1\right)\cos^2\theta$$

$$= \left(\frac{\sin^2\theta - \cos^2\theta}{\cos^2\theta}\right)\cos^2\theta$$

$$= \sin^2\theta\cos^2\theta$$

$$= 1 - \cos^2\theta\cos^2\theta$$

$$= 1 - 2\cos^2\theta$$

$= RHS$

**2. Prove that $\left(\frac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta}\right)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$**

**Solution:**

$$\left(\frac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta}\right)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$$

LHS : $\left(\frac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta}\right)^2 = \frac{(1 + \sin\theta - \cos\theta)^2}{(1 + \sin\theta + \cos\theta)^2}$

$$= \frac{1 + \sin^2\theta + \cos^2\theta + 2\sin\theta - 2\sin\theta\cos\theta - 2\cos\theta}{1 + \sin^2\theta + \cos^2\theta + 2\sin\theta + 2\sin\theta\cos\theta + 2\cos\theta}$$

$[\because\ (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

$$= \frac{1 + 1 + 2\sin\theta - 2\sin\theta\cos\theta - 2\cos\theta}{1 + 1 + 2\sin\theta + 2\sin\theta\cos\theta + 2\cos\theta}$$
$$= \frac{2 - 2\cos\theta + 2\sin\theta - 2\sin\theta\cos\theta}{2 + 2\cos\theta + 2\sin\theta + 2\sin\theta\cos\theta}$$
$$= \frac{2(1 - \cos\theta) + 2\sin\theta(1 - \cos\theta)}{2(1 + \cos\theta) + 2\sin\theta(1 + \cos\theta)}$$
$$= \frac{(1 - \cos\theta)(2 + 2\sin\theta)}{(1 + \cos\theta)(2 + 2\sin\theta)}$$
$$= \frac{(1 - \cos\theta)}{(1 + \cos\theta)}$$
$= \text{RHS}$

**3. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, then prove that $x^2 + y^2 = 1$.**

**Solution:**

$$x\sin^3\theta + y\cos^3\theta = \sin\theta\cos - - - -(1)$$
$$x\sin\theta = y\cos\theta - - - -(2)$$
$$x\sin\theta(\sin^2\theta) + y\cos\theta(\cos^2\theta) = \sin\theta\cos\theta$$
$$x\sin\theta(\sin^2\theta) + x\sin\theta(\cos^2\theta) = \sin\theta\cos\theta$$
$[\,From\ (2) : x\sin\theta = y\cos\theta]$
$$x\sin\theta[\sin^2\theta + \cos^2\theta] = \sin\theta\cos\theta$$
$$x\sin\theta(1) = \sin\theta\cos\theta$$
$$x = \cos\theta - - - -(3)$$
$$(2) \Rightarrow x\sin\theta = y\cos\theta$$
$$\cos\theta\sin\theta = y\cos\theta$$
$$y = \sin\theta - - - -(4)$$
Add $(3)^2 + (4)^2$
$$x^2 + y^2 = \cos^2\theta + \sin^2\theta$$
$$\boxed{x^2 + y^2 = 1}$$

**4. If $a\cos\theta - b\sin\theta = c$, then prove that $(a\sin\theta + b\cos\theta) = \pm\sqrt{a^2 + b^2 - c^2}$.**

**Solution:**

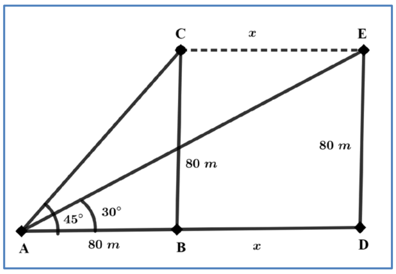
$$a\cos\theta - b\sin\theta = c$$
Squaring on Both Side
$$(a\cos\theta - b\sin\theta)^2 = c^2$$
$$a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta = c^2$$
$$a^2(1 - \sin^2\theta) + b^2(1 - \cos^2\theta) - 2ab\cos\theta\sin\theta = c^2$$
$$a^2 - a^2\sin^2\theta + b^2 - b^2\cos^2\theta - 2ab\cos\theta\sin\theta = c^2$$
$$a^2\sin^2\theta + b^2\cos^2\theta + 2ab\cos\theta\sin\theta = a^2 + b^2 - c^2$$
$$(a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$$
$$(a\sin\theta + b\cos\theta) = \pm\sqrt{a^2 + b^2 - c^2}$$

**5. A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such away that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Determine the speed at which the bird flies. ($\sqrt{3} = 1.732$)**

**Solution:**

Let the bird is sitting initially at $C$ which is 80 $m$ high.

The angle of elevation of $C$, i.e. $\angle BAC = 45^\circ$

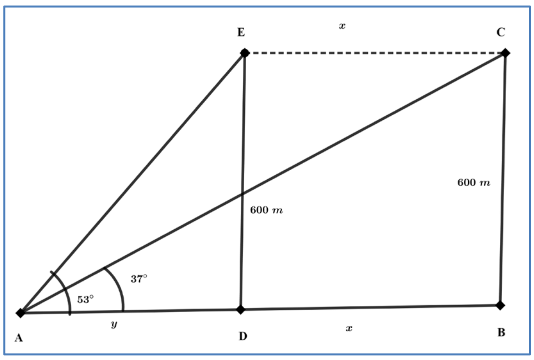



$$\tan 45^\circ = \frac{BC}{AB}$$
$$1 = \frac{80}{AB}, AB = 80\ m$$
Then the bird is flying horizontally $x\ m$ in 2 seconds
$$CE = BD = x\ m$$
Now the angle of elevation of $E$ i.e. $\angle DAE = 30^\circ$
$$\tan 30^\circ = \frac{DE}{AD} = \frac{DE}{AB + BD}$$
$$\frac{1}{\sqrt{3}} = \frac{80}{80 + x}$$
$$80 + x = \sqrt{3} \times 80$$
$$x = \sqrt{3} \times 80 - 80$$
$$x = 80(\sqrt{3} - 1) = 80(1.732 - 1)$$
$$x = 80 \times 0.732 = 58.56m$$
Distance travelled by the bird $x = 58.56\ m$ ;

Time taken for it = 2 seconds
$$\text{Speed of the bird} = \frac{Distance\ travelled}{Time\ taken}$$
$$= \frac{58.56}{2}$$
$$\boxed{= 29.28\ m/sec}.$$

**6. An aeroplane is flying parallel to the Earth's surface at a speed of 175 m/sec and at a height of 600 m . The angle of elevation of the aeroplane from a point on the Earth's surface is 37°. After what period of time does the angle of elevation increase to 53°? ($\tan 53^\circ = 1.3270, \tan 37^\circ = 0.7536$)**

**Solution:**




Let the plane be at $C$ initially which is 600 $m$ high.

The angle of elevation $\angle BAC = 37^\circ$
$$\tan 37^\circ = \frac{BC}{AB}$$

$$0.7536 = \frac{600}{x+y}$$

$$x+y = \frac{600}{0.7536} = 796.18\ m$$

Then the plane is flying horizontally $x\ m$ to the point E.

$$CE = BD = x\ m$$

Now the angle of elevation $\angle DAE = 53^\circ$;

$DE = BC = 600\ m$

$$\tan 53^\circ = \frac{DE}{AD}$$

$$1.3270 = \frac{600}{y}$$

$$y = \frac{600}{1.327} = 452.15\ m$$

$$x + y = 796.18\ m$$

$$x = 796.18 - y$$

$$x = 796.18 - 452.15 = 344.03\ m$$

Distance travelled by the plane $x = 344.03\ m$ ;

Speed of the plane $= 175\ m/seconds$

$$Time\ taken = \frac{Distance\ travelled}{Speed} = \frac{344.03}{175} = 1.97\ seconds$$

**7. A bird is flying from $A$ towards $B$ at an angle of $35°$, a point 30 km away from $A$. At $B$ it changes its course of flight and heads towards $C$ on a bearing of $48°$ and distance 32 km away.**

**(i) How far is $B$ to the North of $A$ ?**

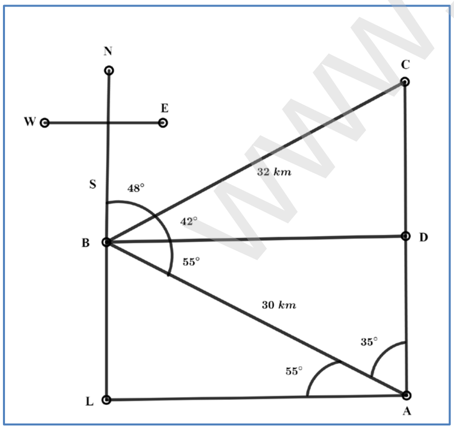
**(ii) How far is $B$ to the West of $A$ ?**

**(iii) How far is $C$ to the North of $B$ ?**

**(iv) How far is $C$ to the East of $B$ ?**

**$\left(\begin{array}{l}\sin 55^\circ = 0.8192, \cos 55^\circ = 0.5736,\\ \sin 42^\circ = 0.6691, \cos 42^\circ = 0.7431\end{array}\right)$**

**Solution:**




Let A be a bird Starting Position .

Let B from A to ∠ 35° Distance of Bird is 30 km.

Let C be a Distance from A point 32 km of bird ∠ 48°

(i) In Right angle triangle ABD

$$\sin 55^\circ = \frac{AD}{AB}$$

$$0.8192 = \frac{AD}{30}$$

$$AD = 30 \times 0.8192$$

$$AD = 24.58\ km$$

The distance of B to the North of A $= 24.58\ km$

(ii) In Right angle triangle ALB

$$\cos 55^\circ = \frac{AL}{AB}$$

$$0.5736 = \frac{AL}{30}$$

$$AL = 30 \times 0.5736$$

$$AL = 17.21\ km$$

The distance of B to the West of A $= 17.21\ km$

(iii) In Right angle triangle BDC

$$\sin 42^\circ = \frac{CD}{BC}$$

$$0.6691 = \frac{CD}{32}$$

$$CD = 32 \times 0.6691$$

$$CD = 21.41\ km$$

The distance of C to the North of B $= 21.41\ km$

(iv) In Right angle triangle BCD

$$\cos 42^\circ = \frac{BD}{BC}$$

$$0.7431 = \frac{BD}{32}$$

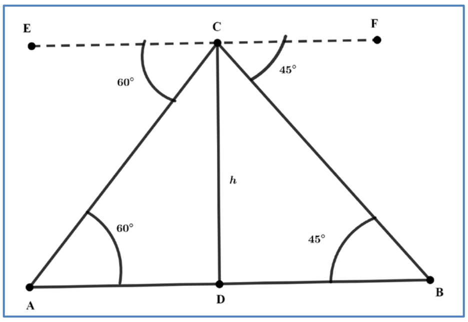
$$BD = 32 \times 0.7431$$

$$\boxed{BD = 23.78\ km}$$

The distance of C to the East of B $= 23.78\ km$

**8. Two ships are sailing in the sea on either side of the lighthouse. The angles of depression of two ships as observed from the top of the lighthouse are $60°$ and $45°$ respectively. If the distance between the ships is $200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$ metres, find the height of the lighthouse. of the lighthouse.**

**Solution:**




Let A and $B$ be the two ships on the either side of the light house CD

Distance between two ships $AB = 200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) m$

The angle of depressions from the top light house are $60^\circ$ and $45^\circ$

The angle of elevation from A is $60^\circ$

The angle of elevation from $B$ is $45^\circ$

Let the height of light house CD be $h\ m$.

In the right angle $\Delta ADC$, $\tan 60^{\circ} = \frac{h}{AD}$

$$\sqrt{3} = \frac{h}{AD} \quad AD = \frac{h}{\sqrt{3}}$$

In the right angle $\Delta BDC$, $\tan 45^{\circ} = \frac{h}{BD}$

$$1 = \frac{h}{BD}, BD = h$$

$$AD + BD = \frac{h}{\sqrt{3}} + h$$

$$AB = h\left(\frac{1}{\sqrt{3}} + 1\right)$$

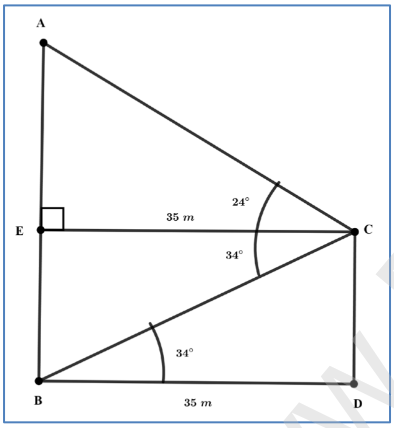
$$200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) = h\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$$

$$\boxed{h = 200\ m}$$

The height of the light house $= 200\ m$

**9. A building and a statue are in opposite side of a street from each other 35 m apart. From a point on the roof of building the angle of elevation of the top of statue is 24° and the angle of depression of base of the statue is 34°. Find the height of the statue. $\left(\begin{matrix}\tan 24^{\circ} = 0.4452, \\ \tan 34^{\circ} = 0.6745\end{matrix}\right)$**

**Solution:**




AB is the width of the street $= 35\ m$;

AD is the Building; BC is the height of the statue.

From the top of the building,

The angle of elevation to the top of the statue $= 24^{\circ}$

The angle of depression to the bottom of the statue $= 34^{\circ}$

The angle of elevation from $B$ to the top of the building $= 34^{\circ}$

In the *right* $\Delta BAD$, $\tan 34^{\circ} = \frac{AD}{AB}$

$$0.6745 = \frac{AD}{35}$$

$$AD = 35 \times 0.6745 = 23.61\ m$$

$$BE = AD = 23.61\ m$$

In the *right* $\Delta DEC$, $\tan 24^{\circ} = \frac{EC}{DE}$

$$0.4452 = \frac{EC}{35}$$

$$EC = 35 \times 0.4452 = 15.58\ m$$

Height of the statue $= BE + EC$

$$= 23.61 + 15.58$$

$$\boxed{= 39.19\ m.}$$

The height of the Statue $= 39.19\ m$

## CHAPTER – 7 (MENSURATION)

**1. The barrel of a fountain-pen cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen will be used for writing 330 words on an average. How many words can be written using a bottle of ink containing one fifth of a litre?**

**Solution:**

Given: Pen's Cylindrical barrel length = 7 cm,

Diameter = 5 mm (or) 0.5 cm, Radius = 0.25 cm

Volume of the ink bottle $= \left(\frac{1}{5}\right)^{th}$ of 1 litre;

Number of words written in 1 barrel = 330

Vol. of Cylindrical barrel $= \pi r^2 h$

$$= \frac{22}{7} \times 0.25 \times 0.25 \times 7 = 22 \times 0.25 \times 0.25\ cm^3$$

$$\text{Vol. of ink bottle} = \left(\frac{1}{5}\right)^{th} \text{of } 1\ litre = \frac{1}{5} \times 1000$$

$$= 200\ ml\ (or)\ 200\ cm^3$$

Number of barrels to be filled up

$$= \frac{\text{Vol.of ink bottle}}{\text{Vol.of 1 Cylindrical barrel}} = \frac{200}{22 \times 0.25 \times 0.25}$$

Number of words to be written = Number of barrels × Number of words written in 1 barrel

$$= \frac{200}{22 \times 0.25 \times 0.25} \times 330$$

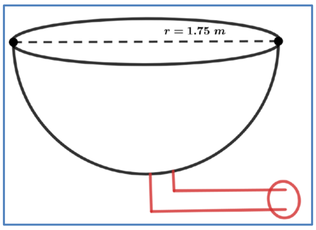
$$= \frac{200 \times 100 \times 100}{22 \times 25 \times 25} \times 330$$

$$\boxed{= 48000}$$

Total number of words to be written by using the ink bottle = 48000

**2. A hemi-spherical tank of radius 1.75 m is full of water. It is connected with a pipe which empties the tank at the rate of 7 litre per second. How much time will it take to empty the tank completely?**

**Solution:**



Given : Radius of the hemispherical tank $= 1.75\ m$;
Emptying speed of the pipe = 7 lit. per second

Vol. of hemispherical tank $= \frac{2}{3}\pi r^3\ m^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4}$$
$$= \frac{539}{48}$$
$$= 11.229\ m^3$$
$$= 11.229 \times 1000$$
$$= 11229\ l$$

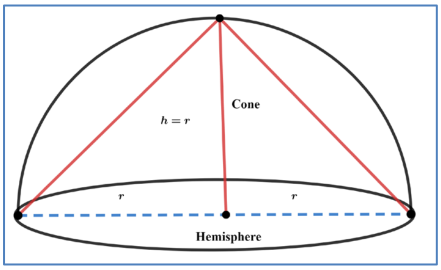
Time taken to empty the tank $= \frac{\text{Vol.of hemispherical tank}}{\text{Emptying speed of the pipe}}$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{1.75 \times 1.75 \times 1.75 \times 1000}{7}$$
$$\boxed{= 1604\ seconds}$$

Time taken for emptying the tank $= 1604\ seconds\ (or)$
$\frac{1604}{60} = 26\ min\ 44\ sec\ (or) \cong\ 27$ .

**3. Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius $r$ units.**

**Solution:**




Given: Let solid hemisphere radius $= r\ units$.
solid hemisphere radius = Radius of cone
Cone radius $= r\ units$.
solid hemisphere radius = Height of the Cone
Height of cone $h = r$ units.

Volume of Cone $= \frac{1}{3}\pi r^2 h\ cu.Units$

$$= \frac{1}{3}\pi r^2 \times r$$
$$= \frac{1}{3}\pi r^3\ cu.units$$

Maximum Vol. of Cone $= \frac{1}{3}\pi r^3\ cu.units$

**4. An oil funnel of tin sheet consists of a cylindrical portion 10 cm long attached to a frustum of a cone. If the total height is 22 cm , the diameter of the cylindrical portion be 8 cm and the diameter of the top of the funnel be 18 cm , then find the area of the tin sheet required to make the funnel.**

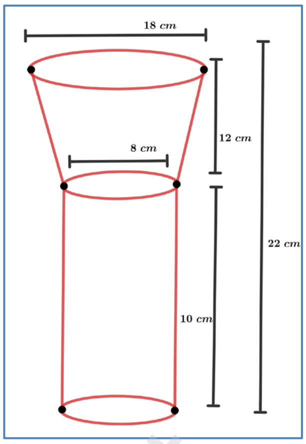
**Solution:**

Given: **Cylinder**

Diameter $= 8\ cm\ ; \rightarrow\ r = 4\ cm;\ h = 10\ cm$

**Frustum**

Top Diameter $d = 8\ cm \rightarrow r = 4\ cm$
Bottom diameter $D = 18\ cm\ \rightarrow R = 9\ cm$
$h_2 = 12\ cm$




Frustrum Slant height $= \sqrt{h_2^2 + (R - r)^2}$

$$= \sqrt{12^2 + (9-4)^2}$$
$$= \sqrt{144 + 25}$$
$$= \sqrt{169} = 13\ cm$$

CSA of Funnel = CSA of Cylinder + CSA of Frustrum

$$= 2\pi r h_1 + \pi(R + r)l$$
$$= \pi[2rh_1 + (R + r)l]$$
$$= \frac{22}{7} \times [2 \times 4 \times 10 + (9 + 4) \times 13]$$
$$= \frac{22}{7} \times\ [80 + 169]$$
$$= \frac{22}{7} \times 249$$
$$\boxed{= 782.57\ cm^2}$$

The area of the tin sheet required to make the funnel $= 782.57\ cm^2$

**5. Find the number of coins, 1.5 cm in diameter and 2 mm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm .**

**Solution:**

Given : **Cylinder**

$D = 4.5\ cm\ \ R = \frac{4.5}{2} = \frac{9}{2}\ cm,\ H = 10\ cm$

**Coin (Cylinder)**

$d = 1.5, r = \frac{1.5}{2} = \frac{3}{2}\ cm,$

Thickness= Height, $h = 2\ mm = \frac{2}{10}\ cm$

Number of coins required to the cylinder $= \frac{\text{Volume of the cylinder}}{\text{Volume of a coin}}$

$$= \frac{\pi R^2 H}{\pi r^2 h}$$
$$= \frac{\frac{9}{2} \times \frac{9}{2} \times 10}{\frac{3}{2} \times \frac{3}{2} \times \frac{2}{10}}$$

$$= \frac{9}{2} \times \frac{9}{2} \times 10 \times \frac{2}{3} \times \frac{2}{3} \times \frac{10}{2}$$

$$\boxed{= 450}$$

The Number of Coins = 450.

**6. A hollow metallic cylinder whose external radius is 4.3 cm and internal radius is 1.1 cm and whole length is 4 cm is melted and recast into a solid cylinder of 12 cm long. Find the diameter of solid cylinder.**

**Solution:**

Given: **Hollow Cylinder**

$R = 4.3\ cm; r = 1.1\ cm; Height\ h = 4\ cm$

**Solid Cylinder**

Height $h_1 = 12\ cm; r_1 = ?$

Volume of the Solid Cylinder = Volume of the Hollow Cylinder

$$\pi r_1^2 h_1 = \pi(R^2 - r^2)h$$

$$r_1^2 \times 12 = (4.3^2 - 1.1^2) \times 4$$

$$r_1^2 = \frac{(18.49 - 1.21)}{12} \times 4$$

$$r_1^2 = \frac{17.28}{3}$$

$$r_1^2 = 5.76$$

$$\boxed{r_1 = 2.4}$$

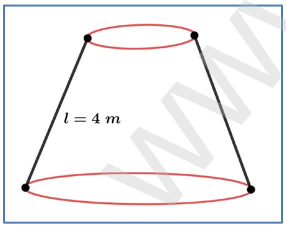
$$d = 4.8\ cm$$

Diameter of the Solid Cylinder

$$= 2r = 2 \times 2.4 = 4.8\ cm$$

**7. The slant height of a frustum of a cone is 4 m and the perimeter of circular ends are 18 m and 16 m . Find the cost of painting its curved surface area at ₹100 per sq. m .**

**Solution:**




Given : **Frustrum**

Top Perimeter = $18\ m, 2\pi R = 18$

Top Radius $R = \frac{9}{\pi}\ m$

Bottom Perimeter = $16\ m, 2\pi r = 16$

Bottom Radius $r = \frac{9}{\pi}\ m$

Slant height $l = 4\ m$

CSA of Frustrum = $\pi(R + r)l\ \ sq.units$

$$= \pi \times \left(\frac{9}{\pi} + \frac{8}{\pi}\right) \times 4$$

$$= \pi \times \frac{17}{\pi} \times 4$$

$$= 68\ m^2$$

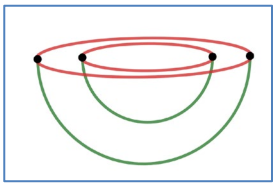
Cost of Painting 1 sq.m = ₹ 100

Cost of Painting for 68 sq. m = ₹ 100 × 68

$$\boxed{Cost\ of\ Painting\ = ₹\ 6{,}800}$$

**8. A hemi-spherical hollow bowl has material of volume $\frac{436\pi}{3}$ cubic cm . Its external diameter is 14 cm . Find its thickness.**

**Solution:**



Given: **Hemi Spherical Hollow bowl**

External diameter $D = 14\ cm \rightarrow R = 7\ cm$

Internal Radius =?.

Volume of the Hollow Hemi Sphere= $\frac{436\pi}{3}\ cm^3$

$$\frac{2}{3}\pi(R^3 - r^3) = \frac{436\pi}{3}$$

$$\frac{2}{3} \times (7^3 - r^3) = \frac{436}{3}$$

$$343 - r^3 = 218$$

$$r^3 = 125$$

$$r^3 = 5^3$$

$$\boxed{r = 5\ cm}$$

Thickness of the Bowl = $R - r = 7 - 5 = 2\ cm$

Thickness = $2\ cm$.

**9. The volume of a cone is $1005\frac{5}{7}$ cu. cm. The area of its base is $201\frac{1}{7}$ sq. cm . Find the slant height of the cone.**

**Solution:**

Given : **Cone**

Base Area of Cone = $201\frac{1}{7}\ sq.units$

Volume of Cone = $1005\ \frac{5}{7}\ cu.units$

Slant height $l$ =?.

Base Area of Cone = $201\frac{1}{7}\ sq.units$

$$\pi r^2 = \frac{1408}{7} ----(1)$$

$$\frac{22}{7} \times r^2 = \frac{1408}{7}$$

$$r^2 = \frac{1408}{7} \times \frac{7}{22} = 64$$

$$r = 8\ cm$$

Volume of Cone = $1005\ \frac{5}{7}\ cub.units$

$$\frac{1}{3}\pi r^2 h = \frac{7040}{7}$$

$$\frac{1}{3} \times \frac{1408}{7} \times h = \frac{7040}{7}\ [From\ (1)]$$

$$h = \frac{7040}{7} \times \frac{7}{1408} \times 3$$

$$h = 15\ cm$$

Slant height $l = \sqrt{r^2 + h^2}$

$$= \sqrt{8^2 + 15^2}$$

$$= \sqrt{64 + 225}$$

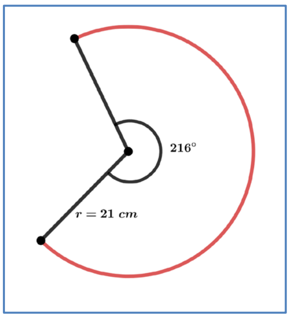
$$= \sqrt{289}$$

$$\boxed{l = 17\ cm}$$

Hence Slant height of the Cone $= 17\ cm$ .

**10. A metallic sheet in the form of a sector of a circle of radius 21 cm has central angle of 216°. The sector is made into a cone by bringing the bounding radii together. Find the volume of the cone formed.**

**Solution:**




Given: **Cone**

The radius of the sector of the circular sheet $R = 21\ cm$

Its central angle $= 216^\circ$

Slant height of the cone=Radius of the sector

Slant height of the Cone $l = 21\ cm$

Perimeter of the Cone = Arc length of the Sector

$$2\pi r = \frac{\theta}{360} \times 2\pi R$$

Radius of cone $r = \frac{216}{360} \times 21$

$$= \frac{63}{5}$$

$$r = 12.6\ cm$$

Height of Cone $h = \sqrt{l^2 - r^2}$

$$= \sqrt{21^2 - (12.6)^2}$$

$$= \sqrt{441 - 158.76}$$

$$= \sqrt{282.24}$$

$$h = 16.8\ cm$$

Volume of Cone $= \frac{1}{3}\pi r^2\ cu.units$

$$= \frac{1}{3} \times \frac{22}{7} \times 12.6 \times 12.6 \times 16.8$$

$$= 22 \times 4.2 \times 1.8 \times 16.8$$

$$= 2794.18\ cm^3$$

Volume of the Cone $\boxed{= 2794.18\ cm^3}$

## CHAPTER – 8 (STATISTICS AND PROBABILITY)

**1. The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50 . Compute the missing frequencies $f_1$ and $f_2$.**

| Class Interval | 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 | 100 – 120 |
|---|---|---|---|---|---|---|
| Frequency | 5 | $f_1$ | 10 | $f_2$ | 7 | 8 |

**Solution:**

Given : Mean $\bar{x} = 62.8$

Sum of Frequency $\sum f = 50$

$$f_1 + f_2 + 30 = 50$$

$$f_1 + f_2 = 20$$

$$f_2 = 20 - f_1 \text{ ----(1)}$$

$A = 50$ $\quad C = 20$

| Class Interval | Mid Point $(x)$ | Frequency $(f)$ | $d = \frac{x-A}{c}$ | $fd$ |
|---|---|---|---|---|
| 0 – 20 | 10 | 5 | −2 | −10 |
| 20 – 40 | 30 | $f_1$ | −1 | $-f_1$ |
| 40 – 60 | 50 | 10 | 0 | 0 |
| 60 – 80 | 70 | $20 - f_1$ | 1 | $20 - f_1$ |
| 80 – 100 | 90 | 7 | 2 | 14 |
| 100 – 120 | 110 | 8 | 3 | 24 |
| | | $\sum f = 50$ | | $\sum fd = 48 - 2f_1$ |

Mean $\bar{x} = A + \frac{\Sigma fd}{\Sigma f} \times c$

$$62.8 = 50 + \frac{48 - 2f_1}{50} \times 20$$

$$62.8 - 50 = \frac{2}{5} \times 48 - 2f_1$$

$$12.8 \times \frac{5}{2} = 48 - 2f_1$$

$$32 = 48 - 2f_1$$

$$2\ f_1 = 48 - 32 = 16$$

$$f_1 = 8$$

$$\boxed{(1) \text{ ----}\rightarrow f_2 = 12}$$

**2. The diameter of circles (in mm ) drawn in a design are given below.**

| Diameters | 33 − 36 | 37 − 40 | 41 − 44 | 45 − 48 | 49 − 52 |
|---|---|---|---|---|---|
| Number of circles | 15 | 17 | 21 | 22 | 25 |

**Calculate the standard deviation.**

**Solution:**

Assumed Mean $= 42.5, \quad c = 4$

| Class Interval | Mid Point $(x)$ | Frequency $(f)$ | $d = \frac{x-A}{c}$ | $d^2$ | $fd$ | $fd^2$ |
|---|---|---|---|---|---|---|
| 32.5 − 36.5 | 34.5 | 15 | −2 | 4 | −30 | 60 |
| 36.5 − 40.5 | 38.5 | 17 | −1 | 1 | −17 | 17 |
| 40.5 − 44.5 | 42.5 | 21 | 0 | 0 | 0 | 0 |
| 44.5 − 48.5 | 46.5 | 22 | 1 | 1 | 22 | 22 |
| 48.5 − 52.5 | 50.5 | 25 | 2 | 4 | 50 | 100 |
| | | $N = 100$ | | | $\sum fd = 25$ | $\sum fd^2 = 199$ |

$$\text{Standard Deviation } \sigma = \sqrt{\frac{\sum f d^2}{N} - \left(\frac{\sum f d}{N}\right)^2} \times C$$

$$= \sqrt{\frac{199}{100} - \left(\frac{25}{100}\right)^2} \times 4$$

$$= \sqrt{\frac{19900 - 625}{10000}} \times 4$$

$$= \sqrt{\frac{19275}{10000}} \times 4$$

$$= \frac{138.83}{1000} \times 4$$

$$= 1.3883 \times 4$$

$$\boxed{\sigma \simeq 5.55}$$

**3. The frequency distribution is given below.**

| $x$ | $k$ | $2k$ | $3k$ | $4k$ | $5k$ | $6k$ |
|---|---|---|---|---|---|---|
| $f$ | 2 | 1 | 1 | 1 | 1 | 1 |

**In the table, $k$ is a positive integer, has a variance of 160 . Determine the value of $k$.**

**Solution:**

Assumed Mean $A = 4k, \quad c = k.$

| $x$ | $(f)$ | $d = \frac{x-A}{c}$ | $d^2$ | $fd$ | $fd^2$ |
|---|---|---|---|---|---|
| $k$ | 2 | −3 | 9 | −6 | 18 |
| $2k$ | 1 | −2 | 4 | −2 | 4 |
| $3k$ | 1 | −1 | 1 | −1 | 1 |
| $4k$ | 1 | 0 | 0 | 0 | 0 |
| $5k$ | 1 | 1 | 1 | 1 | 1 |
| $6k$ | 1 | 2 | 4 | 2 | 4 |
| | $N = 7$ | | | $\sum fd = -6$ | $\sum fd^2 = 28$ |

$$\text{Variance } \sigma^2 = 160$$

$$\frac{\sum f d^2}{N} - \left(\frac{\sum f d}{N}\right)^2 \times c^2 = 160$$

$$\left(\frac{28}{7} - \left(\frac{-6}{7}\right)^2\right) \times k^2 = 160$$

$$\left(4 - \frac{36}{49}\right) \times k^2 = 160$$

$$\frac{160}{49} \times k^2 = 160$$

$$k^2 = 160 \times \frac{49}{160}$$

$$k^2 = 49$$

$$\boxed{k = 7}$$

**4. The standard deviation of some temperature data in degree celsius ( °C) is 5 . If the data were converted into degree Farenheit ( °F ) then what is the variance?**

**Solution:**

Given: The S.D of some temperature data in degree Celsius $(^\circ C) = 5$

Celsius $(°C)$ to Fahrenheit $(°F)$ conversion

$$= \frac{9}{5} \times °c + 32$$

The SD of the temperature data's in Fahrenheit $(°F) = \frac{9}{5} \times 5 = 9$ [*Leaving the constant of* 32]

It's variance $\boxed{\sigma^2 = 9^2 = 81}$

**5. If for a distribution, $\sum(x-5) = 3, \sum(x-5)^2 = 43$, and total number of observations is 18 , find the mean and standard deviation.**

**Solution:**

$\sum(x-5) = 3, \ \sum(x-5)^2 = 43, \ n = 18$

$$\sum(x-5) = 3$$

$$\sum x - \sum 5 = 3$$

$$\sum x - 5\sum 1 = 3$$

$$\sum x - 5 \times 18 = 3 \quad \left[\sum 1 = n = 18\right]$$

$$\sum x - 90 = 3$$

$$\sum x = 93$$

$$\sum (x-5)^2 = 43$$
$$\sum (x^2 - 10x + 25) = 43$$
$$\sum x^2 - 10\sum x + 25\sum 1 = 43$$
$$\sum x^2 - 10 \times 93 + 25 \times 18 = 43$$
$$\sum x^2 - 930 + 450 = 43$$
$$\sum x^2 - 480 = 43$$
$$\sum x^2 = 523$$

Mean $\overline{x} = \frac{\sum x}{n} = \frac{93}{18} = 5.17$

Standard Deviation $\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$

$$= \sqrt{\frac{523}{18} - \left(\frac{93}{18}\right)^2}$$
$$= \sqrt{\frac{9414 - 8649}{324}}$$
$$= \sqrt{\frac{765}{324}}$$
$$= \frac{27.66}{18}$$
$$\boxed{\sigma \simeq 1.54}$$

**6. Prices of peanut packets in various places of two cities are given below. In which city, prices were more stable?**

| | | | | | |
|---|---|---|---|---|---|
| **Prices in city A** | **20** | **22** | **19** | **23** | **16** |
| **Prices in city B** | **10** | **20** | **18** | **12** | **15** |

**Solution:**

Prices in City A:

Mean $\overline{x_1} = \frac{20+22+19+23+16}{5} = \frac{100}{5} = 20$

| $x$ | $d_1 = x - \overline{x_1}$ $= x - 20$ | $d_1^2$ |
|---|---|---|
| 16 | $-4$ | 16 |
| 19 | $-1$ | 1 |
| 20 | 0 | 0 |
| 22 | 2 | 4 |
| 23 | 3 | 9 |
| | $\sum d = 0$ | $\sum d_1^2 = 30$ |

Standard Deviation $\sigma_1 = \sqrt{\frac{\sum d_1^2}{n}} = \sqrt{\frac{30}{5}} = \sqrt{6} = 2.45$

Coefficient of Variation $CV_1 = \frac{\sigma_1}{\overline{x_1}} \times 100\%$

$$= \frac{2.45}{20} \times 100$$
$$= \frac{245}{20}$$
$$= 12.25\%$$

Prices in City B:

Mean $\overline{x_2} = \frac{10+12+15+18+10}{5} = \frac{75}{5} = 15$

| $x$ | $d_2 = x - \overline{x_2}$ $= x - 15$ | $d_2^2$ |
|---|---|---|
| 10 | $-5$ | 25 |
| 12 | $-3$ | 9 |
| 15 | 0 | 0 |
| 18 | 3 | 9 |
| 20 | 5 | 25 |
| | $\sum d = 0$ | $\sum d_2^2 = 68$ |

Standard Deviation $\sigma_2 = \sqrt{\frac{\sum d_2^2}{n}} = \sqrt{\frac{68}{5}} = \sqrt{13.6}$

$= 3.69$

Coefficient of Variation $CV_2 = \frac{\sigma_2}{\overline{x_2}} \times 100\%$

$$= \frac{3.69}{15} \times 100$$
$$= \frac{369}{15}$$
$$= 24.6\%$$

$$\boxed{CV_1 < CV_2}$$

∴ City A is More Consistent.

**7. If the range and coefficient of range of the data are 20 and 0.2 respectively, then find the largest and smallest values of the data.**

**Solution:**

Range $L - S = 20 - - - - - - (1)$

Coefficient of Range $\frac{L-S}{L+S} = 0.2 - - - - (2)$

$$(2) \Rightarrow \frac{20}{L+S} = 0.2$$
$$\frac{20}{0.2} = L + S$$
$$L + S = 100 - - - - (3)$$

$(1) + (3) \Rightarrow 2L = 120 \Rightarrow L = 60$

$$\boxed{Largest\ Value = 60}$$

$(1) \Rightarrow\ 60 - S = 20\ \Rightarrow\ S = 40$

$$\boxed{Smallest\ Value = 40}$$

**8. If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5 .**

**Solution:**

Sample Space of Two Dices Rolled

$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$
$(2,1),(2,2),(2,3),(2\ 4),(2,5),(2,6)$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$n(S) = 36$

Let A be a Event of getting the product value is 6.

$A = \{(1,6),(2,3),(3,2),(6,1)\}, n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36}$$

Let B be a event of getting a difference of 5.

$$B = \{(6,1)\};\ n(B) = 1$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{36}$$

$$A \cap B = \{(6,1)\}; n(A \cap B) = 1$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{4}{36} + \frac{1}{36} - \frac{1}{36}$$

$$= \frac{4}{36}$$

$$\boxed{P(A \cup B) = \frac{1}{9}}$$

**9. In a two children family, find the probability that there is at least one girl in a family.**

**Solution:**

A Family With Two Children One Boy or Girl

Sample Space $S = \{BB, BG, GB, GG\}$, $n(S) = 4$

Let A be a event of getting atleast One girl

$$A = \{BG, GB, GG\}: n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

**10. A bag contains 5 white and some black balls. If the probability of drawing a black ball from the bag is twice the probability of drawing a white ball then find the number of black balls.**

**Solution:**

Given: A bag contains 5 white balls

Number of white balls $n(w) = 5$

Let the number of black balls $n(B) = x$,

$n(S) = 5 + x$

Given $\rightarrow P(B) = 2 \times P(W)$

$$\frac{n(B)}{n(S)} = 2 \times \frac{n(W)}{n(S)}$$

$$\frac{x}{5+x} = 2 \times \frac{5}{5+x}$$

$$x = 2 \times 5$$

$$\boxed{x = 10}$$

$\therefore$ The Number of Black Balls $\boxed{n(B) = 10}$

**11. The probability that a student will pass the final examination in both English and Tamil is 0.5 and the probability of passing neither is 0.1 . If the probability of passing the English examination is 0.75 , what is the probability of passing the Tamil examination?**

**Solution:**

Given :

Probability of passing the English examination :

$$P(E) = 0.75$$

Probability of passing in both English and Tamil :

$$P(E \cap T) = 0.5$$

Probability of passing neither: $P(\overline{E \cup T}) = 0.1$

$$P(\overline{E \cup T}) = 0.1$$

$$1 - P(E \cup T) = 0.1$$

$$P(E \cup T) = 0.9$$

Probability of Passing in Tamil

$$P(E \cup T) = P(E) + P(T) - P(E \cap T)$$

$$0.9 = 0.75 + P(T) - 0.5$$

$$P(T) = 0.9 + 0.5 - 0.75$$

$$P(T) = 0.65 = \frac{65}{100}$$

$$\boxed{P(T) = \frac{13}{20}}$$

***

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All the best Students

**"Experience is the best Teacher"**