



PART - I

Choose the correct answer:

20*1=20

- The range of the function $f(x) = |[x] - x|, x \in \mathbb{R}$ is
 (a) $[0,1]$ (b) $[0, \infty)$ (c) $[0,1)$ (d) $(0,1)$
- The number of roots of $(x+3)^4 + (x+5)^4 = 16$ is
 (a) 4 (b) 2 (c) 3 (d) 0
- If $\pi < 2\theta < \frac{3\pi}{2}$, then $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}}$ equals to
 (a) $-2 \cos\theta$ (b) $-2 \sin\theta$ (c) $2 \cos\theta$ (d) $2 \sin\theta$
- In a triangle ABC , $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then the triangle is
 (a) equilateral triangle (b) isosceles triangle
 (c) right triangle (d) scalene triangle.
- The number of five digit telephone numbers having at least one of their digits repeated is
 (a) 90000 (b) 10000 (c) 30240 (d) 69760
- If $a, 8, b$ are in AP, $a, 4, b$ are in GP, and if a, x, b are in HP then x is
 (a) 2 (b) 1 (c) 4 (d) 16
- The value of $1 - \frac{1}{2} \left(\frac{2}{3}\right) + \frac{1}{3} \left(\frac{2}{3}\right)^2 - \frac{1}{4} \left(\frac{2}{3}\right)^3 + \dots$ is
 (a) $\log\left(\frac{5}{3}\right)$ (b) $\frac{3}{2} \log\left(\frac{5}{3}\right)$ (c) $\frac{5}{3} \log\left(\frac{5}{3}\right)$ (d) $\frac{2}{3} \log\left(\frac{2}{3}\right)$
- The area of the triangle formed by the lines $x^2 - 4y^2 = 0$ and $x = a$ is
 (a) $2a^2$ (b) $\frac{\sqrt{3}}{2} a^2$ (c) $\frac{1}{2} a^2$ (d) $\frac{2}{\sqrt{3}} a^2$

9. If $A = \begin{vmatrix} -1 & 2 & 4 \\ 3 & 1 & 0 \\ -2 & 4 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} -2 & 4 & 2 \\ 6 & 2 & 0 \\ -2 & 4 & 8 \end{vmatrix}$, then B is given by
- (a) $B = 4A$ (b) $B = -4A$ (c) $B = -A$ (d) $B = 6A$
10. One of the diagonals of parallelogram $ABCD$ with \vec{a} and \vec{b} as adjacent sides is $\vec{a} + \vec{b}$. The other diagonal \vec{BD} is
- (a) $\vec{a} - \vec{b}$ (b) $\vec{b} - \vec{a}$ (c) $\vec{a} + \vec{b}$ (d) $\frac{\vec{a} + \vec{b}}{2}$
11. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$, then the angle between \vec{A} and \vec{B} is
- (a) π (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$
12. If $f(x) = x(-1)^{\lfloor \frac{1}{x} \rfloor}$, $x \leq 0$, then the value of $\lim_{x \rightarrow 0} f(x)$ is equal to
- (a) -1 (b) 0 (c) 2 (d) 4
13. The differential coefficient of $\log_{10} x$ with respect to $\log_x 10$ is
- (a) 1 (b) $-(\log_{10} x)^2$ (c) $(\log_x 10)^2$ (d) $\frac{x^2}{100}$
14. The number of points in \mathbb{R} in which the function $f(x) = |x - 1| + |x - 3| + \sin x$ is not differentiable, is
- (a) 3 (b) 2 (c) 1 (d) 4
15. $\int \sqrt{\frac{1-x}{1+x}} dx$ is
- (a) $\sqrt{1-x^2} + \sin^{-1} x + c$ (b) $\sin^{-1} x - \sqrt{1-x^2} + c$
(c) $\log |x + \sqrt{1-x^2}| - \sqrt{1-x^2} + c$ (d) $\sqrt{1-x^2} + \log |x + \sqrt{1-x^2}| + c$
16. A number is selected from the set $\{1, 2, 3, \dots, 20\}$. The probability that the selected number is divisible by 3 or 4 is
- (a) $\frac{2}{5}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$
17. If α and β are the roots of the equation $2x^2 - 3x - 4 = 0$, the value of $\alpha^2 + \beta^2$
- (a) $\frac{41}{4}$ (b) $\frac{\sqrt{14}}{2}$ (c) 0 (d) $\frac{25}{4}$
18. If $100C_r = 100C_{3r}$, then the value of r
- (a) 24 (b) 25 (c) 20 (d) 50
19. If A and B are square matrices of order 3 and $|A| = 5$, $|B| = 3$ then $|3AB|$ is
- (a) 27 (b) 81 (c) 135 (d) 405

20. The function $f(x) = \tan x$ is discontinuous on the set
(a) $n\pi, n \in Z$ (b) $2n\pi, n \in Z$ (c) $(2n + 1)\frac{\pi}{2}, n \in Z$ (d) $\frac{n\pi}{2}, n \in Z$

PART - II

Answer any seven questions: (Q.No. 30 is compulsory) 7*2=14

21. Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, find A and B , where x, y, z are distinct elements.
22. Prove: $\log \frac{a^2}{bc} + \log \frac{b^2}{ca} + \log \frac{c^2}{ab} = 0$.
23. Find the value of $\operatorname{cosec}(-1410^\circ)$.
24. Write any two equations of straight lines
25. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, then compute A^4
26. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ prove that \vec{a} and \vec{b} are perpendicular.
27. Find $\frac{dy}{dx}$ if $x^2 + y^2 = 1$
28. Integrate $\frac{1}{6-4x}$.
29. The odd that the event A occurs is 5 to 7. Find $P(A)$
30. If a and b are any two positive numbers, find its harmonic mean.

PART-III

Answer any seven questions : (Q.No. 40 is compulsory). 7 * 3 = 21

31. Find the range of the function $f(x) = \frac{1}{1-3 \cos x}$
32. Prove that $\frac{\sin 4x + \sin 2x}{\cos 4x + \cos 2x} = \tan 3x$
33. Find the ranks of the word "TABLE".
34. If the roots of the equation $(q - r)x^2 + (r - p)x + p - q = 0$ are equal, then show that p, q and r are in AP.
35. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is twice that of the other, show that $8h^2 = 9ab$
36. Evaluate $\lim_{x \rightarrow 0} \frac{2^x - 3^x}{x}$

37. Find $\frac{dy}{dx}$ if $x = a \cos^3 t$; $y = a \sin^3 t$.
38. Evaluate $\int x e^x dx$.
39. If A and B are two independent events such that $P(A) = 0.4$ and $P(A \cup B) = 0.9$. Find $P(B)$
40. If unit vector \hat{a} makes angles $\frac{\pi}{3}$ with \hat{i} , $\frac{\pi}{6}$ with \hat{j} , then prove that \hat{a} is perpendicular to \hat{k}

PART - IV

Answer all the questions:

7 * 5 = 35

41. a) If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 3x - 5$, prove that f is a bijection and find its inverse. (OR)
- b) Resolve into partial fractions: $\frac{2x}{(x^2+1)(x-1)}$
42. a) If θ is an acute angle, then find $\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$, when $\sin \theta = \frac{1}{25}$. (OR)
- b) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$
43. a) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large. (OR) b) Rewrite $\sqrt{3}x + y + 4 = 0$ into normal form.
44. a) A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box, if at least one black ball is to be included in the draw? (OR) b) Prove that the points whose position vectors $2\hat{i} + 4\hat{j} + 3\hat{k}$, $4\hat{i} + \hat{j} + 9\hat{k}$ and $10\hat{i} - \hat{j} + 6\hat{k}$ form a right angled triangle
45. a) Evaluate (i) $\int e^{x \log 2} e^x dx$ (ii) $\int \frac{2x+4}{x^2+4x+6} dx$. (OR)
- b) If $y = e^{\tan^{-1} x}$, show that $(1+x^2)y'' + (2x-1)y' = 0$
46. a) Evaluate $\int \frac{3x+5}{x^2+4x+7} dx$. (OR)
- b) A factory has two Machines-I and II. Machine-I produces 60% of items and Machine-II produces 40% of the items of the total output. Further 2% of the items produced by Machine-I are defective whereas 4% produced by Machine-II are defective. If an item is drawn at random what is the probability that it is defective?
47. a) If $f(x) = \frac{1+x}{1+x^2}$, find $3f'(2) + 2f'(3)$ (OR)
- b) Find the general solution of $\sin 4x + \cos 2x = 0$.