

MATHEMATICS

Time: 3.00 hrs.

Part - I

Marks: 90

20 x 1 = 20

I. Choose the correct answer:

- For non-empty sets A and B if $A \subset B$, then $(A \times B) \cap (B \times A)$ is equal to
 a) $A \cap B$ b) $A \times A$ c) $B \times B$ d) None of these
- Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 1 - |x|$. The range of f is
 a) \mathbb{R} b) $(1, \infty)$ c) $(-1, \infty)$ d) $(-\infty, 1)$
- The equation whose roots are numerically equal but opposite in sign to the roots of $3x^2 - 5x - 7 = 0$ is
 a) $3x^2 - 5x - 7 = 0$ b) $3x^2 + 5x - 7 = 0$
 c) $3x^2 - 5x + 7 = 0$ d) $3x^2 + x - 7 = 0$
- Find the value of $\sqrt[4]{(-2)^4}$
 a) -2 b) 16 c) 2 d) -16
- Which of the following is not true?
 a) $\sin \theta = -\frac{3}{4}$ b) $\cos \theta = -1$ c) $\tan \theta = 25$ d) $\sec \theta = -\frac{1}{4}$
- $\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A}$ is
 a) $\sin A + \sin B + \sin C$ b) 1 c) 0 d) $\cos A + \cos B + \cos C$
- There are 10 points in a plane and 4 of them are collinear. The number of straight lines joining any two points is
 a) 45 b) 40 c) 39 d) 38
- The expansion of $(1-x)^{-2}$ is
 a) $1 - x + x^2 - \dots$ b) $1 + x + x^2 + \dots$
 c) $1 - 2x + 3x^2 - \dots$ d) $1 + 2x + 3x^2 + \dots$
- The n^{th} term of the sequence 1, 2, 4, 7, 11, is
 a) $n^3 + 3n^2 + 2n$ b) $n^3 - 3n^2 + 3n$ c) $\frac{n(n+1)(n+2)}{3}$ d) $\frac{n^2 - n + 2}{2}$
- Which of the following equation is the locus of $(at^2, 2at)$
 a) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ c) $x^2 + y^2 = a^2$ d) $y^2 = 4ax$
- If $A + I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$, then $(A + I)(A - I)$ is equal to
 a) $\begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$ b) $\begin{bmatrix} -5 & 4 \\ -8 & 9 \end{bmatrix}$ c) $\begin{bmatrix} 5 & 4 \\ 8 & 9 \end{bmatrix}$ d) $\begin{bmatrix} -5 & -4 \\ -8 & -9 \end{bmatrix}$
- A vector makes equal angle with the positive direction of the coordinate axes. Then each angle is equal to
 a) $\cos^{-1}\left(\frac{1}{3}\right)$ b) $\cos^{-1}\left(\frac{2}{3}\right)$ c) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ d) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

13. If \vec{a} , \vec{b} , \vec{c} are the position vectors of three collinear points, then which of the following is true?

- a) $\vec{a} = \vec{b} + \vec{c}$ b) $2\vec{a} = \vec{b} + \vec{c}$ c) $\vec{b} = \vec{c} + \vec{a}$ d) $4\vec{a} + \vec{b} + \vec{c} = \vec{0}$

14. Calculate $\lim_{x \rightarrow x_0} (5)$ for any real number x_0 .

- a) 0 b) 1 c) 5 d) ∞

15. $\lim_{x \rightarrow 0} \frac{xe^x - \sin x}{x}$ is

- a) 1 b) 2 c) 3 d) 0

16. The differential coefficient of $\log_{10} x$ with respect to $\log_x 10$ is

- a) 1 b) $-(\log_{10} x)^2$ c) $(\log_x 10)^2$ d) $\frac{x^2}{100}$

17. If $y = e^{\sin x}$, then $\frac{dy}{dx}$ is

- a) $\sin x e^{\sin x}$ b) $e^{\sin x}$ c) $e^{\cos x}$ d) $\cos x e^{\sin x}$

18. $\int \frac{dx}{e^x - 1}$ is

- a) $\log|e^x| - \log|e^x - 1| + c$ b) $\log|e^x| + \log|e^x - 1| + c$
c) $\log|e^x - 1| - \log|e^x| + c$ d) $\log|e^x + 1| - \log|e^x| + c$

19. $\int e^{\sqrt{x}} dx$ is

- a) $2\sqrt{x}(1 - e^{\sqrt{x}}) + c$ b) $2\sqrt{x}(e^{\sqrt{x}} - 1) + c$
c) $2e^{\sqrt{x}}(1 - \sqrt{x}) + c$ d) $2e^{\sqrt{x}}(\sqrt{x} - 1) + c$

20. If X and Y be two events such that $P(X/Y) = \frac{1}{2}$, $P(Y/X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$, then $P(X \cup Y)$ is

- a) $\frac{1}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{6}$ d) $\frac{2}{3}$

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory)

7 x 2 = 14

21. Find the number of subsets of A if $A = \{x : x = 4n + 1, 2 \leq n \leq 5, n \in \mathbb{N}\}$

22. Find the complete set of values of a for which the quadratic $x^2 - ax + a + 2 = 0$ has equal roots.

23. Prove that $\cos(\pi + \theta) = -\cos\theta$

24. Find the value of n if $\frac{1}{8!} + \frac{1}{9!} = \frac{n}{10!}$

25. Expand $(2x^2 - \frac{3}{x})^3$

26. What is the vector product of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$.

27. Evaluate : $\left(\left((256)^{-\frac{1}{2}} \right)^{\frac{1}{2}} \right)^3$

28. Find $\frac{dy}{dx}$, if $x^2 + y^2 = 1$

29. Evaluate : $\int x^3 \sin x \, dx$

30. Find the sum $A + B + C$ if A, B, C are given by

$$A = \begin{bmatrix} \sin^2 \theta & 1 \\ \cot^2 \theta & 0 \end{bmatrix}, B = \begin{bmatrix} \cos^2 \theta & 1 \\ -\operatorname{cosec}^2 \theta & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory)

7 x 3 = 21

31. Find the domain of $\frac{1}{1-2\sin x}$

32. Compute $\log_3 5 \log_{25} 27$

33. If $a \sin^2 \theta + b \cos^2 \theta = c$, show that $\tan^2 \theta = \frac{c-b}{a-c}$.

34. In how many ways 5 boys and 4 girls can be seated in a row, so that no two girls are together?

35. Nine coins are tossed once, find the probability to get at least two heads.

36. Show that the points $(a, b+c)$, $(b, c+a)$ and $(c, a+b)$ are collinear. *

37. Find the angle between the vectors $(2\hat{i} + \hat{j} - \hat{k})$ and $(\hat{i} + 2\hat{j} + \hat{k})$ using vector product.

38. Find $\frac{dy}{dx}$, if $x = a(t - \sin t)$, $y = (1 - \cos t)$

39. If $f(x) = 3x^2 - 4x + 5$ and $f(1) = 3$, then find $f(x)$.

40. If θ is a parameter, find the equation of the locus of a moving point, whose coordinates $x = a \cos^3 \theta$, $y = a \sin^3 \theta$

Part - IV

IV. Answer all the questions.

7 x 5 = 35

41. a) If $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 3x - 5$, prove that f is a bijection and find its inverse.

(OR)

b) Resolve into partial fractions : $\frac{x+1}{x^2(x-1)}$

42. a) State and prove Napier's formula.

(OR)

b) If the letters of the word IITJEE are permuted in all possible ways and the strings thus formed are arranged in the lexicographic order, find the rank of the word IITJEE.

43. a) Solve for x if
$$\begin{bmatrix} x & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ -1 & -4 & 1 \\ -1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ 2 \\ 1 \end{bmatrix} = 0$$

(OR)

b) If a, b, c are respectively the p^{th} , q^{th} and r^{th} terms of a GP, show that $(q - r) \log a + (r - p) \log b + (p - q) \log c = 0$

44. a) Write any five different forms of an equation of a straight line.

(OR)

b) Using the mathematical induction, show that for any integer $n \geq 2$, $3n^2 > (n + 1)^2$

45. a) Show that the vectors $5\hat{i} + 6\hat{j} + 7\hat{k}$, $7\hat{i} - 8\hat{j} + 9\hat{k}$, $3\hat{i} + 20\hat{j} + 5\hat{k}$ are coplanar.

(OR)

b) Show that $\lim_{x \rightarrow 0^+} x \left[\frac{1}{x} \right] + \left[\frac{2}{x} \right] + \dots + \left[\frac{15}{x} \right] = 120$

46. a) If $y = (\cos^{-1}x)^2$, prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$. Hence find y_2 when $x = 0$

(OR)

b) Using Factor theorem, prove that
$$\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$$

47. a) Evaluate: $\int (3x + 4)\sqrt{3x + 7} dx$

(OR)

b) A consulting firm rents car from three agencies such that 50% from agency L, 30% from agency M and 20% from agency N. If 90% of the cars from L, 70% of cars from M and 60% of the cars from N are in good conditions.

- What is the probability that the firm will get a car in good condition?
- If a car is in good condition, what is probability that it has come from agency N?
