COMMON HALF YEARLY EXAMINATION - 2025

A

Standard XI

Reg.No:

MATHEMATICS

Time : 3.00 hrs **Part - I** *Marks : 90*

20 x 1 = 20

I. Choose the correct answer:

1. If A = {(x,y) : y = sin x, x∈R} and B = {(x,y) : y = cos x, x∈R} then A ∩ B contains
   a) no element b) infinitely many elements
   c) only one element d) cannot be determined
2. The function f : [–3,3] → S by defined $f(x) = x^2$ is onto, then S is
   a) [–9,9] b) R c) [–3,3] d) [0,9]
3. The number of solutions of $x^2 + |x - 1| = 1$ is
   a) 1 b) 0 c) 2 d) 3
4. $\frac{\cos 6x+6\cos 4x+15\cos 2x+10}{\cos 5x+5\cos 3x+10\cos x}$ is equal to
   a) cos 2x b) cos x c) cos 3x d) 2 cos x
5. In $^{2n}C_3 : {}^nC_3 = 11 : 1$, then n is
   a) 5 b) 6 c) 11 d) 7
6. The number of 5 digit numbers all digits of which are odd is
   a) 25 b) 5^5 c) 5^6 d) 625
7. The value of $1-\frac{1}{2}\left(\frac{2}{3}\right)+\frac{1}{3}\left(\frac{2}{3}\right)^2-\frac{1}{4}\left(\frac{2}{3}\right)^3+\ldots\ldots$ is
   a) $\log\left(\frac{5}{3}\right)$ b) $\frac{3}{2}\log\left(\frac{5}{3}\right)$ c) $\frac{5}{3}\log\left(\frac{5}{3}\right)$ d) $\frac{2}{3}\log\left(\frac{2}{3}\right)$
8. If a is the arithmetic mean and g is the geometric mean of two numbers, then
   a) a ≤ g b) a ≥ g c) a = g d) a > g
9. The image of the point (2,3) in the line y = –x is
   a) (–3,–2) b) (–3,2) c) (–2,–3) d) (3,2)
10. If the point (8, –5) lies on the locus $\frac{x^2}{16}-\frac{y^2}{25}=k$, then the value of k is
    a) 0 b) 1 c) 2 d) 3
11. The value of the determinant of $A=\begin{bmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{bmatrix}$ is
    a) –2abc b) abc c) 0 d) $a^2 + b^2 + c^2$
12. If A is a square matrix, then which of the following is not symmetric?
    a) $A+A^T$ b) AA^T c) A^TA d) $A-A^T$
13. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+x\hat{j}+\hat{k}$, $\vec{c}=\hat{i}-\hat{j}+4\hat{k}$ and $\vec{a}.(\vec{b}\times\vec{c})=70$, then x is equal to
    a) 5 b) 7 c) 26 d) 10

14. If $\lim_{x\to 0}\frac{\sin px}{\tan 3x} = 4$, then the value of p is

a) 6 b) 9 c) 12 d) 4

15. The function $f(x)=\begin{cases}\frac{x^2-1}{x^3+1} & x\neq -1\\ p & x=-1\end{cases}$ is not defined for x = –1. The value of f(–1) so that the function extended by this value is continuous is

a) $\frac{2}{3}$ b) $\frac{-2}{3}$ c) 1 d) 0

16. If y = mx + c and f(0) = f'(0) = 1, then f(2) is

a) 1 b) 2 c) 3 d) –3

17. The number of points in R in which the function f(x) = |x – 1| + |x - 3| + sin x is not differentiable is

a) 3 b) 2 c) 1 d) 4

18. $\int\frac{\sqrt{\tan x}}{\sin 2x}dx$ is

a) $\sqrt{\tan x}+c$ b) $2\sqrt{\tan x}+c$ c) $\frac{1}{2}\sqrt{\tan x}+c$ d) $\frac{1}{4}\sqrt{\tan x}+c$

19. A number is selected from the set {1,2,3,.....20}. The probability that the selected number is divisible by 3 or 4 is

a) $\frac{2}{5}$ b) $\frac{1}{8}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

20. If two events A and B are independent such that P(A) = 0.35 and P(A∪B) = 0.6, then P(B) is

a) $\frac{5}{13}$ b) $\frac{1}{13}$ c) $\frac{4}{13}$ d) $\frac{7}{13}$

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. Construct a quadratic equation with the roots 7 and –3
22. Express sin50° + sin20° as a product form.
23. Find the value of n if (n + 1)! = 20(n – 1)!
24. Find the co-efficient of x^6 in the expansion of $(3 + 2x)^{10}$
25. The length of the perpendicular drawn from the origin to a line is 12 and makes an angle 150° with positive direction of the x-axis. Find the equation of the line.
26. Determine the value of B so that the following matrix is singular. $B=\begin{bmatrix}b-1 & 2 & 3\\ 3 & 1 & 2\\ 1 & -2 & 4\end{bmatrix}$
27. Calculate $\lim_{x\to -1}\left(x^2-3\right)^{10}$
28. Integrate the following with respect to x : $\int\frac{1}{x\log x}dx$

29. If two coins are tossed simultaneously, then find the probability of getting atmost two tails.
30. Draw the function f(x) if $f(x) = 2x^2 - 5x + 3$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) 7 x 3 = 21

31. If n(P(A)) = 1024, n(A∪B) = 15 and n(P(B)) = 32, then find n(A∩B)
32. Reslolve into partial fractions : $\frac{x}{(x+3)(x-4)}$
33. Prove that $^{24}C_4 + \sum_{r=0}^{4} {}^{(28-r)}C_3 = {}^{29}C_4$
34. Write the first 6 terms of the sequences whose n^{th} term a_n is given below

$$a_n = \begin{cases} n & \text{if } n \text{ is } 1,2 \text{ or } 3 \\ a_{n-1} + a_{n-2} + a_{n-3} & \text{if } n > 3 \end{cases}$$

35. Show the points $\left(0, -\frac{3}{2}\right)$, (1,–1) and $\left(2, -\frac{1}{2}\right)$ are collinear.
36. Find the value of x if $\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$
37. Show that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$
38. Show that $\lim_{n \to \infty} \frac{1+2+3+.....+n}{3n^2+7n+2} = \frac{1}{6}$
39. Find $\frac{dy}{dx}$ if x = a(t – sin t), y = a(1 – cos t)
40. If P(A) = 0.5, P(B) = 0.8 and P(B/A) = 0.8, find P(A/B) and P(A∪B)

Part - IV

IV. Answer all the questions. 7 x 5 = 35

41. a) If f : R→R is defined by f(x) = 2x – 3, prove that f is a bijection and find its inverse.

(OR)

b) A factory has two machines I and II. Machine I produces 40% of items of the output and Machine II produces 60% of the items. Further 4% of items produced by machine I are defective and 5% produced by machine II are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by machine II.

42. a) Prove that $\log \frac{2}{10} + 16\log_{10} \frac{16}{15} + 12\log_{10} \frac{25}{24} + 7\log_{10} \frac{81}{80} = 1$

(OR)

b) Evaluate : $\int \frac{2x+3}{\sqrt{x^2+x+1}} dx$

43. a) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

(OR)

b) Find $\frac{d^2y}{dx^2}$ if $x^2 + y^2 = 4$

44. a) By the principle of mathematical induction, prove that, for $n \geq 1$

$$1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

(OR)

b) Do the limits of following functions exists as $x \to 0$? State reasons for your answer.

$$\frac{\sin(x - [x])}{x - [x]}$$

45. a) If θ is an acute angle, then find $\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$ when $\sin\theta = \frac{1}{25}$

(OR)

b) Show that the points whose position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$ $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ are coplanar.

46. a) Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines. Find the distance between them.

(OR)

b) If $f, g : R \to R$ are defined by $f(x) = |x| + x$ and $g(x) = |x| - x$, find gof and fog.

47. a) If ABCD is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$

(OR)

b) Show that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$ by using factor theorem.

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