Sri Raghavendra Tuition Center

Matrices and Determinants

11th Standard

Maths

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I. Multiple Choice Question. 15 x 1 = 15

1) If $a_{ij} = \frac{1}{2}(3i-2j)$ and A = $[a_{ij}]_{2x2}$ is

(a) $\begin{bmatrix} \frac{1}{2} & 2 \\ -\frac{1}{2} & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 2 \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ (d) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ 1 & 2 \end{bmatrix}$

2) What must be the matrix X, if 2x + $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$?

(a) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -6 \\ 4 & -2 \end{bmatrix}$

3) If A and B are two matrices such that A + B and AB are both defined, then

(a) A and B are two matrices not necessarily of same order (b) A and B are square matrices of same order

(c) Number of columns of A is equal to the number of rows of B (d) A = B.

4) If A = $\begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$, then for what value of λ, A^2 = O?

(a) 0 (b) ± 1 (c) -1 (d) 1

5) If A = $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, B = $\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then the values of a and b are

(a) a = 4, b = 1 (b) a = 1, b = 4 (c) a = 0, b = 4 (d) a = 2, b = 4

6) If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to

(a) (2, - 1) (b) (- 2, 1) (c) (2, 1) (d) (- 2, - 1)

7) If A = $\begin{bmatrix} a & x \\ y & a \end{bmatrix}$ and if xy = 1, then det $(A A^T)$ is equal to

(a) $(a-1)^2$ (b) $(a^2+1)^2$ (c) a^2-1 (d) $(a^2-1)^2$

8) The value of x, for which the matrix A = $\begin{bmatrix} e^{x-2} & e^{7+x} \\ e^{2+x} & e^{2x+3} \end{bmatrix}$ is singular

(a) 9 (b) 8 (c) 7 (d) 6

9) If the points (x,−2), (5, 2), (8, 8) are collinear, then x is equal to

(a) -3 (b) $\frac{1}{3}$ (c) 1 (d) 3

10) If $\begin{vmatrix} 2a & x_1 & y_1 \\ 2b & x_2 & y_2 \\ 2c & x_3 & y_3 \end{vmatrix} = \frac{abc}{2} \neq 0$, then the area of the triangle whose vertices are $\left(\frac{x_1}{a}, \frac{y_1}{a}\right), \left(\frac{x_2}{b}, \frac{y_2}{b}\right), \left(\frac{x_3}{c}, \frac{y_3}{c}\right)$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{4}abc$ (c) $\frac{1}{8}$ (d) $\frac{1}{8}abc$

11) If $\triangle = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$, then $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix}$ is

(a) $\triangle$ (b) $k\triangle$ (c) $3k\triangle$ (d) $k^3\triangle$

12) A root of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0\, is$

(a) 6 (b) 3 (c) 0 (d) -6

13) If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in geometric progression with the same common ratio, then the points (x_1, y_1), (x_2, y_2), (x_3, y_3) are

(a) vertices of an equilateral triangle (b) vertices of a right angled triangle (c) vertices of a right angled isosceles triangle (d) collinear

14) If $\lfloor . \rfloor$ denotes the greatest integer less than or equal to the real number under consideration and $-1 \leq x < 0$, $0 \leq y < 1$, $1 \leq z < 2$, then the value of the determinant $\begin{vmatrix} \lfloor x \rfloor + 1 & \lfloor y \rfloor & \lfloor z \rfloor \\ \lfloor x \rfloor & \lfloor y \rfloor + 1 & \lfloor z \rfloor \\ \lfloor x \rfloor & \lfloor y \rfloor & \lfloor z \rfloor + 1 \end{vmatrix}$ is

(a) $\lfloor z \rfloor$ (b) $\lfloor y \rfloor$ (c) $\lfloor x \rfloor$ (d) $\lfloor x \rfloor + 1$

15) Let A and B be two symmetric matrices of same order. Then which one of the following statement is not true?

(a) A + B is a symmetric matrix (b) AB is a symmetric matrix (c) $AB = (BA)^T$ (d) $A^T B = AB^T$

II. ANSWER ANY 7 QUESTION. 7 x 2 = 14

16) Suppose that a matrix has 12 elements. What are the possible orders it can have? What if it has 7 elements?

17) Determine 3B + 4C - D if B, C, and D are given by

$B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & -1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} -1 & -2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 4 & -1 \\ 5 & 6 & -5 \end{bmatrix}$

18) Construct an $m \times n$ matrix $A = [a_{ij}]$, where a_{ij} is given by

$a_{ij} = \frac{|3i-4j|}{4} with\ m = 3, n = 4$

19) If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, then compute A^4.

20) Express the following matrices as the sum of a symmetric matrix and a skew-symmetric matrix:

$\begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix}$

21) Find |A| if $A = \begin{bmatrix} 0 & sin\alpha & cos\alpha \\ sin\alpha & 0 & sin\beta \\ cos\alpha & -sin\beta & 0 \end{bmatrix}$.

22) Evaluate $\begin{vmatrix} 2014 & 2017 & 0 \\ 2020 & 2023 & 1 \\ 2023 & 2026 & 0 \end{vmatrix}$

23) Without expanding, evaluate the following determinants:

$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

24) If (k, 2), (2, 4) and (3, 2) are vertices of the triangle of area 4 square units then determine the value of k.

25) Show that $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix} = 0$

III. ANSWER ANY 7 QUESTION. 7 x 3 = 21

26) If $A = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 1 & 5 \\ 0 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & -1 \\ 3 & -1 & 4 \\ -1 & 2 & 1 \end{bmatrix}$ verify $(A+B)^T = A^T + B^T$

27) If $A = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 1 & 5 \\ 0 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & -1 \\ 3 & -1 & 4 \\ -1 & 2 & 1 \end{bmatrix}$

verify $(3A)^T = 3A^T$

28) If A = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $A^3 - 6A^2 + 7A + KI = O$, find the value of k.

29) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$ = (x - y) (y - z) (z - x).

30) Show that $\begin{vmatrix} b+c & bc & b^2+c^2 \\ c+a & ca & c^2+a^2 \\ a+b & ab & a^2+b^2 \end{vmatrix} = 0$

31) Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

32) If a, b, c are p^{th}, q^{th} and r^{th} terms of an A.P, find the value of $\begin{vmatrix} a & b & c \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$

33) Using Factor Theorem, prove that $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix}$ = $(x - 1)^2 (x + 9)$.

34) Verify that | AB | = | A | | B | if A = $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ and B = $\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$

35) For any square matrix A with real number entries, $A + A^T$ is a symmetric matrix and $A - A^T$ is a skew-symmetric matrix.

IV. ANSWER ALL QUESTION. 7 x 5 = 35

36) a) Solve for x if $\begin{bmatrix} x & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ -1 & -4 & 1 \\ -1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ 2 \\ 1 \end{bmatrix}$ =0

(OR)

b) Verify that det(AB) = (det A) (det B) for A = $\begin{bmatrix} 4 & 3 & -2 \\ 1 & 0 & 7 \\ 2 & 3 & -5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 3 & 3 \\ -2 & 4 & 0 \\ 9 & 7 & 5 \end{bmatrix}$.

37) a) Show that $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ab & bc & a^2+b^2 \end{vmatrix}$

(OR)

b) Prove that $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}^2 = \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix}$

38) a) Solve the following problems by using Factor Theorem :

Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix}$ = (a + b + c)(a - b)(b - c)(c - a).

(OR)

b) Show that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$.

39) a) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

(OR)

b) If a, b, c are all positive and are p^{th}, q^{th} and r^{th} terms of a G.P., show that $\begin{vmatrix} log\,a & p & 1 \\ log\,b & q & 1 \\ log\,c & r & 1 \end{vmatrix} = 0$.

40) a)

Prove that $|A| = \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix}$ = 2pqr(p + q + r)^3.

(OR)

b)

Solve the following problems by using Factor Theorem :

Solve $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$.

41) a)

Find the value of the product $\begin{vmatrix} log_3 64 & log_4 3 \\ log_3 8 & log_4 9 \end{vmatrix} \times \begin{vmatrix} log_2 3 & log_8 3 \\ log_3 4 & log_3 4 \end{vmatrix}$

(OR)

b)

Solve the following problems by using Factor Theorem :

Show that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$

42) a)

Using cofactors of elements of second row, evaluate | A |, where A = $\begin{bmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

(OR)

b)

Prove that $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$ = (x - y)(y - z)(z - x)(xy + yz + zx).

ALL THE BEST

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