Class : 12

Register Number

FIRST REVISION EXAMINATION, JANUARY - 2025

Time Allowed : 3.00 Hours] **MATHEMATICS** [Max. Marks : 90

PART - I

I. Answer all the questions: 20x1=20

1. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$ then λ is

   a) 17 b) 14 c) 19 d) 21

2. If $x+y=k$ is a normal to the Parabola $y^2=12x$, then the value of k is

   a) 3 b) -1 c) 1 d) 9

3. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

   a) 2ab b) ab c) $\sqrt{ab}$ d) $\frac{a}{b}$

4. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

   a) cis $\frac{2\pi}{3}$ b) cis $\frac{4\pi}{3}$ c) -cis $\frac{2\pi}{3}$ d) -cis $\frac{4\pi}{3}$

5. If $|z-z_1| = |z-z_2|$, the locus of z is

   a) the perpendicular bisector of line joining z_1 and z_2

   b) a line parallel to the line joining the points z_1 and z_2

   c) a circle, where z_1 and z_2 are the end points of a diameter

   d) a line joining z_1 and z_2

6. The value of the limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is

   a) 0 b) 1 c) 2 d) ∞

7. A is orthogonal and consides the statements and select the suitable option

   a) $A^{-1} = A^T$ b) $AA^T = A^TA = I$

   (a) A and B are true b) A only true c) B only true d) both are false

8. If α, β and γ are the zeros of x^3+px^2+qx+r, then $\Sigma\frac{1}{\alpha}$ is

   a) $\frac{-q}{r}$ b) $\frac{-p}{r}$ c) $\frac{q}{r}$ d) $\frac{-q}{p}$

9. The equation $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

   a) no solution b) unique solution

   c) two solution d) infinite number of solution

10. $\sin(\tan^{-1}x)$, $|x|<1$ is equal to

a) $\frac{x}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{x}{\sqrt{1+x^2}}$

11. The vertical Asymptote of $f(x) = \frac{1}{x}$ is

a) $x = 0$ b) $y = 0$ (c) $x = c$ (d) $y = c$

12. The value of $\int_0^1 x(1-x)^{99}\,dx$ is

a) $\frac{1}{11000}$ b) $\frac{1}{10100}$ c) $\frac{1}{10010}$ d) $\frac{1}{10001}$

13. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}+\vec{c}}{2}$, then the angle between $\vec{a}$ and $\vec{b}$ is

a) $\frac{\pi}{2}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{4}$ d) π

14. If $f(x, y, z) = xy + yz + zx$, then $f_x - f_z$ is equal to

a) $z - x$ b) $y - z$ c) $x - z$ d) $y - x$

15. The non-parametric form of a vector equation passing through two points, whose position vectors are $\vec{a}$ and $\vec{b}$ are parallel to $\vec{u}$ is

a) $[\vec{r}-\vec{u}\ \ \vec{b}-\vec{a}\ \ \vec{u}] = 0$ b) $[\vec{r}-\vec{a}\ \ \vec{u}-\vec{a}\ \ \vec{u}] = 0$

c) $[\vec{r}-\vec{a}\ \ \vec{b}-\vec{a}\ \ \vec{u}] = 0$ d) $[\vec{r}-\vec{a}\ \ \vec{b}-\vec{a}\ \ \vec{u}] = 0$

16. The change in the surface are $s = 6x^2$ of a cube when the edge length varies from x_0 to $x_0 + dx$ is

a) $12x_0 + dx$ b) $12x_0dx$ c) $6x_0dx$ d) $6x_0+dx$

17. The value of $\int_0^a \left(\sqrt{a^2-x^2}\right)^3 dx$ is

(a) $\frac{\pi a^3}{16}$ (b) $\frac{3\pi a^4}{16}$ (c) $\frac{3\pi a^2}{8}$ (d) $\frac{3\pi a^4}{8}$

18. The order and degree of the differential equation $\frac{dy}{dx} + xy = \cot x$ are

a) 1 , 0 b) 1 , 1 c) 0 , 1 d) 0 , 0

19. If $P(x = 0) = 1 - P(x = 1)$. If $E(x) = 3\,var(x)$, then $P(x = 0)$ is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

20. The dual of $\neg(p \vee q) \vee [p \vee (p \wedge \neg r)]$ is

a) $\neg(p \wedge q) \wedge [p \vee (p \wedge \neg r)]$ b) $(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$

c) $\neg(p \wedge q) \wedge [p \wedge (p \wedge r)]$ d) $\neg(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$

PART - II

II. Answer any 7 Questions. Question Nunmber 30 is compulsory. **7x2=14**

21. Find the rank of the matrix by using minor method. $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 2 & -4 \end{bmatrix}$

22. A stone is dropped into a pond causing ripples in the form of concentric circles. The radius 'r' of the outer ripple is increasing at a constant rate at 2cm per second. When the radius is 5cm find the rate of changing of the total area of the disturbed water?

23. Find the volume of the parallelepiped whose coterminus edges (adjacent edges) are given by the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$; $\hat{i} + 2\hat{j} - \hat{k}$ and $3\hat{i} - \hat{j} + 2\hat{k}$.

24. If α and β are the roots of the quadratic equation $17x^2+43x-73=0$, construct a quadrate equation whose roots are $\alpha+2$ and $\beta+2$.

25. Evaluate : $\lim\limits_{(x,y)\to(0,0)} \cos\left(\dfrac{x^3+y^3}{x+y+2}\right)$, if the limit exists

26. Evaluate : $\int_0^{\infty} x^5 e^{-3x}\,dx$.

27. Obtain the equation of the circle for which (3, 4) and (2, -7) are the ends of a diameter.

28. Show that $x^2+y^2=r^2$ where 'r' is a constant, is a solution of the differential equation $\dfrac{dy}{dx} = \dfrac{-x}{y}$

29. For the random variable x with the given probability mass function $f(x) = \begin{cases} 1/10, & x = 2, 5 \\ 1/5, & x = 0,1,3,4 \end{cases}$ find the mean.

30. Show that the points representing complex numbers $7+9i$; $-3+7i$; $3+3i$ form a right angled triangle in the Argand plane.

PART - III

III. **Answer any 7 Questions. Question Number 40 is compulsory.** **7x3=21**

31. Show that the percentage error in the n^{th} root of a number is approximately $1/n$ times the percentage error in the number.

32. Shown that $\left(\dfrac{\sqrt{3}}{2} + \dfrac{i}{2}\right)^5 + \left(\dfrac{\sqrt{3}}{2} - \dfrac{i}{2}\right)^5 = -\sqrt{3}$.

33. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}.

34. Find the foci, vertices and length of major and minor axis of the Conic $4x^2+36y^2+40x-288y+532=0$

35. Find the value of $\sin^{-1}\left[\cos\left(\sin^{-1}\left(\sqrt{3}/2\right)\right)\right]$

36. Find the Absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on [-3, 2]

37. Find the Value of $\int_0^{\pi/2} \sin^4 x \cos^6 x \, dx$

38. Solve : $\dfrac{dy}{dx} = e^{x+y} + x^3 e^y$

39. The Probability density function of x is given by $f(x) = \begin{cases} Kxe^{-2x} & \text{for } x > 0 \\ 0, & \text{for } x \le 0 \end{cases}$ find the Value of K

40. Discuss the real and imaginary roots of $x^5 + x^3 + x^2 + 1 = 0$.

PART - IV

IV. Answer all the questions **7x5=35**

41. a) Let z_1, z_2 and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r>0$ and $z_1+z_2+z_3 \neq 0$. Prove that

$$\left|\frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_1 + z_2 + z_3}\right| = r$$

(OR)

b) Evaluate: $\lim_{x \to 0^+} x^x$, if necessary use *l* Hopital Rule.

42. a) Investigate the values of λ and μ the system of linear equation $2x+3y+5z=9$, $7x+3y-5z=8$, $2x+3y+\lambda z=\mu$, we have

i) No solution ii) A unique solution iii) an infinite number of solution. **(OR)**

b) Prove by Vector method, $\sin(\alpha+\beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

43. a) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, then prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots + \tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right] = \frac{a_n - a_1}{1+a_1a_n}$$

(OR)

b) Find the area of the region bounded by $y=\cos x$, $y=\sin x$, the lines $x = \pi/4$ and $x = 5\pi/4$

44. a) Solve the equations: $6x^4-35x^3+62x^2-35x+6=0$. **(OR)**

b) Find intervals of concavity and points of inflection for the following function: $f(x) = x(x-4)^3$.

45. a) If $V(x, y, z) = x^3+y^3+z^3 + 3xyz$, show that $\frac{\partial^2 V}{\partial y\, \partial z} = \frac{\partial^2 V}{\partial z\, \partial y}$ **(OR)**

b) Find the centre, foci and eccentricity of the hyperbola $11x^2-25y^2-44x+50y-256=0$.

46. a) In a murder investigation, a corpse was found by a detective at exactly 8P.M. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours, later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F and assuming that the body temperature of the person before deathwas 98.6°F, at what time did the murder occur?

[log (2.43) = 0.88789; log(0.5) = -0.69315] **(OR)**

b)(i)Define an operation * on Q as follows: $a * b = \left(\frac{a+b}{2}\right)$; $a, b \in Q$, Examine the closure, commutative, associative, the existence of identify and the existence of inverse for the operation * on Q.

47. a) The probability density function of the random variable x is given by

$$f(x) = \begin{cases} 16x e^{-4x}, & \text{for } x>0 \\ 0, & \text{for } x \leq 0 \end{cases}$$

Find the mean and variance of x.

(OR)

b) Find the vector (parametric and non-parametric) and cartesian Equation of the plane passing through the point (1, -2, 4) and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$