FIRST REVISION EXAM - JANUARY - 2025

STD - 12 **Mathematics**

Time Allowed : 3.00 Hours **Maximum Marks : 90**

Part - I

Note : i) All questions are compulsory. ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer. $20 \times 1 = 20$

1. A zero of $x^3 + 64$ is

   (1) 0 (2) 4 (3) $4i$ (4) -4

2. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive root, if and only if

   (1) $a \geq 0$ (2) $a > 0$ (3) $a < 0$ (4) $a \leq 0$

3. If $\sin^{-1} x = 2\sin^{-1} \alpha$ has a solution, then

   (1) $|\alpha| \leq \frac{1}{\sqrt{2}}$ (2) $|\alpha| \geq \frac{1}{\sqrt{2}}$ (3) $|\alpha| < \frac{1}{\sqrt{2}}$ (4) $|\alpha| > \frac{1}{\sqrt{2}}$

4. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$

   (1) 0 (2) $\sin\theta$ (3) $\cos\theta$ (4) 1

5. In the case of Cramer's rule which of the following are correct?

   (i) $\Delta = 0$ (ii) $\Delta \neq 0$ (iii) the system has unique solution
   (iv) the system has infinitely many solutions

   (1) i andiv (2) Only ii (3) all (4) None of them

6. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

   (1) -2 (2) -1 (3) 1 (4) 2

7. $cis\ \frac{28\pi}{5} =$

   (1) $cis\left(-\frac{2\pi}{5}\right)$ (2) $cis\ \frac{2\pi}{5}$ (3) $cis\ \frac{3\pi}{5}$ (4) $cis\left(-\frac{3\pi}{5}\right)$

8. The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of $x^2 - (a+b)x - 4 = 0$, then the value of $(a+b)$ is

   (1) 2 (2) 4 (3) 0 (4) -2

9. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a} \times \vec{b}] = \frac{\pi}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

   (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

10. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then

    (1) $c = \pm 3$ (2) $c = \pm\sqrt{3}$ (3) $c > 0$ (4) $0 < c < 1$

11. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1,9]$ is

(1) 2 (2) 2.5 (3) 3 (4) 3.5

12. Lanrange mean value theorem becomes Rolls theorem if

(1) $f(a) = f(b)$ (2) $f'(a) = f'(b)$ (3) $f(a) = 0$ (4) $f(b) = 0$

13. If $u(x, y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

(1) $e^{x^2+y^2}$ (2) $2xu$ (3) x^2u (4) y^2u

14. If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then n is

(1) 10 (2) 5 (3) 8 (4) 9

15. The order and degree of the differential equation $\sqrt{sinx}(dx + dy) = \sqrt{cosx}(dx - dy)$are

1) 1 , 2 2) 2 , 2 3) 1 , 1 4) 2 , 1

16. Integrating factor of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+1}$ is

(1) $\frac{1}{x+1}$ (2) $x + 1$ (3) $\frac{1}{\sqrt{x+1}}$ (4) $\sqrt{x+1}$

17. If $P\{X = 0\} = 1 - P\{X = 1\}$. If $E[X] = 3Var(X)$, then $P\{X = 0\}$.

(1) $\frac{2}{3}$ (2) $\frac{2}{5}$ (3) $\frac{1}{5}$ (4) $\frac{1}{3}$

18. Which one of the following is a binary operation on N ?

(1) Subtraction (2) Multiplication (3) Division (4) All the above

19. A random variable$X$is function from

(1) $S \to R$ (2) $R \to S$ (3) $S \to N$ (4) $N \to S$

20. Which one of the following statements has the truth value ?

(1) sin$x$is an even function.
(2) Every square matrix is non-singular
(3) The product of complex number and its conjugate is purely imaginary
(4) $\sqrt{5}$ is an irrational number

Part – II

Note : i) Answer any Seven questions. ii) Question number 30 is compulsory. 7 ×2 = 14

21. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form.

22. If adj$A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$,find A^{-1}.

23. If $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}, \vec{b} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}, \vec{c} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$, find $\vec{a} \cdot (\vec{b} \times \vec{c})$.

24. Explain why Rolle's theorem is not applicable to the following functions in the

respective intervals : $f(x) = \left|\frac{1}{x}\right|, x \in [-1,1]$.

25. Evaluate : $\int_0^{\frac{\pi}{2}} cos^7 x\, dx$

26. If $u(x, y, z) = log(x^3 + y^3 + z^3)$, find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.

27. Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant .

28. Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}$ as a root.

29. Let $A = \{a + \sqrt{5}b : a, b \in \mathbb{Z}\}$. Check whether the usual multiplication is a binary operation on A.

30. Find the general equation of the circle whose diameter is the line segment joining the points $(-4,-2)$ and $(-1,-1)$.

Part – III

Note : i) Answer any Seven questions. ii) Question number 40 is compulsory. $7 \times 3 = 21$

31. Solve the following systems of linear equations by Cramer's rule:

    $5x - 2y + 16 = 0, \quad x + 3y - 7 = 0$

32. If $z = (cos\,\theta + i\,sin\,\theta)$, show that $z^n + \frac{1}{z^n} = 2\,cos\,n\,\theta$ and $z^n - \frac{1}{z^n} = 2i\,sin\,n\,\theta$.

33. Find the value of $sin^{-1}\left(sin\frac{5\pi}{9}cos\frac{\pi}{9} + cos\frac{5\pi}{9}sin\frac{\pi}{9}\right)$.

34. For any vector $\vec{a}$, prove that $\hat{\imath} \times (\vec{a} \times \hat{\imath}) + \hat{\jmath} \times (\vec{a} \times \hat{\jmath}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$.

35. Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

36. Find the linear approximation for $f(x) = \sqrt{1+x}, x \geq -1$, at $x_0 = 3$. Use the linear approximation to estimate $f(3.2)$.

37. Solve *the following* differential equations : $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

38. For the random variable X with the given probability mass function as below, find the mean and variance : $f(x) = \begin{cases} 2(x-1), 1 < x < 2 \\ 0, \text{otherwise} \end{cases}$

39. Using the equivalence property, show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.

40. Prove that $\int_0^{\frac{\pi}{2}} \frac{f(cosx)}{f(sinx)+f(cosx)} dx = \frac{\pi}{4}$.

Part – IV

Note : i) Answer all the questions. 7×5 = 35

41. a)Let A be $A = Q\backslash\{1\}$. Define * on A by $x * y = x + y - xy$ is * binary on A? If so , examine the commutative , associative , the existence of identity and the Existence of inverse properties . **(OR)**

b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

42 . a) A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find (i) $P(2 < X < 6)$ (ii) $P(2 \leq X < 5)$ (iii) $P(X \leq 4)$ (iv) $P(3 < X)$. **(OR)**

b) The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

43. a)Prove by vector method that the perpendiculars (altitudes) from the vertices to the opposite sides of a triangle are concurrent. **(OR)**

b) Solve : $tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

44. a) Test for consistency and if possible, solve the following systems of equations by rank Method :$2x + 2y + z = 5,\ x - y + z = 1,\ \ 3x + y + 2z = 4$ **(OR)**

b) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1,2,0), (2,2,-1)$ and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.

45. a) Solve the equation $z^3 + 27 = 0$. **(OR)**

b) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

46. a) Suppose a person deposits ₹10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later? **(OR)**

b) Find the local extrema of the function $f(x) = 4x^6 - 6x^4$.

47. a)Find the foci, vertices and length of major and minor axis of the conic $4x^2 + 36y^2 + 40x - 288y + 532 = 0$. **(OR)**

b) If $v(x,y) = log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$.