Class : 12

Register Number

FIRST REVISION EXAMINATION, JANUARY - 2025

Time Allowed : 3.00 Hours] **MATHEMATICS** [Max. Marks : 90

PART - I

I. Answer all the questions.

20x1=20

1. If A, B and C are invertible matrices of some order, then which one of the following is not true?
   a) $\text{adj } A = |A| A^{-1}$ b) $\text{adj}(AB) = (\text{adj } A)(\text{adj } B)$
   c) $A^{-1} = (\det A)^{-1}$ d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

2. If $z = \dfrac{(\sqrt{3}+i)^3 (3i+4)^2}{(8+6i)^2}$ then $|z|$ is equal to
   a) 0 b) 1 c) 2 d) 3

3. If f and g are polymomials of degrees m and n respectively, and if h(x) = (fog) (x), then the degree of h is
   a) mn b) m + n c) m^n d) n^m

4. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \dfrac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \dfrac{9}{x^{101} + y^{101} + z^{101}}$ is
   a) 0 b) 1 c) 2 d) 3

5. If $\cot^{-1}(\sqrt{\sin \alpha}) + \tan^{-1}(\sqrt{\sin \alpha}) = u$, then cos 2u is equal to
   a) $\tan^2\alpha$ b) 0 c) -1 d) $\tan 2\alpha$

6. The equation of the normal to the circle $x^2+y^2-2x-2y+1=0$ which is parallel to the line $2x+4y=3$ is
   a) $x+2y=3$ b) $x+2y+3=0$ c) $2x+4y+3=0$ d) $x-2y+3=0$

7. If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$, where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{b} \cdot \vec{c} \neq 0$ and $\vec{a} \cdot \vec{b} \neq 0$, then $\vec{a}$ and $\vec{c}$ are
   a) perpendicular b) parallel
   c) inclined at an angle $\pi/3$ d) inclined at an angle $\pi/3$

8. What is the value of the $\lim\limits_{x \to \infty} \left(\cot x - \dfrac{1}{x}\right)$?
   a) 0 b) 1 c) 2 d) ∞

9. If $w(x, y, z) = x^2(y-z) + y^2(z-x) + z^2(x-y)$, then $\dfrac{\partial w}{\partial x} + \dfrac{\partial w}{\partial y} + \dfrac{\partial w}{\partial z}$ is
   a) $xy + yz + zx$ b) $x(y+z)$ c) $y(z+x)$ d) 0

10. The value of $\int_{\pi/2}^{\pi/2} \sin^2 x \cos x \, dx$ is
   a) $\dfrac{3}{2}$ b) $\dfrac{1}{2}$ c) 0 d) $\dfrac{2}{3}$

11. The order and degree of the differential equation $\sqrt{\sin x}(dx + dy) = \sqrt{\cos x}(dx - dy)$ is
   a) 1, 2 b) 2, 2 c) 1, 1 d) 2, 1

12. The population P in any year t is such that the rate of increase in the population is proportional to the population. Then

a) $P = Ce^{kt}$ b) $P = Ce^{-kt}$ c) $P = Ckt$ d) $P = C$

13. If $P(x = 0) = 1 - P(x=1)$. If $E(x) = 3\,Var(x)$, then $P(x = 0)$ is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

14. Which one of the following is a binary operation on N?

a) Subtraction b) Multiplication c) Division d) All the above

15. The dual of $\neg(p\vee q) \vee [p\vee(p\wedge\neg r)]$ is

a) $\neg(p\wedge q) \wedge [p\vee(p\wedge\neg r)]$ b) $(p\wedge q) \wedge [p\wedge(p\vee\neg r)]$

c) $\neg(p\wedge q) \wedge [p\wedge(p\wedge r)]$ d) $\neg(p\wedge q) \wedge [p\wedge(p\vee\neg r)]$

16. $z^{-1} = \bar{z}$ then

a) $z = 1$ b) $|z| = 1$ c) $z = -z$ d) $z = -\bar{z}$

17. If $x^2 + 2(k + 2)x + 9k = 0$ has equal roots then values of k is

a) -1 b) -4 c) 4 d) 0

18. $3\vec{i} + 3\vec{j} + 2\vec{k}$ and $\vec{i} - m\vec{j} + 3\vec{k}$ are perpendicular then value of m is

a) -3 b) -1 c) 2 d) 3

19. If the normal is parallel to x- axis then

a) $\frac{dy}{dx} = 1$ b) $\frac{dy}{dx} = 0$ c) $\frac{dx}{dy} = 0$ d) $\frac{dx}{dy} = k$

20. $\Gamma n =$ a) $n!$ b) $(n + 1)!$ c) $(n - 1)\Gamma(n - 1)$ d) $(n + 1)\Gamma(n+1)$

PART - II

II. Answer any 7 Questions. Each question carries 2 marks.

Question Nunmber 30 is compulsory. **7x2=14**

21. Find the differential equation of the family of all the parabolas with latus rectum 4a and whose axes are parallel to the x -axis.

22. If $\frac{z + 3}{z - 5i} = \frac{1 + 4i}{2}$, find the complex number z in the rectangular form

23. Discuss the maximum possible number of positive and negative roots of the polynomial equation.

$9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.

24. If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, find the value of $\cos\theta$.

25. If $A^3 = A$ then find A^{-1}.

26. Find the angles between the straight line $\frac{x + 3}{2} = \frac{y - 1}{2} = -z$ with coordinate axes

27. Find maclaurin's series for $\frac{1}{1 - x}$

28. Evaluate $\lim_{(x,y)\to(1,2)} g(x,y)$, if the limit exists, where $g(x,y) = \dfrac{3x^2 - xy}{x^2+y^2+3}$

29. Evaluate the following integrals using properties of integration: $\int_{-\pi/4}^{\pi/4} \sin^2 x \, dx$

30. Write the Properties of cumulative distribution function.

PART - III

III. Answer any 7 Questions. Each question carries 3 marks.

Question Nunmber 40 is compulsory. **7x3=21**

31. Solve the following system of homogenous equations.

$3x + 2y + 7z = 0,\ 4x - 3y - 2z = 0,\ 5x + 9y + 23z = 0.$

32. Simplify $\left(\sin\dfrac{\pi}{6} + i\cos\dfrac{\pi}{6}\right)^{18}$

33. Find the domain of (i) $f(x) = \sin^{-1}\left(\dfrac{|x| - 2}{3}\right) + \cos^{-1}\left(\dfrac{1 - |x|}{4}\right)$ (ii) $g(x) = \sin^{-1}x + \cos^{-1}x$

34. Examine the position of the point (2, 3) with respect to the circle $x^2+y^2-6x-8y+12=0$

35. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$.

If the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$, show that $[\vec{a}, \vec{b}, \vec{c}]^2 = \frac{1}{4}|\vec{a}|^2 |\vec{b}|^2$.

36. Sketch the graphs of the following functions: $y = x\sqrt{4 - x}$

37. If $w(x, y, z) = x^2y + y^2z + z^2x$, $x, y, z \in R$, find the different dw.

38. Find , by integration, the volume of the solid generated by revolving by revolving about the y-axis, the region enclosed by $x^2=1+y$ and $y=3$.

39. The mean and variance of a binomial variate X are respectively 2 and 1.5.

Find (i) $P(X = 0)$ (ii) $P(X = 1)$ (iii) $P(X \geq 1)$.

40. Find the value of $\int_0^1 x(1 - x)^n \, dx$.

PART - IV

IV. Answer all the questions. Each question carries 5 marks. **7x5=35**

41. a) Solve the following system of linear equations by Cramer's rule:

$\dfrac{3}{x} - \dfrac{4}{y} - \dfrac{2}{z} - 1 = 0,\ \dfrac{1}{x} + \dfrac{2}{y} + \dfrac{1}{z} = 0,\ \dfrac{2}{x} - \dfrac{5}{y} - \dfrac{4}{z} + 1 = 0.$

(OR)

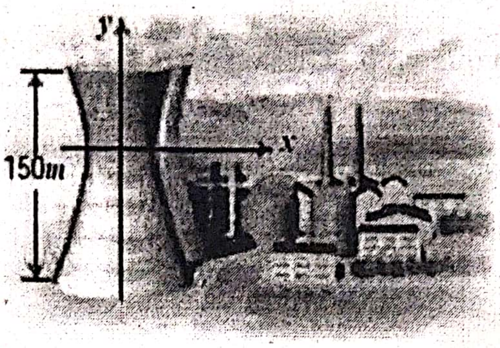
b) Show that $\left(\dfrac{19 + 9i}{5 - 3i}\right)^{15} - \left(\dfrac{8 + i}{1 + 2i}\right)^{15}$ is purely imaginary.

42. a) Solve : $(2x - 1)(x + 3)(x - 2)(2x + 3) + 20 = 0$

(OR)

b) Prove that $\tan(\sin^{-1} x) = \dfrac{1}{\sqrt{1 - x^2}}$, $-1 < x < 1$

43. a) Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150m tall and the distance from the top of the tower to the centre of the hyper bola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter o the top and base of the tower.



(OR)

b) If $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} - \hat{j} - 4\hat{k}$, $\vec{c} = 3\hat{j} - \hat{k}$ and $\vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$, Verify that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$

44. a) Verify the following points (1,3,1) (1,1,-1), (-1,1,1), (2,2,-1) are coplanar. (OR)

b) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

45. a) A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (Brine is a high -concentration solution of salt (usually sodium chloride) in water) runs in a rate of 10 litres per minute, and each litre contain 5 grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t. (OR)

b) Evaluate: $\lim_{x \to 0^+} x^{\sin x}$

46. a) Father of a family wishes to divide his equare field bounded by x = 0, x = 4, y = 4 and y = 0 along the curve $y^2 = 4x$ and $x^2 = 4y$ into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them. (OR)

b) Solve $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$

47. a) If X is the random variable with probability density function f(x) given by, $f(x) = \begin{cases} x+1 & , -1 \le x < 0 \\ -x+1 & 0 \le x < 1 \\ 0, & \text{otherwise} \end{cases}$

then find (i) the distribution function F(x) (ii) $P = (-0.5 \le X \le 0.5)$ (OR)

b) (i) Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x R - \{0\} \right\}$ and let * be the matrix multiplication. Determine whether M us closed under *. If so, examine the commutative and associative properties satisfied by * on M.

(ii) Also examine the existence of identity, existence of inverse properties for the operation *on M.