$|z_1 + z_2 + z_3| = 1$ show that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$

42. a) Find all Zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$ if its known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros. **(OR)**

b) Solve $\tan^{-1}\left[\frac{x-1}{x-2}\right] + \tan^{-1}\left[\frac{x+1}{x+2}\right] = \frac{\pi}{4}$

43. a) A bridge has a parabolic arch that is 10m height in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on either sides. **(OR)**

b) By vector method, prove that $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

44. a) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally if $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$ **(OR)**

b) Prove that $f(x, y) = x^3 - 2x^2y + 3xy^2 + y^3$ is homogeneous, what is the degree? Verify Euler's Theorem for f.

45. a) A pot of boiling water at 100°c is removed from a stove at time t = 0 and left to cool in the kitchen. After 5 minutes, The water temperature has decreased to 80°c, and another 5 minutes later it has dropped to 65°c. Determine the temperature of the kitchen. **(OR)**

b) The cumulative distribution function of a discrete random variable is given by

$$F(x) = \begin{cases} 0 & \text{for } -\infty < x < 0 \\ 1/2 & \text{for } 0 \le x < 1 \\ 3/5 & \text{for } 1 \le x < 2 \\ 4/5 & \text{for } 2 \le x < 3 \\ 9/10 & \text{for } 3 \le x < 4 \\ 1 & \text{for } 4 \le x < \infty \end{cases}$$

Find (i) the probability mass function ii) $p(x < 3)$ and iii) $p(x \ge 2)$

46. a) i) Define an operation $\bullet$ on Q as follows $a \bullet b = \left(\frac{a+b}{2}\right)$ a, b $\in$ Q. Examine the closure commutative, associative properties satisfied by $\bullet$ on Q.

ii) Define an operation $\bullet$ on Q as follows $a \bullet b = \left(\frac{a+b}{2}\right)$ a, b $\in$ Q. Examine the existence of Indentify and the existence of inverse for the operation $\bullet$ on Q. **(OR)**

b) Find the intervals of increasing, decreasing concavity and point of inflection of the curve $y = x^3 - 3x$.

47. a) Show that the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{3}$ and $\frac{x-1}{-3} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar. Also find the Cartesian equation of the plane containing these lines. **(OR)**

b) Find the area of the region bounded between the parabolas $y^2 = x$ and $x^2 = y$.

FIRST REVISION TEST - 2025

Standard - XII

Reg No

Time: 3.00 hrs. **MATHEMATICS** *Marks: 90*

PART - A

I. **Answer all the questions:** **20×1=20**

1. If A is a 3 × 3 non - singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T =
a) A b) B c) I_3 d) B^T

2. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is
a) 1 b) 2 c) 3 d) 4

3. If z is a complex number such that $z \in C \setminus R$ and $z + \frac{1}{z} \in R$, then $|z|$ is
a) 0 b) 1 c) 2 d) 3

4. If α, β and γ are the roots of $x^3 + px^2 + qx + r = 0$ then $\Sigma \frac{1}{\alpha}$ is
a) $\frac{-q}{r}$ b) $\frac{-p}{r}$ c) $\frac{q}{r}$ d) $-\frac{q}{p}$

5. The domain of the function which is defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is
a) [1, 2] b) [-1, 1] c) [0, 1] d) [-1, 0]

6. If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$, where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{b} \cdot \vec{c} \neq 0$ and $\vec{a} \cdot \vec{b} \neq 0$ then $\vec{a}$ and $\vec{c}$ are
a) perpendicular b) parallel c) inclined at angle $\pi/3$ d) inclined at an angle $\pi/6$

7. $\lim_{x \to 0}\left(\cot x - \frac{1}{x}\right)$ is
a) 0 b) 1 c) 2 d) ∞

8. If $u(x, y) = e^{x^2+y^2}$ then $\frac{\partial u}{\partial x}$ is equal to
a) $e^{x^2+y^2}$ b) 2xu c) x^2u d) y^2u

9. The approximate change in the volume v of a cube of side x metres caused by increasing the side by 1% is
a) 0.3 x dx m^3 b) 0.03 x m^3 c) 0.03 x^2 m^3 d) 0.03 x^3 m^3

10. The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is
a) $y + \sin^{-1}x = c$ b) $x + \sin^{-1}y = 0$ c) $y^2 + 2\sin^{-1}x = c$ d) $x^2 + 2\sin^{-1}y = 0$

11. A random variable x has binomial distribution with n = 25 and p = 0.8 then standard deviation is
a) 6 b) 4 c) 3 d) 2

12. In the set $\mathbb{Q}$ define $a \odot b = a + b + ab$ then the value of y in $3 \odot (y \odot 5) = 7$ is

a) $\frac{2}{3}$ b) $\frac{-2}{3}$ c) $\frac{-3}{2}$ d) 4

13. If P represents the rank and A and B are $n \times n$ matrices, then

a) [illegible] b) [illegible] c) [illegible] d) [illegible]

14. Identify the incorrect statement

a) [illegible] b) [illegible] c) [illegible] d) [illegible]

15. The amplitude and period of $y = a \tan bx$ are respectively

a) $|a|, \frac{\pi}{b}$ b) $a, \frac{\pi}{b}$ c) not defined, $\frac{\pi}{|b|}$ d) not defined, $\frac{\pi}{b}$

16. Which one is meaningful?

a) [illegible] b) [illegible] c) [illegible] d) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$

17. Rolle's constant c for the function $f(x) = |x|, x \in [-1, 1]$ is

a) 0 b) 1 c) -1 d) not existing

18. If $f(2a-x) = f(x)$ then $\int_0^{2a} f(x)\,dx =$

a) $2\int_0^a f(x)\,dx$ b) $\int_{-a}^{a} f(x)\,dx$ c) 0 d) $\int_0^a f(x)\,dx$

19. The integrating factor of $\frac{dx}{dy} + Px = Q$ is

a) [illegible] b) [illegible] c) [illegible] d) [illegible]

20. If $x + y = k$ is a normal to the parabola $y^2 = 12x$ then the value of k is

a) 3 b) -1 c) 1 d) 9

PART - B

II Answer any 7 questions. iii) Q.No.30 is compulsory 7×2=14

21. Find the value of [illegible]

22. Obtain the equation of circle for which (3, 4) and (2, -7) are the ends of a diameter.

23. Find the inverse of $\begin{pmatrix} -2 & 4 \\ 1 & -3 \end{pmatrix}$

24. Evaluate $\int_0^{\pi/2} \sin^{10} x\,dx$

25. Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right)$

26. Find the slope of the tangent to the curve $y = x^4 + 2x^2 - x$ at $x = 1$

27. Find the acute angle between the planes $\vec{r} \cdot (2\hat{i} + 2\hat{j} + 2\hat{k}) = 11$ and $4x - 2y + 2z = 15$

28. Show that $x^2 + y^2 = r^2$, where r is a constant is a solution of the differential equation $\frac{dy}{dx} = \frac{-x}{y}$

29. Prove that in an algebraic structure the identity element (if exists) must be unique.

30. If $f(x, y) =$ [illegible] then [illegible] $= 0$

PART - C

III Answer any 7 questions. iii) Q.No.40 is compulsory 7×3=21

31. Solve by Cramer's rule $5x - 2y + 16 = 0$, $x + 3y - 7 = 0$

32. Show that $\left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$ is real.

33. Prove that the length of the Latus Rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$

34. Find the magnitude and direction cosines of the torque of a force represented by [illegible] about the point (2, 0, -1) and through the origin.

35. If $\lim_{\theta \to 0}\left(\frac{1 - \cos m\theta}{1 - \cos n\theta}\right) = 1$ then prove that $m = \pm n$.

36. Evaluate $\int_0^{\pi/2} \frac{dx}{1 + 5\cos^2 x}$

37. Solve $(1 + x^2)\frac{dy}{dx} = 1 + y^2$

38. Suppose that f(x) given below represents probability mass function.

x	1	2	3	4	5	6
f(x)	c^2	$2c^2$	$3c^2$	$4c^2$	c	2c

Find i) value of C ii) Mean & variance

39. Construct the truth table for [illegible]

40. If α & β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$ then show that $\alpha^2 + \beta^2 + 3\alpha\beta = \frac{75}{4}$

PART - D

IV Answer all questions 7×5=35

41. a) A boy is walking along the path $y = ax^2 + bx + c$ through the points (-6, 8), (-2, -12) and (3, 8). He wants to meet his friend at (7, 60). Will he meet his friend? (Use Gaussian elimination method)

(OR)

b) If z_1, z_2 and z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and

12th Maths

M. Keerthana. MSc (Maths)

part-I

1. c I_3
2. a 1
3. b 1
4. a $\frac{-9}{r}$
5. a $[1,2]$
6. b parallel
7. a 0
8. b 2×4
9. d $0.03x^3m^3$
10. a $y+\sin^{-1}x = c$
11. d 2
12. b $-2/3$
13. d $P(A+B) \le n$
14. c $||z_1|-|z_2|| \ge ||z_1+z_2|$
15. c not defined, $\frac{\pi}{|b|}$
16. d $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$
17. d not existing
18. c 0
19. b $e^{\int p\,dx}$
20. d 9

Part-II

21. $\left|\frac{2+i}{-1+2i}\right| = \frac{\sqrt{2^2+1^2}}{\sqrt{1^2+2^2}} = 1$

22. $(3,4)$ and $(2,-7)$

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$(x-3)(x-2)+(y-4)(y+7)=0$

23. $A = \begin{pmatrix} -2 & 4 \\ 1 & -3 \end{pmatrix}$ $|A| = 6-4 = 2 \neq 0$

$\text{adj}\,A = \begin{pmatrix} -3 & -4 \\ -1 & -2 \end{pmatrix}$

$A^{-1} = \frac{1}{2}\begin{pmatrix} -3 & -4 \\ -1 & -2 \end{pmatrix}$

24. $\int_0^{\pi/2} \sin^{10}x\,dx = \frac{9}{10}\times\frac{7}{8}\times\frac{5}{6}\times\frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2}$

$= \frac{63\pi}{512}$

25. $\tan^{-1}(-1)+\cos^{-1}(\tfrac{1}{2})$

$-\pi/4 + \pi/3 = \frac{\pi}{12}$

26. $y = x^4+2x^2-x$

$y' = 4x^3+4x-1$

$y'_{x=1} = 4+4-1 = 7$

27. $\vec{n}_1 = 2\hat{i}+2\hat{j}+2\hat{k}$

$\vec{n}_2 = 4\hat{i}-2\hat{j}+2\hat{k}$

$\theta = \cos^{-1}\left(\frac{|\vec{n}_1\cdot\vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}\right)$

$= \cos^{-1}\left(\frac{(2\hat{i}+2\hat{j}+2\hat{k})\cdot(4\hat{i}-2\hat{j}+2\hat{k})}{\sqrt{4+4+4}\ \sqrt{16+4+4}}\right)$

$= \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$

28. $x^2+y^2 = r^2$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

29. identity e_1 & e_2

$e_1 * e_2 = e_2 * e_1 = e_2 \rightarrow (1)$

Interchanging the Role of e_1 and e_2

$e_2 * e_1 = e_1 * e_2 = e_1 \rightarrow (2)$

From (1) & (2) $e_1 = e_2$

30. $f(x,y) = \frac{x^2+y^2+xy}{x^2-y^2}$

$\frac{\partial f}{\partial x} = \frac{-x^2y-4xy^2-y^3}{(x^2-y^2)^2}$

$x\frac{\partial f}{\partial x} = \frac{-x^3y-4x^2y^2-y^3x}{(x^2-y^2)^2} \rightarrow (1)$

$\frac{\partial f}{\partial y} = \frac{4x^2y+xy^2+x^3}{(x^2-y^2)^2}$

$y\frac{\partial f}{\partial y} = \frac{4x^2y^2+xy^3+x^3y}{(x^2-y^2)^2} - (2)$

$(1)+(2) = 0.$

Part - III

31. $5x - 2y + 16 = 0$
$x + 3y - 7 = 0$

$\Delta = 17$, $\Delta x = \begin{vmatrix} -16 & -2 \\ 7 & 3 \end{vmatrix} = -34$

$\Delta y = \begin{vmatrix} 5 & -16 \\ 1 & 7 \end{vmatrix} = 51$

$\boxed{x = -2}$
$\boxed{y = 3}$

32. $\frac{19-7i}{9+i} \times \frac{9-i}{9-i} = \frac{1}{82}(19-7i)(9-i)$
$= 2-i$

$\frac{20-5i}{7-6i} \times \frac{7+6i}{7+6i} = \frac{1}{85}(20-5i)(7+6i)$
$\Rightarrow 2+i$

$z^{12} = r^{12}(\cos 12\theta - i\sin 12\theta)$

$(2-i)^{12} + (2+i)^{12} = r^{12}.2.\cos 12\theta$
$\Rightarrow$ real

33. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $S(ae, 0)$

L is (ae, y_1)

$\frac{a^2e^2}{a^2} + \frac{y_1^2}{b^2} = 1$

$\frac{y_1^2}{b^2} = 1 - e^2$

$y_1^2 = b^2(1-e^2)$

$y_1 = \pm \frac{b^2}{a}$

$\boxed{LL' = \frac{2b^2}{a}}$

34. $\vec{OA} = 2\hat{i} - \hat{k}$ $\vec{r} = \vec{AO} = -2\hat{i} + \hat{k}$

$\vec{F} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{t} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -\hat{i} - 2\hat{k}$

$r = \sqrt{5}$, torque, $\frac{-1}{\sqrt{5}}, 0, \frac{-2}{\sqrt{5}}$

35. $\lim_{\theta \to 0} \left(\frac{1-\cos m\theta}{1-\cos n\theta}\right) = 1.$

$\left(\frac{0}{0}\right)$ Form, using l'Hopital rule'.

$\lim_{\theta \to 0} \left(\frac{1-\cos m\theta}{1-\cos n\theta}\right) = \lim_{\theta \to 0}\left(\frac{m \sin m\theta}{n \sin n\theta}\right)$

$\lim_{\theta \to 0} \left(\frac{m^2}{n^2}\right)$ $m^2 = n^2$
$m = \pm n.$

36. $\int_0^{\pi/2} \frac{dx}{1+5\cos^2 x}$

$\Rightarrow \int_0^{\pi/2} \frac{1/\cos^2 x}{\frac{1}{\cos^2 x} + 5} dx = \int_0^{\pi/2} \frac{\sec^2 x}{6 + \tan^2 x} dx$

$t = \tan x$, $dt = \sec^2 x\, dx$

$x = 0$, $t = 0$, $x = \pi/2$, $t = \infty$

$I = \int_0^{\infty} \frac{dt}{6+t^2} = \frac{1}{\sqrt{6}}(\pi/2 - 0)$
$= \pi/2\sqrt{6}$

37. $(1+x^2)\frac{dy}{dx} = 1+y^2$

$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

$\tan^{-1} y = \tan^{-1} x + c.$

$\tan^{-1} y - \tan^{-1} x = \tan^{-1}\left(\frac{y-x}{1+xy}\right) = c$

$\frac{y-x}{1+xy} = \tan c = a$ (say)

$(y-x) = a(1+xy).$

38. (i) $c^2 + 2c^2 + 3c^2 + 4c^2 + 4c + 2c = 1$
$c = 1/5$ (or) $-1/2$

$\boxed{c = 1/5}$

(ii) Mean: $\Sigma x f(x) = \frac{115}{25} = 4.6$

(iii) variance: $V(x) = E(x^2) - (E(x))^2$
$= \frac{585}{25} - \left(\frac{115}{25}\right)^2$
$\Rightarrow 23.40 - 21.16 = 2.24.$

Mean $= 4.6$, variance $= 2.24.$

39. $(p \vee q) \vee \neg q$

p	q	$p \vee q$	$\neg q$	$(p\vee q)\vee \neg q$
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

40. $\alpha+\beta = 7/2,\ \alpha\beta = 13/2$

$\alpha^2+\beta^2+3\alpha\beta$

$(\alpha+\beta)^2+\alpha\beta = \left(\frac{7}{2}\right)^2+\frac{13}{2}$

$= \frac{49}{4}+\frac{13}{2}$

$= \frac{49+26}{4} = \frac{75}{4}$

$(x^2-2x+5)(x^2-3) = x^4-2x^3+2x^2+6x-15$

$x^6-3x^5-5x^4+22x^3-39x^2-39x+135$

$= (x^4-2x^3+2x^2+6x-15)(x^2+px-9)$

$\boxed{P=-1}$

$x^2-x-9=0$

$x = \frac{1\pm\sqrt{37}}{2}$

Roots $1+2i,\ 1-2i,\ \sqrt{3},\ -\sqrt{3},\ \frac{1\pm\sqrt{37}}{2}$

(b) $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right) = \pi/4$

$2x^2-4=3$

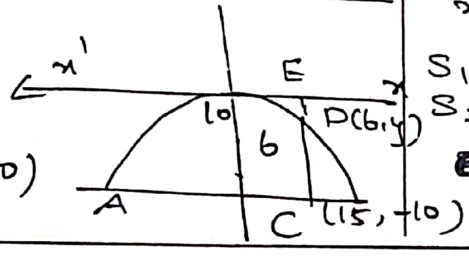
$x^2 = 1/2,\ x = \pm 1/\sqrt{2}$

43(a)

$x^2 = -4ay$

$(15)^2 = -4a(-10)$

$4a = \frac{225}{10}$



Part-IV

41(a) $36a-6b+c=8$

$4a-2b+c=-12$

$9a+3b+c=8$

$[A\ B] = \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 4 & -2 & 1 & -12 \\ 36 & -6 & 1 & 8 \\ 9 & 3 & 1 & 8 \end{array}\right]$ $R_1 \leftrightarrow R_2$

$\sim \left[\begin{array}{ccc|c} 4 & -2 & 1 & -12 \\ 0 & 12 & -8 & 116 \\ 0 & 30 & -5 & 140 \end{array}\right]$ $R_2 \to R_2 - 9R_1$, $R_3 \to 4R_3 - 9R_1$

$\sim \left[\begin{array}{ccc|c} 4 & -2 & 1 & -12 \\ 0 & 3 & -2 & 29 \\ 0 & 6 & -1 & 28 \end{array}\right]$ $R_2 \to R_2/4$, $R_3 \to R_3/5$

$\sim \left[\begin{array}{ccc|c} 4 & -2 & 1 & -12 \\ 0 & 3 & -2 & 29 \\ 0 & 0 & 3 & -30 \end{array}\right]$ $R_3 \to R_3 - 2R_2$

$c = -10$

$b = 3,\ a = 1$

$\boxed{y = x^2+3x-10}$ $P(7, 60)$

(b) $z_1 = \frac{1}{\bar{z}_1},\ z_2 = \frac{4}{\bar{z}_2},\ z_3 = \frac{9}{\bar{z}_3}$

$|9z_1z_2+4z_1z_3+z_2z_3| = \left|(z_1z_2z_3)\left(\frac{9}{\bar{z}_3}+\frac{4}{\bar{z}_2}+\frac{1}{\bar{z}_1}\right)\right|$

$\Rightarrow |z_1||z_2||z_3||z_1+z_2+z_3|$

$= 1\times2\times3\times1 = 6.$

42(a) $S_1 = (1+2i)+(1-2i) = 2$

$S_2 = (1+2i)(1-2i) = 5$

$x^2-2x+5=0.$

$S_1 = \sqrt{3}+(-\sqrt{3}) = 0$

$S_2 = \sqrt{3}\times-\sqrt{3} = 3$

$x^2+3=0$

$x^2 = -\frac{225}{10} y$

$36 = -\frac{225}{10} y$

$\boxed{y = -1.6}$

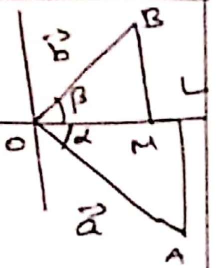
$DE = 1.6m$

$CD = CE - DE = 10 - 1.6 = 8.4m$

(b) $\vec{a} = \vec{OA} = \vec{OL} + \vec{LA}$

$= \cos\alpha \hat{i} - \sin\alpha \hat{j}$

$\vec{b} = \cos\beta \hat{i} + \sin\beta \hat{j}$



$\vec{a} \cdot \vec{b} = (\cos\alpha \hat{i} - \sin\alpha \hat{j})$

$(\cos\beta \hat{i} + \sin\beta \hat{j}) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\hat{a} \cdot \hat{b} = |\hat{a}||\hat{b}| \cos(\alpha+\beta) = \cos(\alpha+\beta)$

$\therefore \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

44(a) $ax^2 + by^2 = 1$ $\quad \frac{dy}{dx} = \frac{-ax}{by}$

$cx^2 + dy^2 = 1$, $\quad \frac{dy}{dx} = \frac{-cx}{dy}$

$\left(\frac{-ax_0}{by_0}\right) \times \left(\frac{-cx_0}{dy_0}\right) = -1$

$\frac{a-c}{ac} = \frac{b-d}{db}$

$\frac{1}{c} - \frac{1}{a} = \frac{1}{d} - \frac{1}{b}$

Hence proved

44(b) $f(x,y) = x^3 - 2x^2y + 3xy^2 + y^3$

$x\frac{\partial f}{\partial x} = 3x^3 - 4x^2y + 3y^2x \rightarrow (1)$

$y\frac{\partial f}{\partial y} = -2x^2y + 6xy^2 + 3y^3 \rightarrow (2)$

$(1) + (2)$ $\quad x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 3f.$

$f(x\lambda, y\lambda) = \lambda^3(x^3 - 2x^2y + 3xy^2 + y^3)$

$= \lambda^3 f(x,y)$

$\therefore$ It is homogeneous Function, with degree 3.

45(a) $T = S + Ce^{kt}$

$t = 0, T = 100,$

$100 = S + C$

$C = 100 - S$

$T = S + (100 - S)e^{kt}$

$t = 5, T = 80$

$80 = S + (100 - S)e^{5k}$

$t = 10, T = 65$

$65 = S + (100 - S)\left(\frac{80 - S}{100 - S}\right)^2$

$65(100 - S) = S(100 - S) + (80 - S)^2$

$\boxed{S = 20^\circ C}$

(b) (i)

0	1	2	3	4	x
$1/2$	$3/5$	$4/5$	$9/10$	1	$X = x$
$5/10$	$1/10$	$2/10$	$1/10$	$1/10$	$P(X = x)$

(ii) $P(X < 3) = \frac{5}{10} + \frac{1}{10} + \frac{2}{10}$

$= 4/5$

(iii) $P(X \geq 2) = \frac{2}{10} + \frac{1}{10} + \frac{1}{10}$

$= \frac{2}{5}$

46(a) Closure:

a, b are rational then $\frac{a+b}{2}$ is also rational.

Commutative:

$a * b = \frac{a+b}{2} = \frac{b+a}{2}$

$b * a$

$*$ is commutative

Associative:

$a * (b * c) = a * \left(\frac{b+c}{2}\right)$

$= \left(\frac{2a+b+c}{4}\right)$

$(a*b)*c = \left(\frac{a+b}{2}\right)*c$

$= \frac{a+b+2c}{4}$

Associative is not true.

Identity: $a*e = a$

$a*e = \frac{a+e}{2}$

by the defn of * $\frac{a+e}{2} = a$

$a+e = 2a \Rightarrow a = e$

Identity is unique

Inverse:

If identity is not satisfied we can't discuss inverse axiom.

(b) $f(x) = x^3 - 3x$ $f'(x) = 3x^2 - 3$

Interval $(-\infty, \infty)$ $f''(x) = 6x$

$(-\infty, 0) \cup (0, \infty)$

$f''(x) =$ negative, at $(-\infty, 0)$

$f''(x) \Rightarrow$ positive at $(0, \infty)$

Interval	Function	$f''(x)$	concave
$(-\infty, 0)$	$x^3 - 3x$	$6x$	$-(\downarrow)$
$(0, \infty)$	$x^3 - 3x$	$6x$	$+(\uparrow)$

Interval	$f'(x)$	Increasing (or) decreasing	
$(-\infty, 0]$	$3x^2 - 3$	$-$	$\swarrow$
$[0, \infty)$	$3x^2 - 3$	$+$	$\nearrow$

47(a) $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

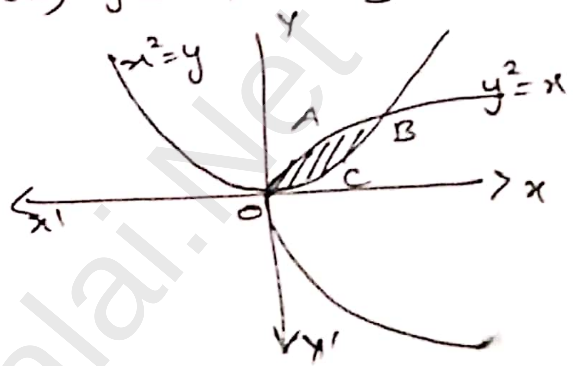
$\begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & 3 \\ -3 & 2 & 1 \end{vmatrix} = 0.$ Coplanar

$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

$\begin{vmatrix} x-2 & y-2 & z-4 \\ 1 & 1 & 3 \\ -3 & 2 & 1 \end{vmatrix} = 0$

$\boxed{x + 2y - z - 4 = 0}$

(b) $y^2 = x$, $x^2 = y$



$x^2 = y \rightarrow (1)$

$y^2 = x \rightarrow (2)$

put (1) in (2)

$y^2 = x$

$(x^2)^2 = x$

$x^4 - x = 0$

$x = 0$ | $x^3 = 1$

$y = x^2$ | $x = 1$

$y = 0, (0,0)$ | $y = x^2$ $(1,1)$

Area of ABC

$= OABD - OCBD$

$\int_0^1 y\,dx \Rightarrow \int_0^1 \sqrt{x}\,dx$

$\Rightarrow \int_0^1 x^{1/2}\,dx = 2/3$

$OCBD \Rightarrow \int_0^1 y^2\,dx = 1/3$

$OABC = \frac{2}{3} - \frac{1}{3} = 1/3$ Sq. unit