RS-1

FIRST REVISION EXAMINATION – 2025

12 - STD

MATHEMATICS

MARK : 90 TIME : 3.00 Hrs

PART – A

CHOOSE THE CORRET ANSWER :- 20 X 1 = 20

1) If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

(a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

2) If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

3) The number of real numbers in $[0,2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

(a) 2 (b) 4 (c) 1 (d) ∞

4) $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

5) The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is

(a) $4(a^2+b^2)$ (b) $2(a^2+b^2)$ (c) a^2+b^2 (d) $\frac{1}{2}(a^2+b^2)$

6) The volume of the parallelepiped with its edges represented by the vectors $\hat{\imath}+\hat{\jmath}, \hat{\imath}+2\hat{\jmath}, \hat{\imath}+\hat{\jmath}+\pi\hat{k}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

7) The maximum value of the function $x^2e^{-2x}, x > 0$ is

(a) $\frac{1}{e}$ (b) $\frac{1}{2e}$ (c) $\frac{1}{e^2}$ (d) $\frac{4}{e^4}$

8) The change in the surface area $S = 6x^2$ of a cube when the edge length varies from x_0 to $x_0 + dx$ is

(a) $12x_0 + dx$ (b) $12x_0dx$ (c) $6x_0dx$ (d) $6x_0 + dx$

9) The volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about x-axis is

(a) πa^3 (b) $\frac{\pi a^3}{4}$ (c) $\frac{\pi a^3}{5}$ (d) $\frac{\pi a^3}{6}$

10) The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through $(-1,1)$. Then the equation of the curve is

(a) $y = x^3 + 2$ (b) $y = 3x^2 + 4$ (c) $y = 3x^3 + 4$ (d) $y = x^3 + 5$.

11) A random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is

(a) 6 (b) 4 (c) 3 (d) 2

12) The operation $*$ defined by $a * b = \frac{ab}{7}$ is not a binary operation on

(a) $\mathbb{Q}^+$ (b) $\mathbb{Z}$ (c) $\mathbb{R}$ (d) $\mathbb{C}$

13) If $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is

(a) 1 (b) 2 (c) 3 (d) 4

14) The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

(a) 1 (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$

15) The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is

(a) $-4\sqrt{3}$ (b) -4 (c) $\frac{\sqrt{3}}{12}$ (d) $4\sqrt{3}$

16) The angle between the lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 4\hat{k}) + t(2\hat{\imath} + 2\hat{\jmath} + \hat{k})$ and the straight line passing through the points (5,1,4) and (9,2,12) is

(a) $\cos^{-1}\left(\frac{3}{2}\right)$ b) $\sin^{-1}\left(\frac{3}{2}\right)$ (c) $\cos^{-1}\left(\frac{2}{3}\right)$ d) $\sin^{-1}\left(\frac{2}{3}\right)$

17) The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x \sin^2 x \, dx$ is

(a) 0 (b) 1 (c) $\frac{\pi}{4}$ (d) $\frac{1}{\sqrt{2}}$

18) The probability density function of X is $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 \le x < 2 \\ 0, & \text{otherwise} \end{cases}$ then the value of $P(X = 1)$ is

(a) 0 (b) 1 (c) 2 (d) -1

19) The solution of the differential equation $xdy - y\,dx = 0$ represents

(a) parabola whose vertex is at origin (b) circle whose center is at origin

(c) a rectangular hyperbola (d) straight line passing through origin

20) The system of linear equations $x + y + z = 2$, $2x + y - z = 3$ and $3x + 2y + kz = 4$ has unique solution, if k is not equal to

(a) 4 (b) -4 (c) 0 (d) 3

PART – B

ANSWER ANY SEVEN QUESTIONS (Q.NO : 30 IS COMPULSORY) :- **7 X 2 = 14**

21. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

22. If $|z| = 3$, show that $7 \le |z + 6 - 8i| \le 13$.

23. Find a polynomial equation of minimum degree with rational coefficients, having $2+\sqrt{3}i$ as a root.

24. Find the distance between the parallel planes $x+2y-2z+1=0$ and $2x+4y-4z+5=0$.

25. Evaluate : $\lim_{x\to\infty}\left(\frac{e^x}{x^m}\right), m\in N$.

26. An egg of a particular bird is very nearly spherical. If the radius of the inside of the shell is 5mm and radius to the outside of the shell is 5.3mm, find the volume of the shell approximately.

27. Find the differential equation corresponding to the family of curves represented by the equation $y=Ae^{8x}+Be^{-8x}$, where A and B are arbitrary constants.

28. Find the constant C such that the function $f(x)=\begin{cases}Cx^2, 1<x<4\\ 0, \text{Otherwise}\end{cases}$ is a density function.

29. Let $A=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ be any two boolean matrices of the same type. Find $A\vee B$ and $A\wedge B$.

30. Find the equation of circle passing through the focus of the parabola $y^2=8x$ and having center at (0,3).

PART – C

ANSWER ANY SEVEN QUESTIONS (Q.NO : 40 IS COMPULSORY) :- 7 X3=21

31. Find the rank of the matrix $\begin{bmatrix}2 & -2 & 4 & 3\\ -3 & 4 & -2 & -1\\ 6 & 2 & -1 & 7\end{bmatrix}$.

32. Find the least value of the positive integer n for which $(\sqrt{3}+i)^n$ i) real (ii) purely imaginar

33. Solve $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$, for $x>0$.

34. If the normal at the point 't_1' on the parabola $y^2=4ax$ meets the parabola again at the point 't_2' then prove that $t_2=-\left(t_1+\frac{2}{t_1}\right)$.

35. Find the local extremum of the function $f(x)=x^4+32x$.

36. Evaluate : $\int_0^{\frac{1}{\sqrt{2}}}\frac{e^{\sin^{-1}x}\sin^{-1}x}{\sqrt{1-x^2}}dx$.

37. Solve : $\frac{dy}{dx}+2y\cot x=3x^2\operatorname{cosec}^2x$.

38. If $X\sim B(n,p)$ such that $4P(X=4)=P(x=2)$ and $n=6$. Find the distribution.

39. Define an operation $*$ on $\mathbb{Q}$ as follows: $a*b=\left(\frac{a+b}{2}\right); a,b\in\mathbb{Q}$. Examine the commutative, the existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$.

40. If $\hat{a},\hat{b},\hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a}\times(\hat{b}\times\hat{c})=\frac{\sqrt{3}}{2}(\hat{b}+\hat{c})$, find the angle between $\hat{a}$ and $\hat{c}$.

PART – D

ANSWER ALL THE QUESTIONS :- $7 \times 5 = 35$

41. a. Test for consistency of the following system of linear equations and if possible, solve: $4x + 3y + 6z = 25, x + 5y + 7z = 13, 2x + 9y + z = 1$. *(OR)*

b. If $v(x, y) = log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$.

42. a. If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$. *(OR)*

b. At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75m from the point of origin.

43. a. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution. *(OR)*

b. Prove that among all the rectangles of the given area square has the least perimeter.

44. a. Prove that $p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table. *(OR)*

b. Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.

45. a. Prove by vector method that $sin(\alpha - \beta) = sin\,\alpha\,cos\,\beta - cos\,\alpha\,sin\,\beta$. *(OR)*

b. A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find (i) $P(2 < X < 6)$ (ii) $P(2 \leq X < 5)$ (iii) $P(X \leq 4)$ (iv) $D(3 < X)$.

46. a. Find the parametric form of vector and Cartesian form of the equations of the plane passing through the three non-collinear points $(3, 6, -2), (-1, -2, 6)$, and $(6, 4, -2)$. **(OR)**

b. A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where [illegible] ≥ 0. (i) At what times the particle changes direction?

(ii) Find the total distance travelled by the particle in the first 4 seconds.

(iii) Find the particle's acceleration each time the velocity is zero.

47. a. If $sin^{-1} x + sin^{-1} y + sin^{-1} z = \frac{\pi}{2}$ and $0 < x, y, z < 1$, then show that $x^2 + y^2 + z^2 + 2xyz = 1$. **(OR)**

b. Find the area of the region bounded between the parabola $y^2 = x$ and the curve $y = |x|$.

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12^{th} – MATHS

1^{ST} REVISION ANSWER KEY FULL SOLVED 2024-25

SALEM DT.

A. DINESH BABU
PGT MATHS
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1) d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

2) b) 1

3) a) 2

4) d) $\tan^{-1}\left(\frac{1}{2}\right)$

5) b) $2(a^2+b^2)$

6) c) π

7) c) $\frac{1}{e^2}$

8) b) $12\, x_0\, dx$

9) d) $\frac{\pi a^2}{b}$

10) a) $y = x^3 + 2$

11) d) 2

12) b) z

13) b) 2

14) c) $\sqrt{10}$

15) c) $\frac{\sqrt{3}}{12}$

16) c) $\cos^{-1}\left(\frac{2}{3}\right)$

17) a) ∞

18) a) 0

19) d) Straight line passing through origin

20) c) 0.

(21) $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ $A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Orthogonal is $AA^T = A^TA = I$, To prove

$\therefore AA^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$AA^T = I$ H.P.

$\therefore A$ is orthogonal.

(22). Given $|z| = 3$

Using upper and lower bound form is

$||z| - |6-8i|| \le |z+6-8i| \le |z| + |6-8i|$

$|3 - \sqrt{36+64}| \le |z+6-8i| \le 3 + \sqrt{36+64}$

$|3 - \sqrt{100}| \le |z+6-8i| \le 3 + \sqrt{100}$

$|3-10| \le |z+6-8i| \le 3+10$

$|-7| \le |z+6-8i| \le 13$

$7 \le |z+6-8i| \le 13$ H.P.

(23). One root is $2+\sqrt{3}i$ other root $2-\sqrt{3}i$

$S_1 = 2+\sqrt{3}i + 2-\sqrt{3}i = 4$

$S_2 = 4+3 = 7$.

Required equation of polynomial.

$x^2 - S_1x + S_2 = 0$

$x^2 - 4x + 7 = 0$

(24) plane 1 : $x+2y-2z+1=0$

plane - 2

$2x+4y-4z+5=0$

$(\div 2)$ $x+2y-2z+\frac{5}{2}=0$

$\therefore$ Parallel planes are.

$a=1$ $b=2$ $c=-2$ $d_1=1$ $d_2=\frac{5}{2}$

Distance $\delta = \dfrac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}} = \dfrac{|1-\frac{5}{2}|}{\sqrt{1+4+4}} = \dfrac{|\frac{-3}{2}|}{\sqrt{9}}$

$= \dfrac{\frac{3}{2}}{3}$

$= \frac{3}{2} \times \frac{1}{3}$

$\delta = \frac{1}{2}$

(25). $\lim\limits_{x\to\infty} \dfrac{e^x}{x^m} = \dfrac{\infty}{\infty}$ Apply for L-Hopital rule

$= \lim\limits_{x\to\infty} \dfrac{e^x}{m x^{m-1}} = \dfrac{\infty}{\infty}$ again

$= \lim\limits_{x\to\infty} \dfrac{e^x}{m!}$

$= \infty$

(26). $r = 5 mm$ $dr = 0.3 mm$

$$V = \frac{4}{3}\pi r^3$$

$$dV = \frac{4}{3}\pi \, 3r^2 . dr$$

$$dV = 4\pi (5)^2 (0.3)$$

$$\boxed{dV = 30\pi} \; mm^3.$$

(27)

$$y = Ae^{8x} + Be^{-8x}$$

$$y' = 8Ae^{8x} - 8Be^{-8x}$$

$$y'' = 64Ae^{8x} + 64Be^{-8x}$$

$$= 64\left[Ae^{8x} + Be^{-8x}\right]$$

$$y'' = 64y$$

$$y'' - 64y = 0.$$

(28). P.D.F $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$$\int_1^4 c x^2 . dx = 1$$

$$c\left[\frac{x^3}{3}\right]_1^4 = 1$$

$$c\left[\frac{64}{3} - \frac{1}{3}\right] = 1$$

$$c\left[\frac{63}{3}\right] = 1$$

$$\boxed{c = \frac{1}{21}}$$

(29) $A \vee B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0\vee 1 & 1\vee 1 \\ 1\vee 0 & 1\vee 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A \wedge B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0\wedge 1 & 1\wedge 1 \\ 1\wedge 0 & 1\wedge 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

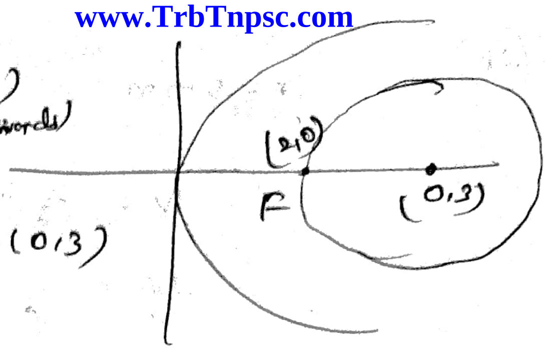
(30)

$y^2 = 8x$ $\rightarrow (y^2 = 4ax)$ right words

$4a = 8$

$a = 2$

Centre $(h, k) = (0, 3)$



The equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-0)^2 + (y-3)^2 = r^2 \quad \text{--- (1)}$$

at (2,0) $\quad (2-0)^2 + (0-3)^2 = r^2$

$$(2)^2 + (3)^2 = r^2$$

$$4 + 9 = r^2$$

$$r^2 = 13 \quad \text{in (1)}$$

$$\therefore \quad x^2 + (y-3)^2 = 13$$

$$x^2 + y^2 + 9 - 6y - 13 = 0$$

$$x^2 + y^2 - 6y - 4 = 0$$

(31).

$$A = \begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$$

$R_2 \rightarrow 2R_2 + 3R_1$, $R_3 \rightarrow R_3 - 3R_1$ $\sim \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & -2 & 8 & 7 \\ 0 & 8 & -13 & -2 \end{bmatrix}$

$R_3 \rightarrow R_3 + 4R_2$ $\sim \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & -2 & 8 & 7 \\ 0 & 0 & 19 & 27 \end{bmatrix}$

Last equivalent matrix It's echelon form

$$\rho(A) = 3$$

(32) $\sqrt{3}+i = r(\cos\theta + i\sin\theta)$

$r = |\sqrt{3}+i| = \sqrt{3+1} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{1}{\sqrt{3}}\right| = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

θ Lies on 1^{st} Quadrant. $\theta = \alpha$

$\theta = \frac{\pi}{6}$

$\sqrt{3}+i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$(\sqrt{3}+i)^n = 2^n\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ — (1)

Let $n=6$ only Real value in (1), Imaginary part zero

Let $n=3$ only Imaginary value in (1), real part zero

(33). $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$

$\tan^{-1}(1) - \tan^{-1}x = \frac{1}{2}\tan^{-1}x$

$\frac{\pi}{4} = \frac{1}{2}\tan^{-1}x + \tan^{-1}x$

$\frac{\pi}{4} = \tan^{-1}x\left(\frac{1}{2}+1\right)$

$\frac{\pi}{4_2} = \tan^{-1}x\left(\frac{3}{2}\right)$

$\frac{\pi}{6} = \tan^{-1}(x)$

$\tan\frac{\pi}{6} = x$

$\boxed{x = \frac{1}{\sqrt{3}}}$

(34) The equation of the normal at 't_1' on the parabola $y^2=4ax$

$$y + xt_1 = 2at_1 + at_1^3$$

at passing through $(at_2^2, 2at_2)$

$$2at_2 + at_2^2 t_1 = 2at_1 + at_1^3$$

$$2at_2 - 2at_1 = at_1^3 - at_2^2 t_1$$

$$2a(t_2 - t_1) = t_1 a(t_1^2 - t_2^2)$$

$$2(t_2 - t_1) = t_1(t_1 + t_2)(t_1 - t_2)$$

$$2(t_2 - t_1) = -t_1(t_1 + t_2)(t_2 - t_1)$$

$$\div t_1 \qquad \frac{2}{t_1} = -(t_1 + t_2)$$

$$\frac{2}{t_2} = -t_1 - t_2$$

$$t_2 = -t_1 - \frac{2}{t_2}$$

$$t_2 = -\left(t_1 + \frac{2}{t_2}\right) \text{ H.P.}$$

(35). $f(x) = x^4 + 32x$.

$$f'(x) = 4x^3 + 32$$

$$f''(x) = 12x$$

$f'(x) = 0$

$4x^3 + 32 = 0$

$4x^3 = -32$

$x^3 = -2^3$

$x = -2$ The critical point.

at $x=-2$ in $f''(x) = 12x^2 > 0$ The function has local minimum at $x=-2$

$\therefore \quad f(x) = x^4 + 32x$

at $x=-2 \quad f(-2) = 16 - 64 = -48$

$\therefore$ The local minimum value is $(-2, -48)$.

(36). $$\int_0^{\frac{1}{\sqrt{2}}} \frac{e^{\sin^{-1}x} \sin^{-1}x}{\sqrt{1-x^2}} \, dx$$

Let $t = \sin^{-1}x$

$dt = \frac{1}{\sqrt{1-x^2}} \, dx$

x	0	$\frac{1}{\sqrt{2}}$
t	0	$\frac{\pi}{4}$

$$\int_0^{\pi/4} t e^t \, dt$$

$$= \left[t e^t - (1) e^t \right]_0^{\pi/4} + c.$$

$$= \left(\frac{\pi}{4} e^{\frac{\pi}{4}} - e^{\frac{\pi}{4}} \right) - (0 - 1)$$

$$= e^{\pi/4} \left(\frac{\pi}{4} - 1 \right) + 1.$$

(37). $\frac{dy}{dx} + 2\cot x \, y = 3x^2 \operatorname{cosec}^2 x$ $\left(\because \frac{dy}{dx} + Py = Q\right)$

$P = 2\cot x \qquad Q = 3x^2 \operatorname{cosec}^2 x$

$I \cdot F = e^{\int p \cdot dx} = e^{\int 2\cot x \cdot dx} = e^{2\int \cot x \cdot dx} = e^{2\log|\sin x|} = e^{\log|\sin x|^2}$

$\boxed{I \cdot F = \sin^2 x}$

G.S

$y \cdot (I \cdot F) = \int Q \cdot (I \cdot F) \cdot dx + c$

$y \cdot \sin^2 x = \int 3x^2 \operatorname{cosec}^2 x \cdot \sin^2 x \cdot dx + c$

$= \int 3x^2 \frac{1}{\sin^2 x} \cdot \sin^2 x \cdot dx + c$

$= 3\left[\frac{x^3}{3}\right] + c$

$y \sin^2 x = x^3 + c$

(38). Given $4P(x=4) = P(x=2)$ $\quad n = 6$

$P(x=x) = nC_x p^x q^{n-x}$

$4\left(6C_4 \, p^4 q^2\right) = 6C_2 \, p^2 q^4$

$4p^2 = q^2$

$4p^2 = (1-p)^2$

$4p^2 = 1 + p^2 - 2p$

$3p^2 + 2p - 1 = 0$

$(p+1)(p - 1/3) = 0$

$p = -1$ not pos $\quad \boxed{p = 1/3} \quad \boxed{q = 2/3}$

distribution function

$p(x=x) = nC_x \, p^x q^{n-x}$

$p(x=x) = 6C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{6-x} \quad x = 0, 1, 2 \ldots 6.$

b)

(39) Closure $a*b = \frac{a+b}{2}$ binary on Q.

$a \in R$, $b \in R$, $\frac{a+b}{2} \in R$ ∴ $*$ is binary on Q.

Commutative:- $a*b = b*a$.

$\frac{a+b}{2} = \frac{b+a}{2}$ $*$ is commutative on Q.

Idenity:- $a*e = e*a = a$

$a*e = a$

$\frac{a+e}{2} = a$

$a+e = 2a$

$\boxed{e = a}$ not uique $*$ $e = 1$

∴ $*$ is not identity on Q.

inverse:- $a*a' = a'*a = e$

If idenity axiom is not satisfied we can't discuss inverse axiom.

(40). Given $\hat{a} \times (\hat{b} \times \hat{c}) = \frac{\sqrt{3}}{2}(\hat{b}+\hat{c})$

$(\vec{a}.\vec{c})\hat{b} - (\hat{a}.\hat{b})\hat{c} = \frac{\sqrt{3}}{2}\hat{b} + \frac{\sqrt{3}}{2}\hat{c}$

Comparing co-efficent of $\hat{b}$

$\vec{a}.\hat{b} = -\frac{\sqrt{3}}{2}$

$|\vec{a}|\,|\vec{b}| \cos\theta = -\frac{\sqrt{3}}{2}$

$(1)(1) \cos\theta = -\cos\frac{\pi}{6}$

$\cos\theta = \cos(\pi - \pi/6)$

$\cos\theta = \cos 5\pi/6$

$\theta = \frac{5\pi}{6}$

(41) a) $[A|B] = \left[\begin{array}{ccc|c} 4 & 3 & 6 & 25 \\ 1 & 5 & 7 & 13 \\ 2 & 9 & 1 & 1 \end{array}\right]$

$R_1 \leftrightarrow R_2 \sim \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 4 & 3 & 6 & 25 \\ 2 & 9 & 1 & 1 \end{array}\right]$

$R_2 \to R_2 - 4R_1$
$R_3 \to R_3 - 2R_1$ $\sim \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & -17 & -22 & -27 \\ 0 & -1 & -13 & -25 \end{array}\right]$

$R_3 \to 17R_3 - R_2 \sim \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & -17 & -22 & -27 \\ 0 & 0 & -199 & -398 \end{array}\right]$

Last row echelon form $\rho(A|B) = \rho(A) = 3$

It's consistent unique solution.

$x + 5y + 7z = 13$ — (1)

$-17y - 22z = -27$ — (2)

$-198z = -398$ — (3)

$z = 2$

sub $z = 2$ in

$-17y - 44 = -27$

$-17y = -27 + 44$

$-17y = +17$

$y = -1$

sub $z = 2$, $y = -1$ in (1)

$x - 5 + 14 = 13$

$x = 13 - 9$

$x = 4$

$(x, y, z) = (4, -1, 2)$

b).

b) $v = \log\left(\frac{x^2+y^2}{x+y}\right)$

$$e^v = \frac{x^2+y^2}{x+y} = f \text{ (say)}$$

$$f(\lambda x, \lambda y) = \frac{\lambda^2 x^2 + \lambda^2 y^2}{\lambda x + \lambda y} = \frac{\lambda^2 (x^2+y^2)}{\lambda (x+y)}$$

$$= \lambda^1 f \quad \text{degree } p = 1$$

by Euler's theorem:-

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = pf$$

$$x \frac{\partial (e^v)}{\partial x} + y \frac{\partial (e^v)}{\partial y} = 1\, e^v$$

$$x\, e^v \frac{\partial v}{\partial x} + y\, e^v \frac{\partial v}{\partial y} = e^v$$

$\div e^v$ $$x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} = 1 \quad \text{H.P.}$$

(42) Given $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$

$$\therefore \frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$\frac{z-1}{z+1} = \frac{(x-1)(x+1) - iy(x-1) + iy(x+1) + y^2}{(x+1)^2 + y^2}$$

$$\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$$

$$\therefore \tan^{-1}\left(\frac{-y(x-1)+y(x+1)}{x^2-1+y^2}\right) = \frac{\pi}{2}$$

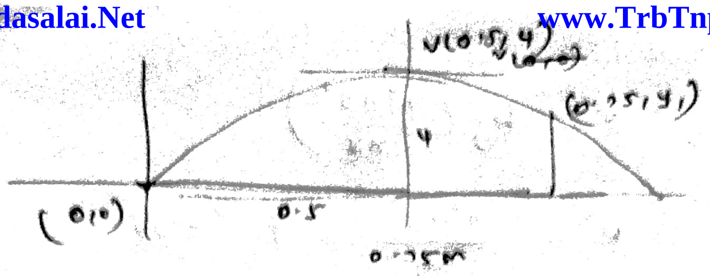
$$\frac{-xy+y+xy+y}{x^2+y^2-1} = \tan\frac{\pi}{2}$$

$$\frac{2y}{x^2+y^2-1} = \frac{1}{0}$$

$$\boxed{x^2+y^2-1=0}$$

H.P.

b).



The equation of parabola downwards

$(x-h)^2 = -4a(y-k)$

$(x-0.5)^2 = -4a(y-4)$ — ①

at (0,0) $(-0.5)^2 = -4a(-4)$

$\frac{0.25}{4} = 4a$ in ①

$(x-0.5)^2 = \frac{-0.25}{4}(y-4)$ — ②

at $(0.75, y_1)$ $(0.75-0.5)^2 = \frac{-0.25}{4}(y_1-4)$

$(0.25)^2 = -\frac{0.25}{4}(y-4)$

$1 = -y+4$

$+y = 4-1$

$+y = 3\ m$

43 a) Given $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$

$\div x^2$ $6x^2 - 5x - 38 - \frac{5}{x} + \frac{6}{x^2} = 0$

$6(x^2 + \frac{1}{x^2}) - 5(x + \frac{1}{x}) - 38 = 0$

Let $y = x + \frac{1}{x}$ $y^2 - 2 = x^2 + \frac{1}{x^2}$

$6(y^2-2) - 5y - 38 = 0$

$6y^2 - 12 - 5y - 38 = 0$

$$\left(x+\frac{5}{2}\right)\left(x-\frac{10}{3}\right)=0$$

Replace

$x+\frac{1}{x}+\frac{5}{2}=0$

$\frac{x^2+1}{x}=-\frac{5}{2}$

$2x^2+2=-5x$

$2x^2+5x+2=0$

$\left(x+\frac{1}{2}\right)(x+2)=0$

$x=-\frac{1}{2},-2$

$x+\frac{1}{x}-\frac{10}{3}=0$

$\frac{x^2+1}{x}=\frac{10}{3}$

$3x^2+3=10x$

$3x^2-10x+3=0$

$(x-3)\left(x-\frac{1}{3}\right)=0$

$x=3,\frac{1}{3}$

$\therefore$ Total roots are $3, \frac{1}{3}, -\frac{1}{2}, -2$. (known as one root)

b) x and y are dimension

$P=2x+2y$ $\quad y=\frac{P-2x}{2}$

maximum Area $A=xy$

$$A=x\left(\frac{P-2x}{2}\right)$$

$$A=\frac{Px}{2}-x^2$$

$$A'(x)=\frac{P}{2}-2x \Rightarrow$$

$A'(x)=0$

$-2x=-\frac{P}{2}$

$x=\frac{P}{4}$

$$A''(x)=-2<0$$

at $x=\frac{P}{4}$ in $A''(x)=-2<0$ Area is maximum.

$$y=\frac{P-2\left(\frac{P}{4}\right)}{2}=\frac{\frac{2P-P}{2}}{2}=\frac{P}{4}$$

$\therefore x=\frac{P}{4}, y=\frac{P}{4}$ It's Square.

(44) a) Total Row $= 2^3 = 8$

p	q	r	~p	~q	~q∨r	p→(~q∨r)	~p∨(~q∨r)
T	T	T	F	F	T	T	T
T	T	F	F	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	F	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

(1) (2)

from (1) + (2) we get

$p \to (\sim q \vee r) \equiv \sim p \vee (\sim q \vee r)$ Hb.

b). P be The population of a city at any time t

$\frac{dp}{dt} \alpha p$

$\frac{dp}{dt} = kp$

$p = ce^{kt}$ —(1)

(i) $t = 0$ $P = 300000$ in (1)

$30000 = ce^{k(0)}$

$c = 300000$

$p = 300000\, e^{kt}$ —(2)

(ii) $t = 40$ $P = 400000$ in (2)

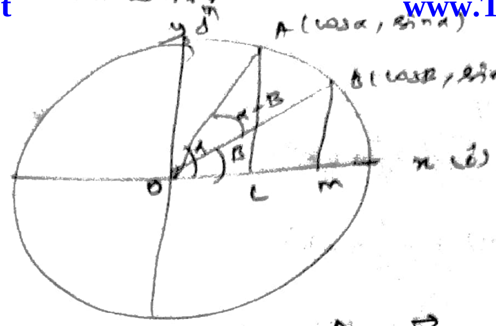
$400000 = 300000\, e^{k(40)}$

$\frac{4}{3} = e^{40k}$

$e^k = \left(\frac{4}{3}\right)^{\frac{1}{40}}$

at any Time $p = 300000\left(\frac{4}{3}\right)^{\frac{t}{40}}$

(45) a)



$\vec{a} = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$, $\vec{b} = \cos\beta\,\hat{i} + \sin\beta\,\hat{j}$

$$\vec{b}\times\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\beta & \sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix} = \hat{k}\,(\sin\alpha\cos\beta - \cos\alpha\sin\beta) \quad -(1)$$

$$\vec{b}\times\vec{a} = |\vec{b}|\,|\vec{a}|\,\sin\theta_{(\alpha-\beta)}\,\hat{k} = (1)(1)\sin(\alpha-\beta)\,\hat{k}$$

$$= \hat{k}\,(\sin(\alpha-\beta)) \quad -(2)$$

from (1) & (2) we get

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta. \quad \text{H.P}$$

b). $\sum_0^n f(x) = 1$

$k + 2k + 3k + 5k + 6k + 10k = 1$

$30k = 1$

$k = \frac{1}{30}$

(i) $P(2 < x < 6) = f(3) + f(4) + f(5) = 6k + 5k + 6k$

$= 17k$

$= \frac{17}{30}$

(ii) $P(2 \le x < 5) = f(2) + f(3) + f(4) = 2k + 6k + 5k$

$= 13k$

$= \frac{13}{30}$

(iii) $P(x \le 4) = f(1) + f(2) + f(3) + f(4) = k + 2k + 6k + 5k$

$= 14k$

$= \frac{14}{30}$

(iv) $P(3 < x) = f(4) + f(5) + f(6) = 5k + 6k + 10k = 21k$

$= \frac{21}{30}$

(46). a) $\vec{a} = 3\hat{i}+6\hat{j}-2\hat{k}$

$\vec{b} = -\hat{i}-2\hat{j}+6\hat{k}$

$\vec{c} = 6\hat{i}+4\hat{j}-2\hat{k}$

$\vec{b}-\vec{a} = -4\hat{i}-8\hat{j}+8\hat{k}$

$\vec{c}-\vec{a} = 3\hat{i}-2\hat{j}+0\hat{k}$.

(i) Parametric form:-

$\vec{r} = \vec{a} + s(\vec{b}-\vec{a}) + t(\vec{c}-\vec{a})$ $s,t \in R$

$\vec{r} = (3\hat{i}+6\hat{j}-2\hat{k}) + s(-4\hat{i}-8\hat{j}+8\hat{k}) + t(3\hat{i}-2\hat{j})$ $s,t \in R$

(ii) Cartesian form:-

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-3 & y-6 & z+2 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$

$((x-3)(0+16)) - ((y-6)(-24+0)) + ((z+2)(8+24)) = 0$

$((x-3)16) - ((y-6)(-24)) + (z+2)(32) = 0$

$16x-48+24y-144+32z+64 = 0$

$16x+24y+32z-128 = 0$

$(\div 8)$ $2x+3y+4z-16 = 0$

b). Given

$s(t) = 128t - 16t^2$

(i) The partical rest at maximum height is velocity is zero

$v(t) = \frac{ds}{dt} = 128 - 32t$

$v(t) = 0$

$128 - 32t = 0$

$-32t = -128$

$t = 4$

$s(t) = 128t - 16t^2$

at $t = 4$ $\quad s(4) = 128(4) - 16(4)^2$

$= 256$ ft

(r) $\quad s = 0 \quad 128t - 16t^2 = 0$

$-16t^2 = 128t$

$t = 8$ sec

$\therefore V(8) = 128 - 32(8)$

$= 128 - 256$

$V(8) = -128$ ft/sec

47 a) $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{\pi}{2}$

Let $a = \sin^{-1}x$ $b = \sin^{-1}y$ $c = \sin^{-1}z$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \sqrt{1-\sin^2\theta}$

$$a + b + c = \frac{\pi}{2}$$

$$a + b = \frac{\pi}{2} - c$$

both side add cos

$$\cos(a+b) = \cos\left(\frac{\pi}{2} - c\right)$$

$$\cos a \cos b - \sin a \sin b = \sin c$$

$$\sqrt{1-\sin^2 a}\sqrt{1-\sin^2 b} - \sin a \sin b = \sin c$$

$$\sqrt{1-x^2}\sqrt{1-y^2} - xy = z$$

$$\sqrt{1-x^2}\sqrt{1-y^2} = (z + xy)$$

Square on both side

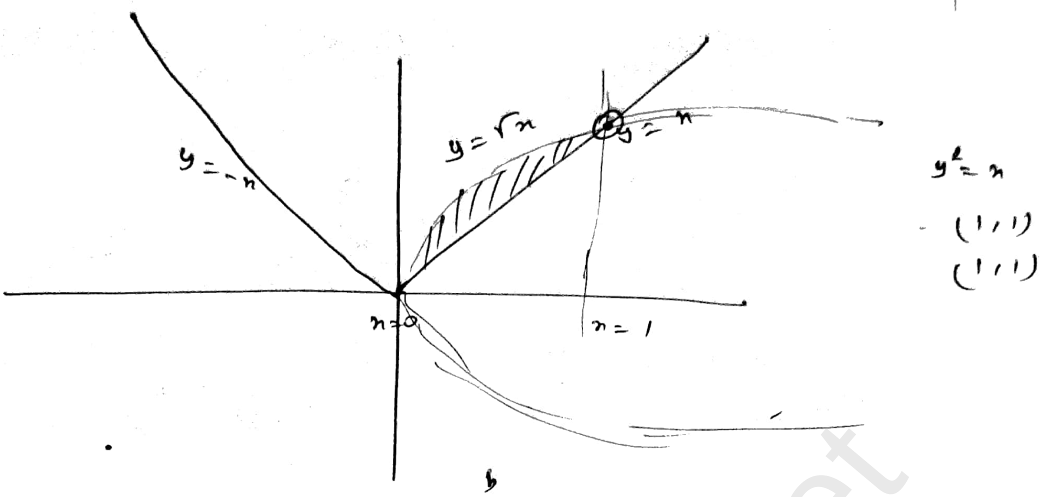
$$(1-x^2)(1-y^2) = (z + xy)^2$$

$$1 - y^2 - x^2 + x^2y^2 = z^2 + x^2y^2 + 2xyz$$

$$1 = x^2 + y^2 + z^2 + 2xyz$$

$$\therefore x^2 + y^2 + z^2 + 2xyz = 1 \quad \text{H.P.}$$

b). $y^2 = x$ Right words Power $y = |x|$



Required Area $= \int_a^b (\text{upper} - \text{Lower}) \cdot dx$

$= \int_0^1 (\sqrt{x} - x) \cdot dx$

$= \int_0^1 (x^{1/2} - x) \cdot dx$

$= \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2} \right]_0^1$

$= \left(\frac{1}{\frac{3}{2}} - \frac{1}{2} \right) - 0$

$= \frac{2}{3} - \frac{1}{2}$

$= \frac{4-3}{6}$

$= \frac{1}{6}$ Sq. units.